



INSTITUTE FOR DEFENSE ANALYSES

## **Portfolio Selection Challenges in Defense Applications**

David M. Tate  
Paul M. Thompson

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#### For More Information:

David M. Tate, Project Leader  
[dtate@ida.org](mailto:dtate@ida.org), (703) 575-1409

David J. Nicholls, Director, Cost Analysis and Research Division  
[dnicholl@ida.org](mailto:dnicholl@ida.org), (703) 575-4991

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## **1. Defense Portfolio Selection**

### **A. Statement of the Problem**

Throughout the federal government, decision makers at multiple levels face the problem of choosing a portfolio of investments (e.g., projects, activities, or programs) to fund, given current and projected budget restrictions. The Department of Defense (DoD) in particular faces a wide range of portfolio selection problems, ranging from investment in basic research programs to decisions about which weapon systems to purchase, maintain, or retire. These are extremely difficult problems. Various methodologies and tools to support decision makers facing such questions have been proposed and implemented in the past, with mixed success. In this paper, we describe and illustrate the particular challenges of defense portfolio problems. We then propose a framework for decision support in this context, and identify the key gaps in current capabilities that would need to be overcome to implement the framework.

### **B. Distinctive Characteristics of Defense Investment Decisions**

A number of features make defense investment selection problems particularly difficult to address. To begin with, these are *portfolio* selection and management problems—we are trying to choose sets of activities to pursue simultaneously, from a potentially large set of candidate activities, in a resource-constrained environment. As we will see, this means that techniques for identifying the best available activity, or even for ranking all candidate activities, may not be sufficient to support good decisions. The following subsections describe the principal sources of difficulty.

## **1. Multiple, Non-Commensurate Objectives**

The first key feature of defense portfolio selection problems is that DoD is trying to accomplish many different things that cannot be easily reduced to a single dimension. This makes defense portfolios very different from financial portfolios, in which utility to the investor can generally be summarized by expected return and some measure of risk. Risk and return can be further combined into a “risk-adjusted expected return” single-dimension measure, given information about the investor’s risk preferences.

DoD, in contrast, is concerned with several distinct goals, such as:

- Project force in support of US foreign policy objectives
- Defend the homeland and our allies from potential attack
- Respond to humanitarian crises around the world
- Maintain a healthy defense industrial base
- Foster good military-to-military relations with US allies

Each of these basic goals involves dozens of specific scenarios, desired capabilities, and potential materiel and non-materiel solutions, both today and for the foreseeable future. There are literally hundreds of specific mission needs, with corresponding current and projected capability gaps that must be accounted for. In addition, there are sharp limits on how much capability in one mission area can be traded for capability in an unrelated mission area—DoD has to be able to do many things to at least a minimum required level. It is impractical to reduce these many and varied objectives to a single “military capability” score.



## **2. Polytomous (Non-Binary) Alternatives**

Many portfolio decision methods assume binary (dichotomous) alternatives in which one either makes that investment or doesn't. Financial models are slightly more general, in that one must also decide how much to invest in each alternative.

Defense portfolio problems typically involve a wide range of distinguishable ways to pursue a given goal. These are *polytomous* alternatives, characterized by many partially exclusive alternative investments intended to meet a specific mission need or mitigate a particular capability gap.

Consider the current Air Force program to develop and field a new long-range bomber. They will need to decide minimum acceptable values for the range and speed of the aircraft, the size of its crew, the number and types of munitions it should be able to deploy, its stealth level, the number of aircraft to build, an affordable cost for the fleet, the urgency of fielding, and so forth. These decisions are strongly coupled with both operational considerations and engineering constraints; they cannot be treated separately. At the portfolio management level, the question of how much (or whether) to fund a new long-range bomber cannot be sensibly separated from the question of *which* long-range bomber program would be funded. In some cases, depending on how uncertain the outcomes of various alternative approaches might be, it might even make sense to fund more than one alternative approach simultaneously, as a hedge against poor program outcomes.

## **3. Interdependencies among Alternatives**

In addition to dependencies within polytomous alternatives, there are also generally interdependencies—positive or negative synergies—across the higher-level alternatives as well.

For example, if the Army is considering buying additional or improved 155 mm artillery pieces, and also considering developing improved 155 mm artillery shells, the value of completing both projects may well be higher than the sum of the values of the individual projects. This would be a positive synergy. Conversely, if they are considering funding development of a new surface-to-surface missile or a new precision bomb, the combined value of funding both projects might be less than the sum of their individual values, since there is significant overlap in the new capabilities that these projects would provide. This would be a negative synergy.

In extreme cases, we can imagine there being many-way interdependencies among alternatives. New platforms, new munitions, upgrades to existing platforms and munitions, new command and control capabilities, and improved training may all interact in complex ways to determine the resulting overall improvement in value to DoD. Even multiplicative value functions may not be able to describe this degree of interdependence.

#### **4. Complex Nonlinear Cost Functions**

In most financial and industrial resource allocation contexts, costs are approximately linear and additive. For example, the cost of buying 1000 shares of a security is almost exactly ten times the cost of buying 100 shares. Similarly, the cost of drilling two wells in geographically separated areas is roughly equal to the sum of the costs of the individual wells.

In defense procurement, there are several ways in which the costs of alternatives are neither linear nor additive. Many defense contracts are performed on a cost-plus basis, where the contractor's compensation is equal to the legitimate costs of production, plus an agreed-upon fee. In many cases, a significant fraction of those costs of production in a given year are relatively fixed, and not proportional to the number of units produced. In addition, many defense systems exhibit "learning curve" effects, where the marginal cost of additional units decreases as a

function of the number of past units produced. As a result, the cost of producing 1000 trucks in ten years might be considerably less than ten times the cost of producing 100 trucks over that period. At the same time, due to fixed costs, the total cost of producing 1000 trucks in five years might be significantly lower than the cost of producing those trucks over ten years.

