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Vehicle Blast Protection Efficiency Analysis and Evaluation

Jeremy A. Teichman
Yevgeny Macheret

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For More Information

Jeremy A. Teichman, Project Leader
jteichma@ida.org, 703-578-2975

Leonard J. Buckley, Director, Science and Technology Division
lbuckley@ida.org, 703-578-2800

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**Vehicle Blast Protection Efficiency
Analysis and Evaluation**

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Executive Summary

This paper analyzes the response of vehicles to an explosive detonated under the vehicle body. We suggest that two factors provide the main contribution to a vehicle's response: (1) damping that controls acceleration between the vehicle seat and its underbody and (2) the vehicle underbody seat stroke available. We define acceleration and stroke efficiencies and describe how to quantify and measure the effectiveness of their contributions to overall blast-protection performance relative to the theoretical upper bound. Decomposing the performance into independent acceleration and stroke responses isolates the causes of suboptimal performance for vehicle analysis and blast-resistance improvement. In addition, the decomposition further indicates how to utilize vehicles blast simulations and tests to determine design changes for improving vehicle performance.

Contents

1.	Introduction	1
	A. Background	1
	B. Perfect Damping	1
	C. Acceleration Efficiency	5
	D. Stroke Efficiency	5
	E. Momentum Reduction	9
	F. Evaluating Efficiencies	11
	G. Limits of Instantaneous Blast and Low Passenger Mass	12
	H. Example Calculation	13

1. Introduction

A. Background

In our past work, we derived an upper bound for vehicle underbody blast resistance, particularly for an instantaneous transfer of momentum to a vehicle.¹ Assuming that the blast does not breach the vehicle hull and that passenger acceleration produces injuries, we showed that for the simple injury criterion of a fixed threshold of passenger acceleration, the perfect vehicle generates constant passenger acceleration marginally below the injury threshold. While that theoretical upper bound to performance may prove useful in understanding the art of the possible, here we address how vehicle designers, analysts, and program managers can exploit the upper bound analysis in a practical fashion to assess vehicle designs, simulations, and experiments to evaluate performance in a way that allows comparison among diverse vehicle designs, quantifies the room for improvement, and helps identify sources of inefficiency (potential targets for improvement) in underbody blast protection performance.

We define and describe a set of efficiencies to quantify the effectiveness of various contributions to overall blast protection performance relative to the upper bound. Decomposing the performance into independent contributions isolates the causes or sources of suboptimal performance for analysis and improvement. Understanding the blast protection efficiency of existing vehicles will indicate how much room there is for improvement. The decomposition will further indicate which contributors to blast protection performance have room for improvement.

In the present work, we also add to our previous analysis, which had assumed an instantaneous transfer of blast momentum to the vehicle, to account for the contribution of time-varying vehicle acceleration to blast protection performance.

B. Perfect Damping

Since the efficiencies explored in this report are defined relative to the absolute upper bound for performance, we first briefly derive the upper bound to performance for a time-varying explosive force by applying conservation of momentum and kinematic constraints on the stroke. The stroke is defined as the displacement of the passenger relative to the vehicle center of mass.

¹ J. Macheret and J. Teichman, “Blast Protection Enhancement,” IDA Paper P-4919 (Alexandria, VA: Institute for Defense Analyses, forthcoming).

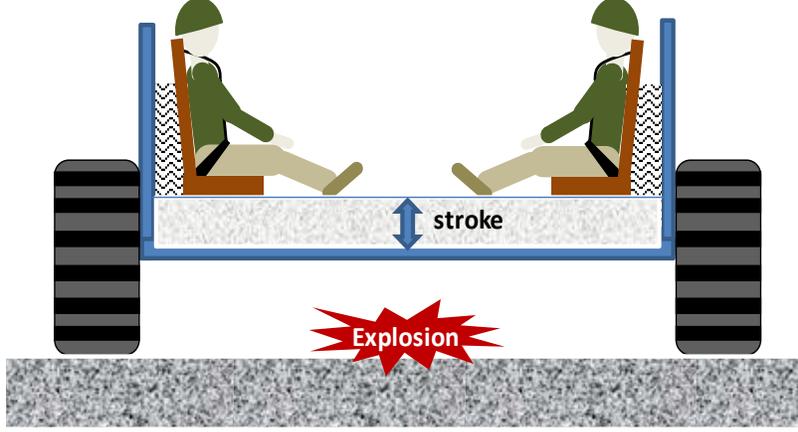


Figure 1. Schematic of Vehicle. The mechanical compliance of the vehicle is labeled “stroke.” We will distinguish between the mechanical stroke and the center-of-mass stroke, but for a perfectly rigid vehicle, they will be identical.

