Strategic Material Shortfall Risk Mitigation Optimization Model

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The Defense Logistics Agency Strategic Materials office recommends strategies to Congress for managing risk that arises from shortages of strategic and critical materials that could occur during military conflicts. Strategies typically considered include material stockpiling, substitution, and spot market purchases. Heuristic methods traditionally used to select the strategies do not allow these strategies to be optimized under budgetary or nonzero expected risk constraints. We developed linear and nonlinear programming models to identify strategies that minimize expected total risk subject to upper bounds on expected total cost (budget) and to upper bounds on expected risks for individual materials.

The model solves a mathematical programming problem to identify an optimal set of strategies for mitigating the shortfalls, within cost and risk constraints set by the user.

Background and Issues

Section 14 of the Strategic and Critical Materials Stock Piling Act requires the Department of Defense (DoD), specifically, Defense Logistics Agency (DLA) Strategic Materials, to assess periodically the potential for shortfalls of strategic and critical non-fuel materials that could occur in the context of a national emergency planning scenario. The scenario consists of one or more major regional military conflicts followed by a period of military force recovery and regeneration. DoD then recommends to Congress mitigation strategies for materials that could potentially suffer shortfalls during the scenario. This paper presents the Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM), which identifies shortfall mitigation strategies that would minimize expected total risk while satisfying an expected total cost constraint and constraints on the expected risks arising from possible shortfalls in individual materials.

OPTIM-SM is part of an assessment procedure that moves beyond the traditional National Defense Stockpile (NDS) planning process of estimating material shortfalls and recommending that the shortfall amounts be acquired and stored in the NDS, to a risk-based process of evaluating stockpiling along with other cost-effective alternatives for mitigating material shortfall risk.

The assessment procedure consists of the following steps:

1. Select materials of interest.
2. Estimate material shortfalls in the planning scenario.
3. Assess shortfall risk.
4. Identify promising shortfall mitigation options.

5. Assess the options' relative costs and mitigation effectiveness.
6. Identify/recommend the most promising option set—potentially within a budget constraint.
7. Begin the cycle again, as appropriate.

OPTIM-SM addresses step 6. Steps 1 and 2 constitute the traditional NDS process. Steps 3 through 5 are evaluated as part of the assessment process, and the results of steps 1 through 5 become inputs to OPTIM-SM.

The assessment procedure was implemented in support of the *Strategic and Critical Materials 2013 Report on Stockpile Requirements* (Department of Defense 2013, referred to hereafter as the 2013 report). Step 2 found 23 materials that had shortfalls, of which 19 were analyzed via OPTIM-SM. For step 4, five different possible mitigation strategies were identified and studied:

- *Stockpiling:* acquisition and storage in the U.S. NDS.
- *Buffer Stocks:* acquisition by vendor and storage in vendor-managed buffer stock inventories.
- *Export Guarantee:* reduced government guarantees of supplies of material used to produce goods to be exported during the scenario.
- *Substitution:* use of substitute materials or goods during the scenario.
- *Extra Buy:* increased U.S. buys of foreign supplies from reliable suppliers during the scenario.

Each mitigation strategy acts as an effective source of supply (or, equivalently, reduction in demand) for one or more materials in shortfall. Each strategy has a different capacity (maximum supply provided or demand reduced) and a different expected cost for each material. The effectiveness of each strategy in reducing risk depends on how much risk is created by each material shortfall and how much each strategy reduces each shortfall. The probability of the emergency scenario occurring, the negative consequences of unmitigated shortfalls, the extent to which each strategy can reduce each shortfall, and the cost of each strategy were evaluated in the preparation of the 2013 report.

Model

The OPTIM-SM model solves a mathematical programming problem to identify an optimal set of strategies for mitigating the shortfalls, within cost and risk constraints set by the user. The model is generally regarded as a linear programming problem, but there is an option for a nonlinear formulation, depending on the form of the assumed relationship between shortfall size and shortfall consequences.

Risk Measure

In the model, risk is expected risk and is defined as follows:

Expected risk = Initial shortfall risk
× Expected shortfall remaining risk factor,

where:

Initial shortfall risk is the product of *Probability of war* and *Shortfall consequences.*

Probability of war is the annual probability that the emergency scenario causing the shortfalls will occur. It was estimated by subject matter experts (SMEs) for the 2013 report.

Shortfall consequences are the consequences to the nation of each material shortfall projected to occur by the DoD planning process. These consequences were estimated for the 2013 report by SMEs using a common ratio scale that focused on economic effects.

Expected shortfall remaining risk factor is *(Expected shortfall remaining/ Initial shortfall)^{Exponent}*.

Expected shortfall remaining is *Initial shortfall* minus the supply increase or demand decrease resulting from the mitigation strategies, each of which has a different capacity and effectiveness.

Initial shortfall is determined by supply and demand modeling for each material.