Significant cost nonlinearities are challenging both when formulating the portfolio selection problem and when trying to solve it numerically. Techniques that can accommodate significant nonlinearities are generally less powerful, less computationally efficient, and less well-supported by commercial software than purely linear models. As a result, the size of the problem that is practically tractable is much smaller for nonlinear problems than for linear problems.

## **5. Resource Constraints**

Every DoD funding decision incurs not only the direct cost of the investment, but also an opportunity cost of forgoing other projects. These opportunity costs are measured in lost capability. When the costs and benefits of individual alternatives are well known, and costs are small relative to the overall budget, the decision problem can be formulated as a knapsack problem. The greedy heuristic that selects investments in decreasing order of benefit-to-cost ratio will usually produce optimal or near-optimal portfolios.

There are many DoD portfolio decisions for which this approach does not work. In particular, it can be difficult to make sense of the notion of “benefit-to-cost ratio” when alternatives consume multiple distinct resources (e.g., dollars, personnel, and time) and stakeholders have multiple or divergent objectives (e.g., reduce capability gaps for various different mission needs, maintain a healthy industrial base, and promote improved relations with various allies). The problem is made even worse when the benefits of a given investment depend on which other investments have been selected.

Given multiple objectives, multiple resource constraints, and interdependent investment outcomes, there is no standard method for even ranking potential investments, much less identifying optimal investment sets. In particular, the relative desirability of two alternatives can depend on the budget. Perhaps even more so than with traditional financial investment portfolios, a high-cost, high-quality program to eliminate a specific capability gap might be preferred when resources are relatively plentiful, whereas a lower-cost, lower-quality alternative that partially mitigates the gap might be preferred when resources are scarce, due to the higher opportunity cost of funding the high-quality option.

## **6. Discrete Alternatives**

In contrast with the financial investment world, where one can invest any continuous amount in any alternative, defense portfolio selection projects only come in predetermined discrete sizes. This “either/or” nature of these projects leads to “knapsack-problem” structures with their concomitant discontinuous objective functions and difficulty in achieving full usage of scarce resources. In some cases, individual projects can consume a large fraction of the available budget, as in pharmaceutical research or other new product development contexts.

## **7. Uncertainty**

All of the aspects cited above are challenging enough when alternatives have known resource requirements and predictable outcomes. In practice, however, both the resource costs and the capability benefits of proposed investment combinations are uncertain. This has profound implications for portfolio selection decisions.

### **a. Uncertainty of Outcomes**

When a military Component requests funding for a major defense acquisition program (MDAP), no one knows for certain exactly how much (if at all) the system actually produced will improve the Component's ability to perform various missions. It is not certain that the program will produce any capability at all—some MDAPs are cancelled before ever fielding any systems. There is even more uncertainty regarding the future benefits of DoD's extensive investments in basic research and exploratory science or of changes to recruiting and retention incentives for military personnel.

It is worth emphasizing that the best portfolio of investments or activities to fund can depend strongly on the uncertainty in the outcomes of the available alternatives. In a study commissioned by the Secretary of the Army to assess the Army's acquisition policies and outcomes in the 2000s (Decker & Wagner 2011), the authors concluded that a major contributing factor to the Army's poor acquisition track record in that decade was a failure to balance the level of outcome uncertainty across the entire acquisition portfolio. In essence, the Army had invested exclusively in high-risk, high-return alternatives, many of which either failed outright or delivered far less than anticipated for the price.

### **b. Uncertainty of Costs and Schedules**

Outcome uncertainty is often coupled with cost and/or schedule uncertainty, in part because portfolio managers may have the ability to trade away potential benefits in order to reduce cost or tighten the schedule during the execution of a program. In such cases, cost and schedule uncertainty become key drivers of outcome uncertainty, and vice versa.<sup>1</sup>

Because portfolio selection is resource constrained, uncertainty in cost has a different impact from uncertainty in benefits realized. Where uncertainty in benefits can sometimes be treated using expected values and riskless equivalents, uncertainty in cost makes it difficult to determine whether or not a given proposed set of investments is *affordable* within a specified budget. Methods for dealing with this “feasibility uncertainty” are typically more mathematically complicated (and less transparent to decision makers) than methods for dealing with uncertainty in benefits obtained. This is not to say that benefit uncertainty is easy to handle; the existence of minimum (threshold) levels of capability requires the decision maker to think not in terms of expected outcomes, but in terms of the probability of simultaneously achieving all minimum standards.

Monte Carlo simulation methods are often used to address feasibility uncertainty. These models have the disadvantage that they are difficult to optimize, compared to analytical value functions and constraints. Finding the set of decisions that produces the most desired distribution of potential outputs from a simulation model is a field of its own, and the best known methods are computationally intensive and do not scale well to large numbers of decision variables.

## **8. Risk Attitudes**

Traditional financial portfolio theory, and much of the Decision Analysis literature, assumes that decision makers should obey the axioms of classical utility theory. Unfortunately, classical utility theory is often not a good model for how people make decisions in real life. Real decision makers exhibit asymmetric risk aversion with respect to gains and losses, sacrifice expected utility to avoid the possibility of a large loss, and otherwise behave in ways that the classical theory would label “irrational.”

In DoD, the attitude toward risk is complex and inconsistent. Decisions that are seen as putting lives at risk are made very conservatively, while decisions that put dollars and time at risk show a preference for higher-risk alternatives. Alternatives that might result in a reduction in some capability relative to the current level are often considered unacceptable, regardless of how much additional capability might be obtained in other areas. In part, this is driven by the recognition that US forces face intelligent adversaries who can adapt their tactics to exploit the largest capability gaps. Because of this, the value to the warfighters of a portfolio of uncertain alternatives is not increasing in the expected values of the resulting capabilities, but is more nearly decreasing as a nonlinear (and possibly discontinuous) function of the remaining capability gaps. Modeling this complex dependency, and accounting for it when choosing portfolios with uncertain outcomes, is a daunting task.