Let τ be the time over which the explosive impulse is delivered to the vehicle via the time-varying force function $F(t)$. The total impulse is given by

$$I = \int_0^{\tau} F(t)dt = v_f M_{\Sigma}, \quad (1)$$

where v_f is the final equilibrium velocity of the system and M_{Σ} is the total vehicle-passenger system mass,

$$M_{\Sigma} = M_v + M_p, \quad (2)$$

where M_v and M_p are the vehicle and passenger mass, respectively. Momentum is conserved, so

$$\int_0^t F(t)dt = v_p(t)M_p + v_v(t)M_v, \quad (3)$$

where v_p and v_v are the passenger and vehicle center-of-mass velocities. Let t^* be the time at which the passenger velocity matches the velocity of the vehicle center of mass,

$$\frac{I}{M_{\Sigma}} = v_v(t^*) = v_p(t^*) = \int_0^{t^*} a_p(t)dt, \quad (4)$$

where $a_p(t)$ denotes the passenger acceleration. The center-of-mass stroke d_{CM} is given by the difference between the vehicle and passenger center-of-mass displacements, here expressed as integrals of their respective velocities,

$$d_{CM} = \int_0^{t^*} \left(v_v(t) - \int_0^t a_p(t')dt' \right) dt, \quad (5)$$

where the passenger velocity is expressed as the integral of the passenger acceleration. Combining Eqs. 3 and 5 to eliminate the vehicle velocity,

$$d_{CM} = \int_0^{t^*} \left(\frac{1}{M_v} \int_0^t F(t') dt' - \frac{M_\Sigma}{M_v} \int_0^t a_p(t') dt' \right) dt. \quad (6)$$

Switching the order of integration and simplifying yields

$$M_v d_{CM} = - \int_0^\tau t F(t) dt + M_\Sigma \int_0^{t^*} t a_p(t) dt. \quad (7)$$

Let us define two measures with units of time. Conceptually, one could think of them as force-averaged time and passenger-acceleration-averaged time,

$$\tilde{t}_F \equiv \frac{\int_0^\tau t F(t) dt}{\int_0^\tau F(t) dt} \quad (8)$$

$$\tilde{t}_a \equiv \frac{\int_0^{t^*} t a_p(t) dt}{\int_0^{t^*} a_p(t) dt}. \quad (9)$$

In terms of these times, Eq. 7 (with the help of the relationships in Eqs. 1 and 4) becomes

$$I = \frac{M_v d_{CM}}{\tilde{t}_a - \tilde{t}_F}. \quad (10)$$

Here \tilde{t}_a and \tilde{t}_F represent a relationship between instantaneous and gradual loading. A real vehicle, accelerated over time by an explosion, and in turn accelerating the passengers over time, will exhibit the same stroke as a vehicle absorbing the explosive impulse instantaneously at time \tilde{t}_F and instantaneously coming to velocity equilibrium with the passenger at time \tilde{t}_a (see Figure 2). Note that the more gradually the explosive impulse is applied to the vehicle (higher \tilde{t}_F), the higher the sustainable impulse because there is more time to accelerate the passenger as the vehicle initially closes the gap more slowly. In the limit of rapidly applied explosive impulse, \tilde{t}_F goes to zero. Note that \tilde{t}_F does not depend on the magnitude of the force function, only its shape, so the right-hand side of Eq. 10 does not depend on the impulse.

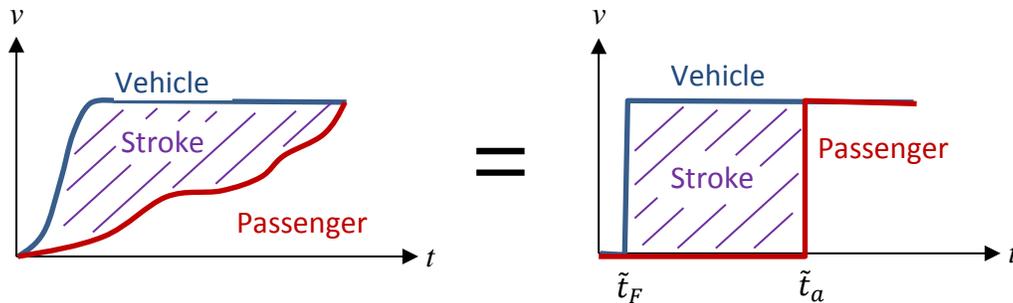


Figure 2. Illustration of \tilde{t}_F and \tilde{t}_a as Times for Equivalent Application of Instantaneous Vehicle and Passenger Accelerations at Equal Total Stroke

Equation 10 shows that the highest tolerable impulse for a given stroke, vehicle mass, and force profile is given by the earliest acceleration time \tilde{t}_a subject to the integrated acceleration equaling the final vehicle velocity (Eq. 4) and the instantaneous acceleration never exceeding the injury threshold a_c . In our prior paper, we demonstrated that a constant passenger acceleration at the threshold of injury yields the maximum blast resistance. Consider the largest such tolerable impulse. Let us demonstrate graphically that no other acceleration profile could do better. The acceleration profile of the presumed best performing vehicle is shown in Figure 3 as a solid line. Two types of area-conserving (invariant impulse) deviations are possible, shown as dashed and dotted lines. Those like the dashed line are injurious because they exceed the critical acceleration. Those like the dotted line bias the acceleration later, increasing \tilde{t}_a and requiring a longer stroke for the same impulse (drawing out the acceleration of the passenger keeps the passenger at lower velocity longer and allows the stroke to be consumed more rapidly). Thus, no acceleration profile can outperform the one depicted by the solid line.