Exponent, which can be equal to or greater than 1, is a factor that is capable of accounting for the effect of *Shortfall consequences* that increase nonlinearly with shortfall amount. (Applications of a material that are less important would tend to be forgone before applications that are more important.) In the linear programming formulation, *Exponent* is set equal to 1.

Cost of Mitigation Options

Expected net cost formulations are devised for each mitigation strategy evaluated by the model. These formulations vary linearly with the amount of material planned to be acquired by the strategy. Discount factors are applied to all future costs and benefits. Costs given here are incurred by the U.S. government, so the Export Guarantees and Substitution strategy options have costs of zero. Net cost is particularly important for the Stockpiling strategy because it accounts for recoupment—the sale of a stockpiled material after it is no longer needed to mitigate shortfall risk. In the 2013 report, recoupment was assumed to take place after 20 years. Expected cost is particularly important for the Buffer Stocks and Extra Buy strategies because the costs of acquiring materials using those options would not be incurred unless the scenario were to occur. Several strategies have a limited capacity to mitigate shortfalls, so even a zero-cost option cannot necessarily be counted upon to eliminate any given shortfall.

Optimization Problem Formulation

The decision variables of the mathematical programming problem are the amounts of each material planned to be provided by each mitigation strategy. For the 2013 report, this means 19 materials times 5 mitigation strategies (i.e., 95 decision variables). In accordance with the formulas stated previously, each decision variable will induce its own amount of cost. Together, the decision variables for a material will lead to an expected risk for that material. The total cost of mitigation is simply the sum (over materials and mitigation strategies) of the individual costs, and the overall remaining risk is regarded as the sum of the expected risks for the individual materials.

Upper limits can be imposed on each decision variable, corresponding to the maximum amount of material that can be obtained by a given mitigation strategy. The total cost must be less than a given budget amount. The *Expected shortfall remaining risk factor* values for each given material, which generally corresponds to the fraction of *Initial shortfall* left unmitigated, can be constrained to be less than an upper limit for that material.

Overall, the optimization problem is to choose the decision variables to minimize overall remaining risk subject to:

- Upper bounds on each decision variable,
- Budget constraint on total cost of mitigation, and
- Upper bounds on *Expected shortfall remaining risk factor* values for each material.

The constraints on the *Expected shortfall remaining risk* values for each given material can be imposed to address the concern that if some shortfalls were left unmitigated or undermitigated (out of a desire to pursue the most cost-effective overall shortfall solution), those shortfalls might prevent certain industries from producing important goods. Constraining *Expected shortfall remaining risk factor* values for each separate material forces all—or at least most—shortfalls to be reduced more evenly among the different materials, which reduces the likelihood that a shortfall that might be more costly to mitigate would end up preventing the production of important goods during a crisis scenario.

Although the analysis for the 2013 report considered five specific mitigation strategies, a mitigation strategy should be recognized as any activity that can increase material supply or decrease material demand and is characterized by its cost and by the change it generates in supply or demand. Risk, upon the application of any strategy, is a function of the still-unsatisfied shortfall. Thus, other shortfall mitigation strategies, such as increased material production, material recycling, and futures contracts, can also be modeled in OPTIM-SM, as long as their attributes are characterized in the terms set out here.

Model Results

Three initial cases are considered, using the data from the 2013 report. Each case is characterized by its own upper bound for the *Expected shortfall remaining risk factor* values for each material. The upper bound values for the different cases are set to 1.00, 0.30, and 0.24, respectively. In each case, total mitigation cost (i.e., budget) is constrained at $50 million, and upper bounds on the capacities of the shortfall mitigation strategies are those used in the 2013 report.

The results show that as residual risk constraints are tightened (in the second and third cases), the shortfalls of some materials must be reduced below the levels to which they are reduced in the unconstrained, minimum total risk (for the given budget) case (the first case). That extra reduction, in turn, requires the diversion of resources that had been spent in the first case to reduce risk arising from the shortfalls of other materials. However, the further shortfall reductions in the second and third cases cost more (in dollars spent per unit of risk reduced) than the original (unconstrained) reductions. Therefore, because total cost is fixed, total risk increases.

In addition to the three optimal solution cases, three experiments were performed to show how the model responds to other changes in input data:

- First, the constraint on total cost is raised from $50 million to $80 million. The overall remaining risk becomes nearly zero.
- Second, *Probability of war*—the occurrence of the scenario—is raised significantly. As expected, the model shows that a higher *Probability of war* raises the expected costs of increased U.S. buys of foreign material supplies at the time of war. This situation can make buys of foreign material at the time of war unattractive relative to other possible shortfall mitigation strategies.
- Third, *Exponent* is set to 1.5 so that *Shortfall consequences* will increase nonlinearly with shortfall amount (representing that less important applications of a material would be forgone before more important applications). As expected, results show further reduced risk as shortfalls are mitigated below their original values.

This paper has developed a mathematical programming model to assess strategies for mitigating shortfalls of strategic and critical materials that could occur during a military conflict. The model identifies an optimal mix of such strategies: one that minimizes risk subject to constraints on total cost.

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