## **9. Long Time Horizons**

Decisions being made by defense portfolio managers often involve long-term commitments of many dollars over periods ranging from a few years to a few decades. A newly launched MDAP will commonly not provide any military capability for at least a decade, and may incur operating and support costs for 40 or 50 years. Such long lead times and planning horizons introduce special challenges for the decision maker.

First, each budget period has its own (uncertain) resource constraints, which must all be satisfied. Using too much of any resource in any year makes a portfolio infeasible, and the probability of violating at least one budget constraint grows quickly with the number of periods. The military Components have only a limited ability to shift funds among years to compensate for unexpected outcomes or needs.

At the same time, it makes sense to be less concerned about exact constraint values far in the future. The future will be surprising, and many of the assumptions of the current planning model will turn out to be irrelevant by the time we get to those later periods. A traditional way of dealing with this is to discount both costs and benefits—but it is not clear how to discount either cost or budget feasibility sensibly. The correct discount rate for costs depends on the value of money in hand that could be used elsewhere. For defense acquisition, this includes potentially reducing the national debt (by not spending on anything), and also includes benefits that could accrue by using the money elsewhere. Office of Management and Budget discount guidance accounts for the former alternative, but not for the latter. For example, no official guidance addresses how to discount the impact of violating a budget constraint eleven years in the future.

The correct discount rate to apply to future benefits is also not obvious. The military value of a given capability at a given time in the future depends on the projected effect on the capability gap, the estimated importance (at that point in the future) of that capability gap, and the rate at which potential adversaries adapt to that new capability. It also should reflect a healthy skepticism about the accuracy of our forecasts that far in the future with respect to all of our inputs to the model. Brown et al. (2004) refer to a *model mischief discount factor* that deliberately reduces the penalties for constraint violation or capability shortfall in the later periods of a planning model, on the grounds that we cannot accurately predict the future that far in advance and that our short-term decisions should not be overly sensitive to those far-future predictions.

Finally, there is a tension between the relative unimportance (or uncertainty) of the far future, and the fact that the state of the world at the end of any planning horizon becomes the initial conditions for the future. Defense planning is an iterated game, and it would not be



acceptable to optimize capability over the planning horizon in a manner that leaves future planners stuck with obsolete and poorly maintained equipment (and nothing new in the pipeline) at the end of the planning horizon. Typically, this is handled by making the planning horizon significantly longer than the decision cycle, so that the latter portions of the plan are always being supplanted by new plans long before those years arrive. There is an obvious danger in this—the hard resource challenges may tend to get pushed into the future, where they pile up.

## **10. Objectives vs. Constraints**

In many DoD portfolio choice decision processes, there is a familiar duality between objectives and constraints. For instance, a decision maker might want to determine the minimum cost portfolio that could achieve at least a specified capability level over the duration of the planning horizon. Upon determining that the cost is untenable, the decision maker might instead want to determine the highest capability levels that are possible given a specified budget flow over the duration of the planning horizon. Thus, the decision maker may want to determine the optimal allocation of a given set of resources to inform their own planning, and may also seek to identify the opportunity cost of the resource constraints in order to better inform the resource authorities. More subtly, there is a similar duality for maximizing the probability of certain outcomes versus optimizing some other objective subject to constraints on the probabilities of those outcomes.

## **11. Multiple Stakeholders**

Portfolio decisions are difficult enough for individual decision makers with multiple objectives. Group decisions involving stakeholders with different preferences or conflicting values pose even greater challenges. In the context of DoD, conflicting goals are the norm. Resource allocations are jointly determined by the military Components, the Office of the Secretary of

Defense (OSD), and the Congress. Even when a single individual or organization has decision authority over allocation of resources, it is rare for that individual to be the only decision stakeholder whose opinion matters. An effective portfolio decision support methodology for DoD investments needs to account for multiple stakeholders, finding the best consensus solutions or, when stakeholder goals are even more divergent, identifying the best compromises. It also needs to account for ongoing changes in personnel in key stakeholder positions due to elections, appointments, reassignments, etc.

## **2. Literature Survey**

The academic literature in Decision Analysis is vast and varied, appearing in the journals of fields as diverse as economics, psychology, finance, operations research, and marketing. There are many high-quality papers, including survey papers, that address or illustrate one or more aspects of the problem described in Subsection 1.B. Even a cursory treatment of that literature would significantly multiply the length of this paper. As an alternative to a traditional literature review, we offer the matrix in Table 1, characterizing which aspects of defense portfolio selection problems are addressed by ~40 important works in the area. These are books and articles that, in our opinion, either introduce key concepts, or provide particularly useful discussion or elaboration of earlier works. Complete citations appear in the References section at the end of the paper.