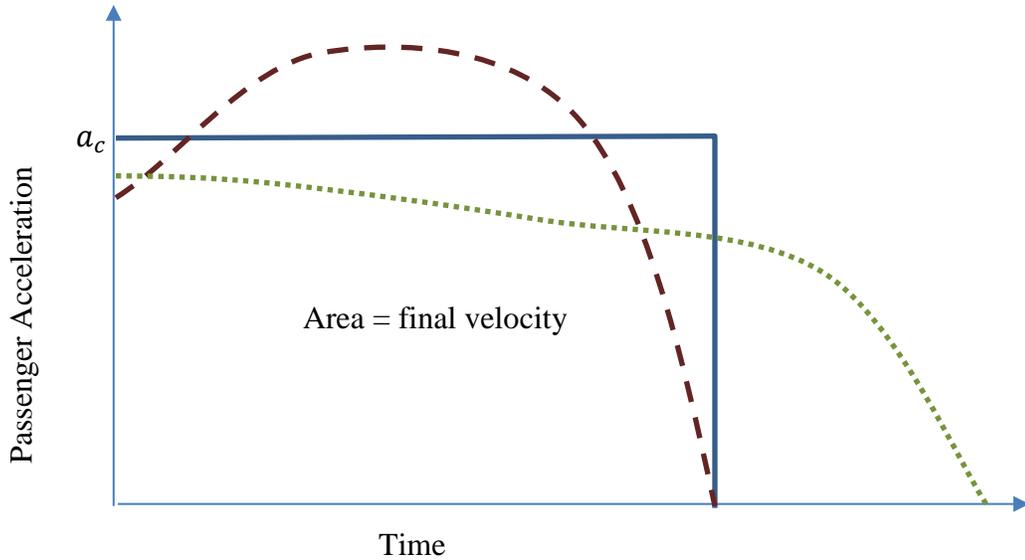


Figure 3. Optimal Acceleration Profile (solid) and Deviations (dashed and dotted)

For optimal blast protection performance

$$a_p(t) = a_c, \quad (11)$$

$$\tilde{t}_a = \frac{t^*}{2}, \quad (12)$$

and

$$t^* = \frac{v_f}{a_c} = \frac{I}{M_\Sigma a_c}. \quad (13)$$

Solving Eq. 10 for the maximum tolerable impulse under perfect damping yields

$$I^* = \sqrt{2M_\Sigma a_c M_v d_{CM} + M_\Sigma^2 a_c^2 \tilde{t}_F^2} + M_\Sigma a_c \tilde{t}_F. \quad (14)$$

Eq. 14 is expressed in terms of the blast characterization (\tilde{t}_F), which may be difficult to measure (further addressed later), and readily measurable quantities in a design.

C. Acceleration Efficiency

Now we introduce the notion of efficiency relative to the ideal. Let η_a be the efficiency of acceleration relative to the perfect system that accelerates the passenger at a constant rate of a_c so that the injurious impulse of the real system is given by

$$I = \sqrt{2M_\Sigma \eta_a a_c M_v d_{CM} + M_\Sigma^2 \eta_a^2 a_c^2 \tilde{t}_F^2} + M_\Sigma \eta_a a_c \tilde{t}_F. \quad (15)$$

By equating Eqs. 10 and 15 we can derive a series of expressions for the efficiency in terms of measurable quantities. The most useful expression will depend on which quantities are most readily measurable,

$$\eta_a = \frac{M_v d_{CM}}{2a_c M_\Sigma \tilde{t}_a (\tilde{t}_a - \tilde{t}_F)} \quad (16)$$

$$\eta_a = \frac{I}{2a_c M_\Sigma \tilde{t}_a} \quad (17)$$

$$\eta_a = \frac{I^2}{2a_c M_\Sigma (M_v d_{CM} + I \tilde{t}_F)}. \quad (18)$$

To more intuitively understand this expression for the acceleration efficiency, it is worthwhile to recast it in more transparent terms,

$$\eta_a = \frac{\frac{1}{2}t^*}{\tilde{t}_a} \frac{\bar{a}}{a_c}, \quad (19)$$

where \bar{a} is the average acceleration. This is a product of two ratios, one of time scales, $\frac{1}{2}t^*$, where $\frac{1}{2}t^*$ is the average time of application for a constant acceleration, and one of accelerations, $\frac{\bar{a}}{a_c}$. Thus, the efficiency depends on how much acceleration is applied and when it is applied. The lower the average acceleration relative to the critical acceleration, the lower the efficiency. The earlier the passenger acceleration is applied, the greater the efficiency.

D. Stroke Efficiency

A given vehicle design will incorporate a certain amount of physical compliance, available stroke, between the vehicle and the passenger (e.g., Figure 1). The stroke is consumed by the motion of the vehicle and conserved by motion (acceleration) of the

passenger. If acceleration efficiency quantifies how well stroke is conserved by accelerating the passenger, stroke efficiency quantifies how well stroke is conserved by minimizing local deformation or velocity of parts of the vehicle that close the stroke.² Figure 4 depicts an analogy to help give a more tangible interpretation to the efficiencies.

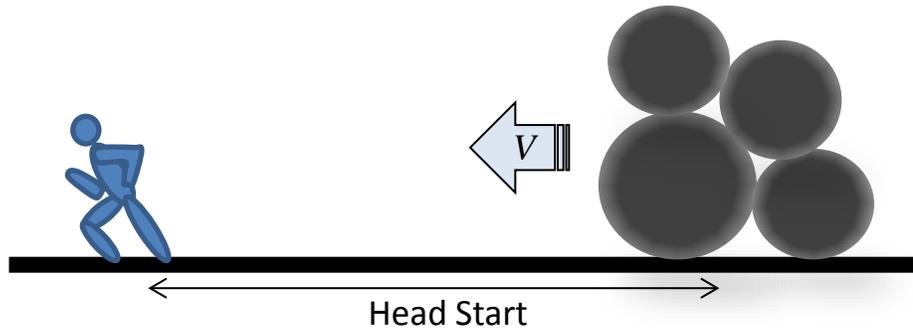


Figure 4. Efficiency analogy: The blast accelerating the vehicle is like setting a cluster of rolling boulders in motion in pursuit of an initially stationary runner (the passenger) with the stroke equivalent to the head start. Acceleration efficiency relates to how well the runner/passenger accelerates to maintain a lead on the boulders. Stroke efficiency relates to how well the boulder momentum is apportioned to slow down the lead boulder.

Figure 5 depicts passengers in a vehicle with multiple stroke components, including a perhaps compliant underbody, a stroking floor, and stroking seats. This type of vehicle is designed to deliberately create relative motion between parts of the vehicle to alter the way momentum is conveyed to the passengers over time. The stroke efficiency can characterize how well such a multistage damping process contributes to improved blast resistance. The stroke efficiency also captures the effects of local deformation such as bending of floors and walls.

² Conservation of momentum dictates the aggregate motion of the vehicle as characterized by the center of mass; however, it does not dictate specifically how that momentum is apportioned over the mass of the vehicle, so some parts may be moving faster than others (e.g., local heave of the floor over the blast).

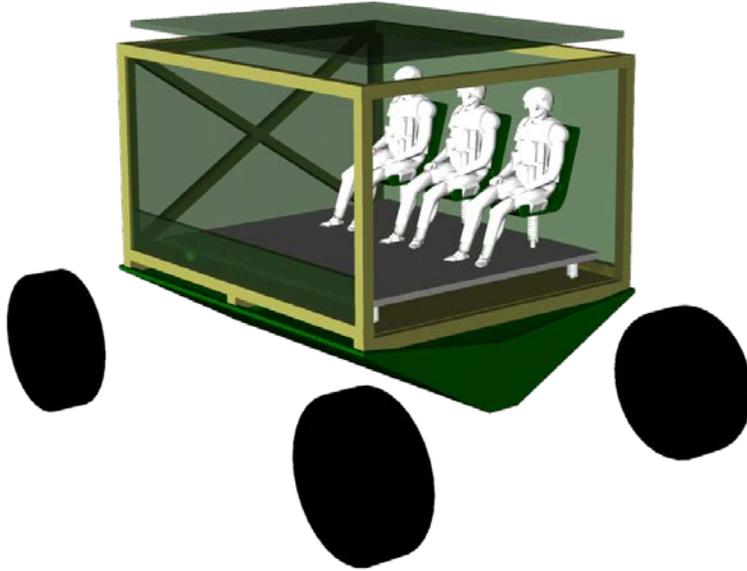


Figure 5. Vehicle with Multipart Stroke

The analysis up to here has considered the stroke of the seat relative to the center of mass of the vehicle. Motion of the vehicle's center of mass is strictly determined by the net momentum transferred to the vehicle irrespective of how that motion is apportioned among the vehicle's components. When the underbody blast occurs, momentum is first transmitted to the underbody of the vehicle. Internal forces in the vehicle spread the underbody momentum. As internal forces recruit additional mass in the vehicle to move with the underbody, the velocity of the moving components decreases because the momentum remains constant. We refer to the set of vehicle components between the point of application of force and the passenger as the stroke path. Decreasing the velocity of parts of the vehicle in the stroke path improves stroke conservation. If vehicle components outside the stroke path (and therefore not impinging on the passengers) can maintain a higher velocity, so much the better. If there are no such components, in the limit, the most effective blast resistance is conferred by the rigid vehicle for which all mass is instantly recruited into motion and relative velocity of the person and the impinging vehicle components is minimized. This reduction of relative motion is none other than kinetic energy dissipation. For a given momentum, kinetic energy is given by

$$\frac{1}{2} \frac{I^2}{M}, \quad (20)$$

which is minimized by maximizing the mass. Effective energy dissipation is required to rapidly recruit vehicle mass into the underbody motion.