**Table 1. Matrix of Literature Coverage**

Reference	Directly addresses portfolio selection?	Survey ?	Problem Characteristics												
			Multiple non-commensurate objectives	Polytomous Alternatives	Inter-dependent alternatives	Complex nonlinear cost functions	Complex nonlinear value functions	Resource constraints	Uncertainty of outcomes	Uncertainty of costs and schedules	Non-quantifiable outputs	Extreme risk aversion	Long time horizons	Primal and dual problems important	Multiple stakeholders
<b>Operations Research Literature</b>															
Birge & Louveaux 1997			X	X	X			X	X	X				X	X
von Neumann & Morgenstern 1944				X				X						X	X
<b>Decision Analysis Literature</b>															
Arrow 1966				X					X						
Edwards & Barron 1994			X	X					X						X
Kahneman & Tversky 1979			X	X					X	X		X			X
Keeney & Raiffa, 1976			X	X	X	X			X	X		X	X		
Keeney 1992			X	X		X			X	X		X			X
Kleinmütz 2007		Yes	X	X	X	X	X	X	X	X			X		X
Liesiö 2014			X	X	X	X	X	X			X				X
Morgan & Henrion 1990									X	X		X			
Ramsey 1931									X	X		X			
Riabacke et al. 2012		Yes	X	X				X	X						
Roy & Vanderpooten 1996		Yes	X	X					X	X					X
Saaty 1980			X	X				X			X				X
Dyer 1990			X	X											
Salo et al. 2011		Yes	X	X	X	X	X	X	X	X	X		X		X
Yoon & Hwang 1995			X	X		X	X				X	X			
<b>Management &amp; Finance Literature</b>															
Black & Scholes 1973				X					X				X		
Fisher 1907													X		
Grenadier & Wang 2007				X					X				X		
Keeney & Oliver 2005			X	X											X
Markowitz 1952				X	X			X	X				X		
OMB Circular A-94, 1992				X									X		

Reference	Directly addresses portfolio selection?	Survey ?	Problem Characteristics												
			Multiple non-commensurate objectives	Polytomous Alternatives	Inter-dependent alternatives	Complex nonlinear cost functions	Complex nonlinear value functions	Resource constraints	Uncertainty of outcomes	Uncertainty of costs and schedules	Non-quantifiable outputs	Extreme risk aversion	Long time horizons	Primal and dual problems important	Multiple stakeholders
<b>Military Domain Applications</b>															
Brown Dell & Newman 2004			X	X	X			X					X		
Burk & Parnell 2011		Yes	X	X	X	X	X	X	X	X	X	X	X		X
Decker & Wagner 2011								X	X	X			X		X
Wall & Felli 2003			X	X			X	X	X	X			X		
<b>Non-Military Applications</b>															
Airoldi & Morton 2011			X	X				X			X				X
Amiri 2010			X	X			X	X	X		X		X		
Blau et al. 2004			X	X			X	X	X	X			X		
Focke & Stummer 2003			X	X	X	X	X	X			X		X		X
Girotra et al. 2007				X	X			X	X				X		
Mild & Salo 2009			X	X			X	X			X		X		X
Paddock et al. 1988				X					X				X		
Paul et al. 2010									X				X		
Rogers et al. 2002				X		X	X	X	X	X			X		
Suslick & Schiozer 2004		Yes	X	X	X	X	X	X	X	X			X		X
Walls 2004				X	X			X	X						
Wang & Hwang 2007				X		X	X	X	X	X					

### **3. Notional Design of Portfolio Selection Decision Support for DoD**

In this section, we propose a general methodology for implementing DoD portfolio selection decision support in practice, along with an architecture and basic design for software tools to support that methodology. The focus is on combining existing best practices where they exist, proposing new approaches only where it is necessary to address what we feel to be the highest-leverage shortfalls of existing methods and tools.

Many descriptions of the end-to-end decision process have been proposed in the literature. For our purposes, we see seven key stages in making good portfolio decisions:

1. Characterizing the alternatives
2. Quantifying the value of portfolios
3. Representing constraints and feasibility
4. Accounting for risk and uncertainty
5. Accounting for multiple stakeholders
6. Identifying good portfolios
7. Performing sensitivity and robustness analyses

We will address these in turn, presenting our vision for how each of these stages might best be accomplished in a practical way, and what software tools would best support that stage. This discussion parallels to some extent that in Burk & Parnell (2011), but with a rather different focus and an emphasis on gaps in existing methods.

## **A. Characterizing the Alternatives**

The most important step in all decision support contexts is to make sure that you are solving the right problem, and have characterized it fully and accurately. This includes framing the problem explicitly, identifying the available alternatives and their attributes, and quantifying those attributes for each alternative to be considered. Of the many proposed techniques for accomplishing this, we tend to favor the Value Focused Thinking (VFT) approach of Keeney (1992).

In the framing phase, VFT gives particular emphasis to distinguishing between stakeholder fundamental objectives (ends) and possible paths to promoting these objectives (means). This distinction is critical to formulating the decision problem in a way that does not introduce unmanageable interactions among various objectives and alternatives. This is often accomplished through a hierarchical model, in which fundamental objectives (ends) reside at the top of the hierarchy, and are in turn associated with enabling objectives (means) that are in turn associated with quantifiable performance measures. This approach depends, of course, on the validity of the mutual preference assumption.

VFT has also proven to be an effective tool for identifying and understanding overlooked alternatives. Indeed, Keeney (1992) considers this to be one of the most important arguments in favor of VFT. One technique he recommends is to consider courses of action that would maximize achievement of each fundamental objective in turn, even if that course of action is not itself an acceptable alternative over all objectives. Typically, some of these courses of action will be quite unlike any proposed alternative, revealing additional modes of possible action or tacit stakeholder desires. Once these new dimensions have been incorporated into the VFT model, the

space of alternatives to be considered can be restricted to those that are actually feasible for purposes of constructing a value function.

If some or all of the alternatives identified are polytomous (as discussed in Section 2), it can be difficult to list them all in a way that is transparent to the stakeholders yet captures all of the possible variations. Decision tree software can be used to represent the alternative space if the number of variations on each primary alternative is small. We are not aware of any software designed to capture complex or semi-continuous sets of polytomous alternatives; this could be a fruitful area of research and development.

The final phase of alternative characterization is quantifying the outcome attributes of the identified alternatives. Keeney (1992) defines an *attribute* as a measure of the degree to which an objective is achieved. Other related terms in military analysis are *measure of effectiveness* (MOE), *performance metric*, or *measure of merit*. Some decision analysts prefer the term *criterion* to mean the same thing. In all cases, the key is that the attributes be both quantitative and measurable (or at least estimable). Attributes provide the linkage between observable outcomes and stakeholder value.