We define the mechanical stroke d_0 as the net compliance available in the path between the passenger and the point of blast force application, which we assume to be the undersurface of the vehicle directly below the passenger. For instance, if as in Figure 6,

which can be thought of as a mechanical schematic of the relevant portions of the vehicle in Figure 5, there is an under-seat damper with available stroke d_1 , a floating floor with available stroke d_2 , and a collapsing underbody crumple zone with available stroke d_3 , then the mechanical stroke would be $d_0 = d_1 + d_2 + d_3$. From a design standpoint, all the stroke contributions should be exhausted simultaneously. In practice, at the threshold of injury, some mechanical stroke elements may be only partly utilized (e.g., $d'_2 < d_2$, where the prime indicates the actual rather than budgeted stroke utilization), and $d_0 = d'_1 + d'_2 + d'_3$.

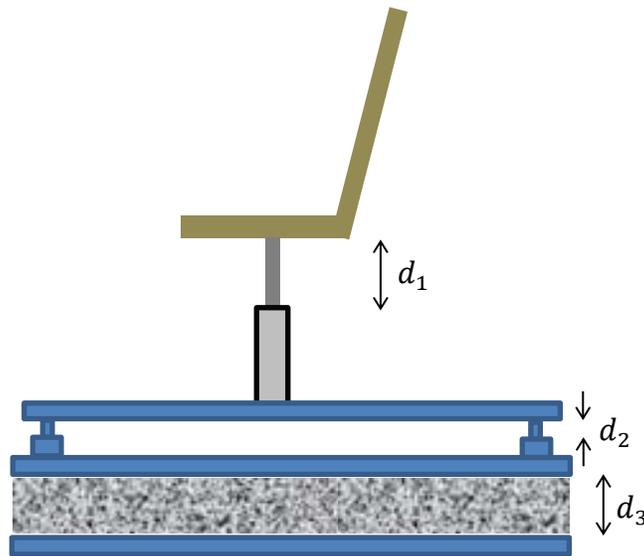


Figure 6. Mechanical Stroke

For instance, suppose of a 10 cm underbody crumple zone, only 5 cm are crumpled when the seat-post stroke is exhausted. The blast resistance will be limited by the complete consumption of the seat-post stroke, and the crumple zone will be underutilized, representing an inefficiency in stroke exploitation relative to the design. This is one source of inefficiency—a shorter mechanical stroke than intended—but it should be measured separately. The stroke efficiency captures the exhibited efficiency of the stroke accounting for the actual motion of the vehicle center of mass relative to the passenger versus the actual consumption of mechanical stroke.

If the full mass of the vehicle is instantly brought into motion in unison, the velocity of every part is equal to the velocity of the center of mass, and the total mechanical stroke is equal to the center-of-mass stroke. If the blast is under the occupants, for most sequences

of relative motion dissipation, $d_{CM} < d_0$.³ We define the stroke efficiency as the ratio of the center-of-mass stroke to the total mechanical stroke:

$$\eta_d = \frac{d_{CM}}{d_0}. \quad (21)$$

Combining this definition with Eq. 14 yields the maximum tolerable impulse under suboptimal stroke use and passenger acceleration,

$$I = \sqrt{2M_\Sigma \eta_a a_c M_v \eta_d d_0 + M_\Sigma^2 \eta_a^2 a_c^2 \tilde{t}_F^2 + M_\Sigma \eta_a a_c \tilde{t}_F}. \quad (22)$$

E. Momentum Reduction

In addition to accommodating momentum in a fashion most protective of a passenger, a vehicle design can attempt to minimize the amount of momentum it receives from the blast. The amount of vertical momentum carried by the blast ejecta I_0 is independent of the vehicle. There are three primary modes of minimizing the amount of momentum carried by the vehicle.

First, a vehicle can be designed to avoid the blast ejecta, which carry the blast momentum. This might be achieved for instance through a narrower hull, a sharp V-hull, or a larger standoff (see Figure 7). All these designs reduce the solid angle subtended by the vehicle from the perspective of the blast center. We refer to this reduction factor as ϕ_{SO} (“so” indicates standoff). Green ejecta paths would miss hulls with low or high standoff. Red ejecta paths would hit both hulls. Blue ejecta paths would hit the low standoff hull but miss the high standoff hull. The high standoff hull has a lower ϕ_{SO} .

³ While it would be advantageous for the center-of-mass stroke to exceed the total mechanical stroke, in practice it would require other parts of the vehicle to rise more than the underbody.

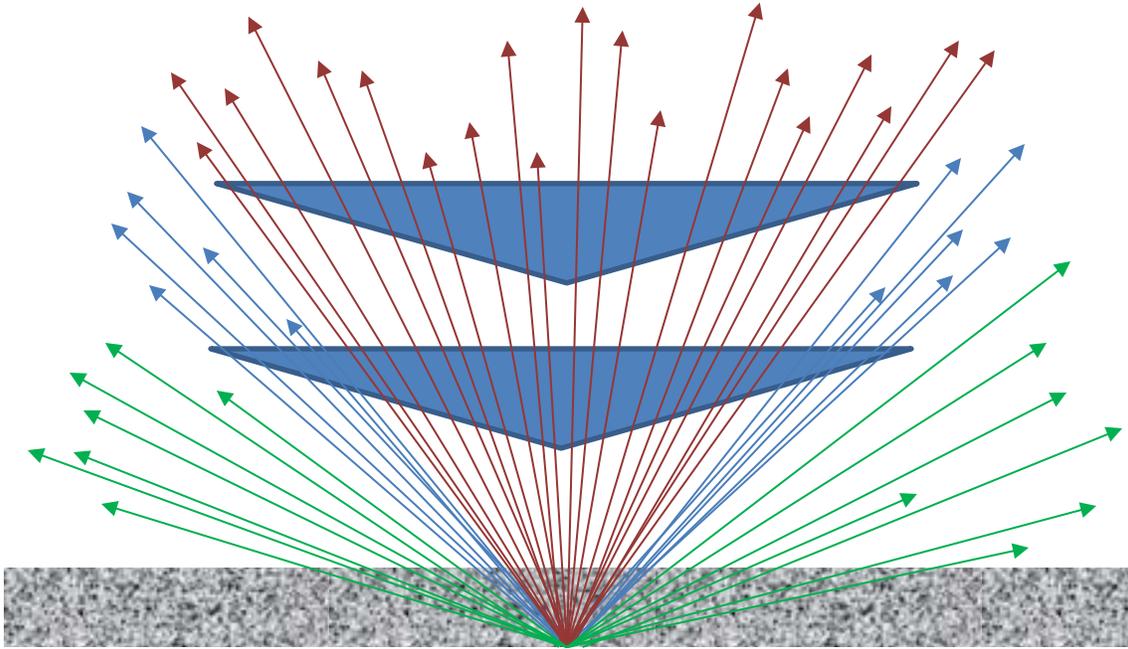


Figure 7. Reduction of Vehicle Blast Momentum Due to Standoff Distance Increase

Second, a vehicle can have a compliant underbody or an angled underbody (see Figure 8) to reduce the effective restitution coefficient of the blast ejecta collision with the hull. The ricochet on the right confers less momentum than the one on the left ($\cos^2 \theta_1 > \cos^2 \theta_2$). Let ϕ_γ indicate the reduction factor due to changing the interaction between the blast ejecta and the hull. A restitution coefficient of zero indicates completely inelastic collision, meaning all the momentum is transferred to the vehicle ($\phi_\gamma = 1$). A restitution coefficient of 1 indicates a completely elastic collision in which no kinetic energy is dissipated. For a flat hull and a perpendicular impact, each particle of ejecta would bounce off to rebound with its initial velocity but in the opposite direction. This reversal of momentum confers twice the initial momentum of the ejecta to the vehicle ($\phi_\gamma = 2$). An angled hull causes the ricocheting ejecta to angle off to the side so that the full initial vertical momentum is not reflected ($\phi_\gamma \sim \cos^2 \theta$, where θ is the angle of the hull relative to horizontal). The less elastic the collision, the less effectively an angled hull deflects momentum.