The attributes of interest to DoD are extremely diverse. Some are easily measured and directly relevant to utility, such as the shipping weight of a ground vehicle or the accuracy (measured by Circular Error Probable) of a munition. Others are extremely complex to measure, such as the marginal impact of a new weapon on friendly force casualties in a specified combat scenario, evaluated through a complex series of simulated force-on-force encounters using different weapon mixes and assumptions. Some are nearly impossible to estimate by any means other than expert judgment—for example, the probability that a program to develop a new radar technology will result in new products that could be fielded within ten years.

While there is no one dominant technique for eliciting and quantifying stakeholder estimates of attribute values, we again feel that VFT provides a sound foundation that can be built on in a portfolio context.

## **B. Quantifying the Value of Portfolios**

Most Decision Analysis applications involve translating the combined attributes of each alternative into a numerical value or rank reflecting the stakeholder goals identified during the decision framing process. Mapping combinations of attributes to values in a way that is consistent and verifiable is a complex process, fraught with potential mistakes. For a formal, repeatable, practical, and mathematically justifiable approach to this task, we prefer the axiomatic framework of Multi-Attribute Value Theory (MAVT), typically implemented using the *swing weight* technique of comparison elicitation. MAVT is based on the axioms of rational preference in the absence of uncertainty.<sup>2</sup> Swing weights are coefficients derived from successive stakeholder judgments of preference or indifference between alternatives that differ in only two attributes. Based on these judgments, the analyst can build up a complete characterization of stakeholder preferences that will support comparison of alternatives differing in many attribute dimensions at once. In contexts in which we are ranking alternatives and selecting a winner, this is the end of the story.

Portfolio selection problems are distinctive in that the value of a portfolio may not be a simple function of the values (in isolation) of its component alternatives. As noted in Section 3, this is one of the most challenging aspects of defense portfolio selection. In essence, the “alternatives” of the portfolio selection problem are really all of the possible portfolios. This is typically a much larger set of available courses of action than could be separately scored by stakeholders, much less compared using swing weights. In practice, it will be necessary to



approximate the value of each possible portfolio of alternatives using some function of the individual alternative attributes. In general, this is difficult unless important interactions are limited (e.g., pairwise only) or if the form of the interaction is consistent.<sup>3</sup> If there are more complex interactions, or interactions with multiple functional forms, other approaches will be necessary.

For the strongest and most important interactions among alternatives, it may be possible (as noted above) to eliminate the interaction by using “subset expansion,” i.e., by replacing the affected alternatives with a derived set of new alternatives corresponding to possible combinations of the original alternatives. For example, given alternatives  $A_1$ ,  $A_2$ , and  $A_3$  with significant and hard-to-model value interactions, define a new set of alternatives  $B_1$  through  $B_7$  corresponding to the seven possible non-empty subsets of  $\{A_1, A_2, A_3\}$  that could be included in the portfolio.<sup>4</sup> Each of the  $B_j$  alternatives would be assessed as a whole by stakeholders and subject matter experts (SMEs) in the usual way. It might even be possible to eliminate some of the subsets *a priori* as undesirable or infeasible, reducing the amount of extra work. However, one must exercise caution here. Keeney and others would argue against eliminating undesirable (but feasible) subsets, in order to avoid constraining the option space too soon. Obviously, this approach cannot be used for large numbers of mutually interacting alternatives, but it is a powerful tool for eliminating a few critical interactions from the model.

Within the context of MAVT, it is also possible to represent some remaining interactions among alternatives through the use of a multiplicative value function (Dyer and Sarin 1979). Formal procedures for eliciting the coefficients of multiplicative value functions exist, and are only mildly more complicated than the procedures for eliciting linear additive model

coefficients. In the presence of significant nonlinear interdependence, the extra work is more than justified by the increased fidelity. Liesiö (2014) presents various techniques for eliciting multilinear (i.e., partially multiplicative) value function parameters.

For any remaining alternative subsets that exhibit more complex many-way interactions among alternatives than can be handled using multiplicative value functions or subset expansion, we would have to model the complex interactions explicitly. One approach would be to use a value composition modeling tool such as the Portfolio Analysis Machine (PALMA) tool developed by MITRE Corporation (Moynihan et al. 2008). PALMA allows users to specify the joint value of a subset of investments very flexibly, as any function of the individual values of the investments. For example, for a set of perfectly substitutable alternatives, the value of any subset might be set to be the maximum value of any member of the subset. The drawbacks to using a tool like PALMA are the labor involved in separately characterizing and implementing composition functions for all of the relevant subsets, and the computational difficulty of optimizing the resulting graph model, subject to all of the necessary constraints.

### **C. Representing Constraints and Feasibility**

Not all portfolios are feasible. As noted above, stakeholders will generally wish to limit consideration to portfolios with a sufficiently high probability of being executable, given the available resources. They will also generally wish to avoid portfolios that are likely to deliver needed capabilities too late or not at all. These restrictions are constraints on the selected portfolio, and must be formulated mathematically in terms of alternative attributes.

There is no single best approach to representing resource constraints in the presence of uncertainty. At one extreme, the mathematical formulation could treat the expected resource consumption and resource availability as certain. This would require extensive sensitivity and

robustness analysis (see Subsection 3.G) to ensure that the eventual portfolio selected had a sufficiently high chance of being executable. Given any appreciable uncertainty in cost, schedule, or funding estimates, this method will generally not be practical.

At the other extreme, the mathematical formulation could explicitly estimate and constrain the probabilities of meeting each of the various resource and schedule constraints. This is also not generally practical. The principal difficulty is that the probabilities of satisfying various constraints are not independent. Estimating the joint distribution of the constraint satisfaction probabilities is itself a very difficult problem.

Stochastic programming, stochastic dynamic programming, and approximate dynamic programming are optimization frameworks intended to address optimization under uncertainty and/or computational complexity of the decision space. They offer possible approaches to representing constraint uncertainty in a computationally tractable way. We discuss them further in Section 3.F.

In addition to accounting for resource and schedule constraints, we can also use constraints to define logical relationships among alternatives. For example, if Project B cannot begin until Project A is completed, we can add constraints that enforce this. Other cases that can be handled using logical constraints include mutually exclusive alternatives, alternatives that must run concurrently, alternatives that are mandatory if certain other alternatives are selected, or alternatives that cannot overlap in time.

## **D. Accounting for Risk and Uncertainty**

As noted in Subsection 1.B.7, there are three distinct categories of uncertainty associated with multi-period project portfolio selection decisions:

- Uncertain project outcomes
- Uncertain project resource requirements
- Uncertain timing of outcomes and resource use

Traditionally, there are two very different ways of dealing with outcome uncertainty. The classical normative approach is to collapse all probability judgments regarding possible outcomes and their relative likelihood into a risk-neutral utility function. This Multi-Attribute Utility Theory (MAUT) approach has the advantages of obeying the classical axioms for rational preferences under uncertainty, and of eliminating some complexity in the formulation of the portfolio optimization problem. However, it also has drawbacks.

The first drawback of MAUT is that it requires a great deal of effort from stakeholders to make judgments about their indifference between various combinations of uncertain outcomes, or “lotteries.” Most people are not good at this, so the process can be both time-consuming and frustrating. The effort involved also grows enormously when there are polytomous alternatives with many variants, or when there are value interactions among alternatives. Both of those will generally be the case in defense portfolio selection contexts.

The second drawback of MAUT is that it reduces transparency. Preferences regarding risk are folded into the utility function, which is estimated by the lengthy and arcane process described above. As a result, it is hard for stakeholders to be confident that their concerns and preferences regarding outcome risks have been correctly incorporated into the model.

Finally, MAUT is not a good model of actual human risk attitudes. In real life, there is an asymmetry between attitudes toward gains and attitudes toward losses (Kahneman & Tversky 1979). In contexts involving very high stakes, such as decisions regarding national defense and force protection, it is not clear that risk neutrality is always appropriate. This asymmetry is real, and will be reflected in decision makers' value judgments. Conversely, the baseline against which gains or losses are determined can be altered by the way judgments are elicited, leading to inconsistent value functions. It is the job of the decision analyst to help the decision makers identify explicit baselines and enforce consistent framing and treatment of gains (or losses) across attributes and alternatives.

For these reasons, we recommend treating outcome uncertainty through explicit project attributes related to risk in a MAVT framework. Where explicit threshold levels of capability have been identified, attributes could be things like "the probability of delivering no new capability," "the probability that capability X will increase by at least Y," or "the probability that the new system will be good enough to permit retiring system Z." Where no explicit target level or gap has been identified, the standard deviation of the gain in capability could be used to quantify the risk.

This approach has the added advantage of being the same approach needed to deal with resource and timing uncertainty. Feasibility is a global attribute of a portfolio that depends on all projects simultaneously based on the amounts and timing of their resource use. Even if we were able to formulate expected utility functions over uncertain outcomes at the portfolio level, we still would not have accounted for the disutility of potential cost and schedule overruns.

Rather than using expected utilities, we recommend treating the range of possible cost and schedule outcomes as attributes at the project level, with resulting values to stakeholders at the

portfolio level, and applying constraints on those attributes when optimizing the portfolio. This not only allows for a more nuanced treatment of various risks, it may also allow the analyst to derive useful sensitivity and opportunity cost information from the optimization process.

Finally, there are also environmental uncertainties that affect the value of portfolios but are not attributes of specific alternatives or combinations of alternatives. “Available funding” is the most common example—future budgets are highly uncertain, yet affordability of the portfolio is a principal consideration. These external uncertainties must be treated outside the MAVT framework, in a way that is consistent with the derived MAVT attributes and value functions. In general, these are handled through constraints in the optimization and/or Monte Carlo methods, followed by sensitivity and robustness analyses as described in Subsection 3.G.

## **E. Accounting for Multiple Stakeholders**

Value and preference elicitation is easier when dealing with only a single decision maker’s preferences and values. In that case, the analyst or facilitator can focus on working with that decision maker to identify values and elicit preferences that are as consistent and accurate as possible. When there are multiple stakeholders, the analyst faces the additional challenge of reconciling divergent values and unequal preferences. At this point, the analyst must decide whether the model being developed will reflect a *consensus* among the stakeholders or a *compromise*.

### **1. Consensus**

If all stakeholders generally share the same values but disagree somewhat in their priorities and preferences, a consensus model is appropriate. Consensus models can be thought of as representing the preferences of an average stakeholder, with some care taken to make sure that

no stakeholder's views are slighted. Developing a consensus value model is very similar to MAVT for a single decision maker. There are some defense portfolio selection contexts, such as investment in basic research programs by a Component, where consensus models may be appropriate.

## **2. Compromise**

If sharply different values or preferences exist between stakeholders, consensus will not be possible. In these cases, the analyst's challenge is to develop a compromise model that equitably balances the divergent values and preferences in a manner that is acceptable (though not ideal) to all. Compromise models do not reflect the preferences of a hypothetical "average" stakeholder. Rather, they attempt to achieve maximum value in dimensions where consensus is possible, while avoiding unacceptable treatment of values that only some stakeholders share.