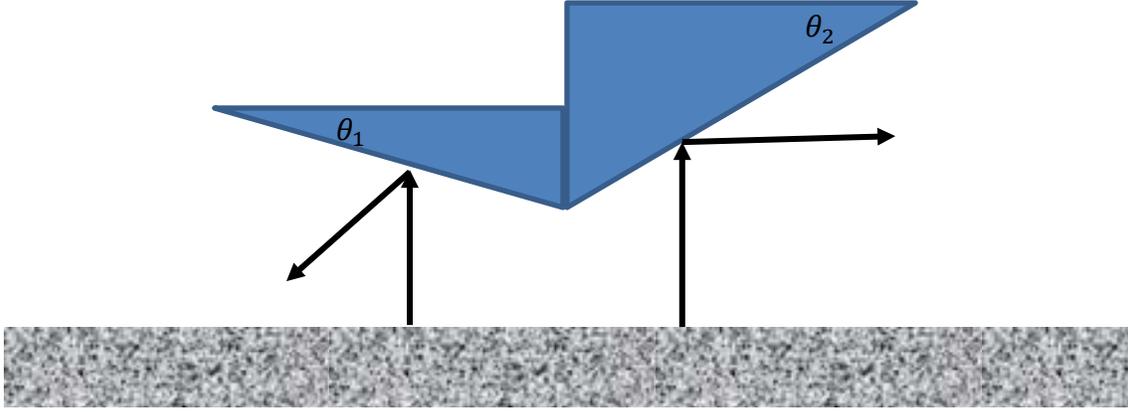


Figure 8. Reduction of Vehicle Blast Momentum Due to Hull Angle Increase

Third, the vehicle can eject mass and associated momentum at high velocity in an upward direction. This class of concepts can be termed counter-momentum; ϕ_{CM} characterizes the fraction of momentum remaining with the vehicle.

The total momentum remaining with the vehicle is

$$I = \phi_{SO}\phi_{\gamma}\phi_{CM}I_0. \quad (23)$$

Thus, extending Eq. 22, the maximum blast momentum tolerable by a vehicle is given by

$$I_0 = \frac{\sqrt{2M_{\Sigma}\eta_a a_c M_v \eta_d d_0 + M_{\Sigma}^2 \eta_a^2 a_c^2 \tilde{t}_F^2} + M_{\Sigma} \eta_a a_c \tilde{t}_F}{\phi_{SO}\phi_{\gamma}\phi_{CM}}. \quad (24)$$

F. Evaluating Efficiencies

We must emphasize that the efficiencies are defined at the injury threshold. Because vehicles may have highly nonlinear responses to changing impulse (e.g., due to internal impacts from excessive relative motion), it may be difficult to accurately predict behavior at the threshold of injury from behavior above or below the threshold.

We want to enable evaluation of efficiencies for existing classes of vehicles and for proposed new vehicles. For existing vehicles, testing may have already established the approximate injury threshold. Instrumenting a physical test or doing computer simulations at the maximum tolerable underbody blast will allow most accurate evaluation of the efficiency.

In experimental studies, Eq. 17 may be the most easily evaluated form of the acceleration efficiency. It requires evaluation of the system mass, momentum delivered, and passenger acceleration as a function of time. System mass should be known. Passenger acceleration is routinely instrumented both in simulation and in an anthropomorphic test dummy. Delivered momentum (impulse) can be measured in simulation by evaluating the

velocities of mass elements or summing the momenta of the component groups associated with the vehicle and its contents; in experiment it can be measured by the relationship of initial velocity to jump height⁴ or time to reach maximum height, which should be readily measurable with off-board sensors.

Stroke efficiency evaluation requires measuring the displacement of stroking elements (readily done in simulation by tracking positions of the stroking element endpoints and presumably instrumented in experiments) and tracking the system center of mass. The latter may be the more challenging of the two. In simulation, the positions and masses of components making up the vehicle and its contents are tracked, but commercial finite-element analysis packages may not make the extraction of such information trivial. In experiment, tracking the center of mass requires tracking the positions of enough vehicle mass components to approximate the overall center of mass. An alternative would be to backtrack from the jump height under the following two presumptions: (1) by the time the passenger velocity equilibrates with the vehicle center-of-mass velocity, the explosion is no longer adding momentum, and (2) by the time the vehicle reaches its zenith, the gross positioning of the vehicle gives a good approximation to the center-of-mass location relative to its pre-explosion location.

Increasing stroke always has the potential to improve a vehicle's underbody blast protection, and incomplete stroke utilization represents an opportunity to increase stroke by improving utilization. Incomplete utilization of stroke will not be apparent in the efficiency measurements but should be measured and identified as an opportunity for vehicle improvement

Momentum-reduction factors would need to be evaluated by comparative experimentation, either physical or in simulation, or by analysis. Other IDA analysis explores the limitations of restitution coefficient reduction ϕ_V .⁵

We should also point out that the maximum tolerable underbody blast may vary with soil type, charge burial depth, etc. However, the structure of the theory surrounding the efficiencies suggests that the time distribution of momentum carried by the blast ejecta will be the unifying characterization.

G. Limits of Instantaneous Blast and Low Passenger Mass

Taking the limit of Eq. 22 as $\tilde{t}_F \rightarrow 0$

⁴ Assuming the vehicle remains intact enough that the gross positioning of the vehicle gives a good approximation of the position of the center of mass of the vehicle when the vehicle reaches zenith.