Labor negotiations are the canonical example of a compromise context. While management and labor can both agree on some values (such as health of the company), there are areas (such as division of profits between management and labor) where values conflict directly. A common instance of compromise in defense planning is in the allocation of fixed budget resources to separately managed sub-portfolios. If a Component decides to spend more money on aviation and networks over a given time frame, they are necessarily deciding to spend less money in other areas that also have advocates. Similarly, allocating funds among development of future capabilities, fielding of current capabilities, and support to fielded forces requires making trades among directly conflicting constituencies.

Compromise models are more challenging to develop than consensus models. This is especially true in adversarial or zero-sum contexts, where one stakeholder's gain is necessarily another stakeholder's loss. It may be necessary to iterate through the set of alternatives under

consideration, eliminating those that are unacceptable to some stakeholders and then recalibrating the value model to reflect the range of obtainable attribute outcomes among the remaining alternatives. Both MAVT and VFT have been successfully adapted to both consensus and compromise contexts. The participation of an objective decision analyst here is crucial, both for ensuring that all stakeholder values are captured and for facilitating communication among stakeholders.

## **F. Identifying Good Portfolios**

Even after the hard work of identifying values and alternatives, defining performance measures on attributes, eliciting preferences from stakeholders, formulating constraints, and accounting for uncertainty has been completed, there remains the problem of finding the best portfolio. In general, this will require solving (at least approximately) a large, nonlinear, discrete stochastic optimization problem. In rare cases, the form of the optimization problem will be such that there is an obvious best algorithm to apply—for instance, if the problem is approximately a linear program. More generally, it will be necessary to resort to general global optimization solvers and heuristic methods.

The value function is not the only potentially challenging aspect of the formulation. In many contexts there is a complex nonlinear dependence between project schedule and costs per period. Such complex nonlinear constraints can be represented explicitly as constraints, or they can be handled using Lagrangean relaxation methods, in which a penalty in the objective serves as a surrogate for the constraint. There is a wide literature on Lagrangean methods, full of examples of successfully embedding Lagrangean techniques within other optimization frameworks.



Feasibility of portfolios is also a thorny problem in general, given the invariable uncertainties regarding cost, schedule, and available funding over time. A given proposed portfolio cannot be said to be feasible or infeasible. At best, we might be able to say that it has a certain probability of delivering a sufficient amount of value within the funding that will be available, while avoiding an unacceptable shortfall in any one capability area.

Optimizing under such a fuzzy notion of feasibility may require more advanced optimization techniques, such as stochastic programming or stochastic dynamic programming. These not only provide a systematic way of representing uncertainty in the optimization formulation, but they also make it possible to take advantage of the sequential nature of portfolio decisions. In some cases, the decision problem can be recast as “find the solution that has the best combination of outcomes in the first period plus opportunities for future outcomes, regardless of what random things happen.” Treated recursively, this allows the decision algorithm to take advantage of future opportunities to re-tune the portfolio in response to unexpected cost, schedule, or funding outcomes. This form of the solution is a good fit for how defense investment decisions are actually implemented year by year.

If the stochastic dynamic programming formulation of the decision problem is computationally intractable (as will generally be the case), approximate dynamic programming (ADP) (Powell 2012) can be an effective technique. ADP offers a potential way around the “curse of dimensionality” of dynamic programming, finding good (if not necessarily optimal) solutions to problems much more complex than can be solved by exact methods.

## **G. Performing Sensitivity and Robustness Analyses**

In general, there will be many feasible portfolios whose values are similar, and are much better than the value of a random portfolio (or even of an expert guess at the best portfolio). The

differences among these portfolios may be small relative to the uncertainties in the underlying input data, preference elicitations, and forecasts of future states of the world. The decision analyst's job is not to find the optimal solution to a mathematical programming problem, but rather to find candidate portfolios that are sufficiently feasible (in the fuzzy sense defined above) and have significantly higher value to the stakeholders than the portfolios they would have selected without formal decision support. In addition, it is vital to give those stakeholders a firm understanding of each candidate portfolio's risk position—that is, to make the stakeholders aware of exactly what bets they will be making if they select a given portfolio.

For any Portfolio Decision Analysis methodology that recommends portfolios, there are two related aspects to understanding this risk position, which we will call *sensitivity* and *robustness*. Sensitivity refers to how the composition and value of the recommended portfolio would change as a function of the input data. For example, if the probability of success of a given program were slightly higher or lower, how would that affect the set of alternatives and overall value of the recommended portfolio? If the stakeholders' assessment of the value of a particular outcome attribute had been slightly higher or lower, how would the composition of the recommended portfolio (and its value) change? Sensitivity measures how much influence specific inputs have on the nature and quality of the eventual solution. That makes it a good guide to where it is important to refine the input data to reduce uncertainty, or to take extra care in eliciting subjective preferences.

Robustness refers to how far off our estimate of the value of a specific portfolio is likely to be, given imperfect knowledge of the input data. We can think of this as a question about potential regret: "If we go with portfolio X, given the uncertainty in our assumptions, how much might we regret choosing X?" A robust recommendation is one that is liable to provide good

value—similar to the best possible portfolio—even though it was made on the basis of imperfect data.

Robustness can vary widely among portfolios with similar assessed values. This usually happens when the two portfolios have very different risk attributes. It is similar to the situation in finance when a high-risk, high-return investment and a low-risk, low-return investment might have the same risk-adjusted utility to an investor. If the risk and return characteristics of the investments are known with certainty, the investor truly is indifferent between them. But the uncertainty in (say) the true risk of default might be much higher (and less symmetric) for the high-risk investment than for the low-risk investment, making the low-risk investment more robust with regard to default risk. If our input data were exact, the two investments would be equally attractive—but our input data are not exact.