⁵ J. Macheret and J. Teichman, "Estimating Potential Benefits of Energy Dissipation Underbody Barrier for Improving Vehicle Blast Protection," IDA Paper P-5285 (Alexandria, VA: Institute for Defense Analyses, forthcoming).

$$I = \sqrt{2M_{\Sigma}\eta_a a_c M_v \eta_d d_0}. \quad (25)$$

If in addition the mass of the passengers is negligible compared to the vehicle mass, $M_{\Sigma} \approx M_v$, and

$$I = M_v \sqrt{2\eta_a a_c \eta_d d_0}, \quad (26)$$

corresponding to our original upper bound when the efficiencies are 100% or

$$I = M_v \sqrt{2a_c d_0}. \quad (27)$$

H. Example Calculation

Just for illustration purposes, we provide an example utilizing a “perfect” under-seat damper with inefficient execution. In this case, as depicted in Figure 9, the underbody locally deforms. Let us assume that the passenger plus the seat have a mass $m_p = 100$ kg; the vehicle has a mass $m_v = 20,000$ kg; and the deforming underbody has 10% of the vehicle mass and undergoes a conical or pyramidal deformation in which its center of mass has 1/3 the displacement of its peak deformation, the seat stroke is $d_0 = 20$ cm, and the under-seat damper applies a constant force of $F = a_c m_p$, the perfect damping law.

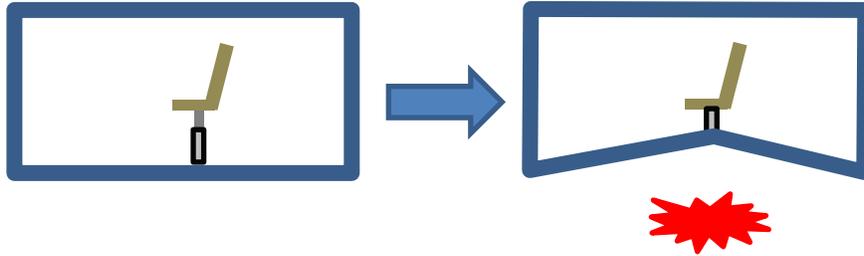


Figure 9. Example 1: Local Underbody Deformation

If the blast force is applied instantaneously, a design study would perhaps have suggested that the vehicle would have an upper bound to tolerable blast momentum of

$$I_{\max} = \sqrt{2M_v M_{\Sigma} a_c d_0} = 190,000 \text{ N} \cdot \text{m}.$$

A simulation or experiment would instead show a maximum tolerable blast momentum of

$$I' = 35,000 \text{ N} \cdot \text{m}.$$

In the marginally injurious simulation, we would measure the momentum of the vehicle, which is the same as I' . In the marginally injurious experiment, we would measure the time to zenith t_z or the jump height z . The delivered impulse is

$$I' = g t_z M_{\Sigma}$$

or

$$I' = M_{\Sigma} \left(\sqrt{2zg + g^2 \tilde{t}_F^2} - g \tilde{t}_F \right) \approx M_{\Sigma} \sqrt{2zg}.$$

Evaluating the center-of-mass stroke and the stroke efficiency would reveal that

$$\eta_d = \frac{d_{CM}}{d_0} = \frac{\frac{0.1M_v \frac{d_0}{3}}{M_v}}{d_0} = 3.3\%.$$

In practice, we would evaluate the velocity of the passenger either by direct extraction from simulation or via accelerometer integration or direct position sensing and differentiation. The equilibration time t^* would be computed by observing when the passenger velocity reached I'/M_{Σ} . The change in position of the passenger at t^* , $z_p(t^*)$, would be given either by direct position sensing or double integration of accelerometer data. The change in position of the center of mass at t^* is given by $z_{CM}(t^*) = t^* I'/M_{\Sigma}$. The center-of-mass stroke would then be given by $d_{CM} = z_{CM}(t^*) - z_p(t^*)$. d_0 would be given by direct measurement in either simulation or experiment.

Evaluating the acceleration efficiency would demonstrate that

$$\eta_a = \frac{I'^2}{2a_c M_{\Sigma} M_v d_{CM}} = 100\%,$$

where all the necessary quantities have already been computed above. One would conclude that a vehicle with this much mechanical stroke has significant room for improvement, and efforts to improve it should leave the under-seat damper alone and focus on making the underbody more rigid in order to more effectively utilize the stroke available.

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14. ABSTRACT This paper addresses how vehicle designers, analysts, and program managers can exploit the upper bound analysis in a practical fashion to assess vehicle designs, simulations, and experiments to evaluate performance in a way that allows comparison among diverse vehicle designs, quantifies the room for improvement, and helps identify sources of inefficiency (potential targets for improvement) in underbody blast protection performance. We define and describe a set of efficiencies to quantify the effectiveness of various contributions to overall blast protection performance relative to the upper bound. Decomposing the performance into independent contributions isolates the causes or sources of suboptimal performance for analysis and improvement. Understanding the blast protection efficiency of existing vehicles will indicate how much room there is for improvement. The decomposition will further indicate which contributors to blast protection performance have room for improvement. We also account for the contribution of time-varying vehicle acceleration to blast protection performance.					
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