Both sensitivity and robustness can be tricky to estimate, and there is both art and science in deciding which inputs to vary, and by how much. Some optimization techniques produce supplemental outputs (such as “shadow prices”) that can be interpreted as sensitivity measurements. In other cases, we would need to generate multiple, slightly perturbed variations of the decision problem, to see how the form and value of the recommended portfolio vary with the inputs. This can be particularly useful for identifying subsets of alternatives that seem to appear in good solutions no matter how the inputs vary. These “stable subsets” can be useful both for finding robust portfolio recommendations and for reducing the complexity of the optimization by preselecting certain alternatives. Ranges for input parameters can be estimated using historical data or expert judgment, or by eliciting ranges of interest from stakeholders.

For proposed portfolios, robustness can be assessed using Monte Carlo methods, varying the input parameters according to estimated uncertainty distributions and evaluating the resulting

value (and feasibility) of the recommended portfolio. Portfolios that are robust with respect to both value and feasibility should be preferred. It may be that there is a significant expected value difference between the “best” portfolio found by optimization and the highest-value portfolio that is robust with respect to both value and feasibility.

Given the sequential and provisional nature of most defense portfolio selection contexts, a case can be made for “discounting” robustness. A portfolio that is robust with respect to near-term changes can be adjusted in the future to adapt to those changes. A portfolio whose value is sensitive to near-term changes or uncertain current parameters does not have that same ability. Consequently, it makes sense to focus more on near-term inputs when evaluating robustness for defense portfolios.

#### **4. Conclusions and Recommendations**

##### **A. The State of the Art**

We have seen that portfolio selection is a complex process. Good decision support for portfolio selection involves a sequential (and iterative) combination of

- Discovering stakeholder values;
- Defining objectives;
- Identifying the set (or space) of available alternatives;
- Quantifying attributes of available alternatives relative to the objectives;
- Eliciting stakeholder preferences;
- Characterizing interactions among attributes and alternatives;
- Quantifying risk and uncertainty in (and among) alternatives;
- Deriving a quantitative stakeholder value function;

- Identifying constraints on allowable portfolios;
- Formulating and solving an optimization model that (at least approximately) captures the values, interactions, and constraints identified above; and
- Performing sensitivity and robustness analysis on the solution.

There has been significant progress over the past few decades in both theory and practical tools for many of these iterative steps. At the front end of the process, VFT and related techniques have bridged the gap between a formal theory of rational preferences and a working tool for helping stakeholders figure out what they really want and why. Preference elicitation techniques have also improved, making it faster and easier to derive appropriate stakeholder value functions in the face of conflicting goals and diverse alternatives. Software now exists to support several of the more common elicitation techniques, such as the method of swing weights (Riabacke, Danielson, & Ekenberg 2012) or the SMART family of techniques (Edwards & Barron 1994).

At the back end of the process, there has been at least as much progress in techniques for formulating and solving large, nonlinear, discrete optimization problems. Formulations that would have been considered hopelessly intractable 20 years ago are now solved routinely. In addition, robust heuristic techniques now exist that make it possible to find good (if not necessarily optimal) solutions to an even broader class of problems. From a portfolio decision support point of view, optimization tools are no longer the limiting factor. Instead, other factors, such as formulating the value function and modeling the uncertainties, have become critical.

For Decision Analysis problems that seek to choose one best alternative, or to rank alternatives in order of preference, these advances may suffice. For defense portfolio selection, though, there remain a few weak links in the chain. The first of these is representing the set of

available alternatives when those alternatives are polytomous. Traditional Decision Analysis and Portfolio Decision Analysis deal primarily with dichotomous (yes or no) decisions over a relatively small set of alternatives, or with polytomous choices over a very small set of attributes (e.g., risk and return). No standard techniques that we are aware of deal with the large set of closely related, mutually exclusive alternatives that can arise from decision questions such as how many of each kind of plane and missile we should produce and field or what the ideal fleet of surface ships is for the Navy over the next 50 years.

A second key area where further work is needed is in representing nonlinear interactions among alternatives, such as partially redundant alternatives, synergistic alternatives, and coupled costs. Nonlinear resource interactions can generally be handled through constraints in the optimization model's feasible space, but nonlinear value interactions are more difficult. There is a limited set of value function forms that are both theoretically justified and amenable to practical elicitation.

Finally, no generally accepted technique exists to account for risk and uncertainty in portfolio selection. This is true both for uncertainty in attributes, which makes the value of a portfolio itself a random variable, and for uncertainty in resource utilization, which makes the feasibility of a portfolio difficult to characterize. The problem is made worse by the covariance among the outcome probabilities within a portfolio. Given the high stakes and relatively high levels of uncertainty in many defense portfolio contexts, this may be the most significant shortcoming of current practice.

## **B. A Challenge**

DoD spends more than \$200 billion each year on research, development, and procurement. Total National Defense expenditures are approaching \$700 billion annually. Some individual

decisions, such as the decision to build and field the F-35 Joint Strike Fighter, may commit as much as \$1 trillion of taxpayer money in a single decision. The opportunity costs of these investments across the rest of government are enormous and growing. Even marginal improvement in the quality of DoD resource allocation decisions would have tremendous value to the nation, and there is room for more than marginal improvement.

What DoD requires is a successful union of Decision Analysis and Operations Research at their natural intersection, which is Portfolio Decision Analysis. While there are several obstacles to be overcome in achieving this union, there is also much that could be done simply by combining existing tools and techniques in a systematic way. This paper attempts to identify those obstacles and to describe one possible framework for achieving an implementable end-to-end portfolio selection methodology. Section 3 provides our first attempt at Step #1—identifying best practices for each of the necessary stages. This is only a first hesitant step, however, and we challenge the Decision Science community to fill in the gaps, or show an alternative path, to be able to grapple effectively with the complexities of DoD portfolio selection. We also challenge the Decision Analysis community to develop and implement formal decision support methodologies for defense portfolio planning and management that can:

1. Identify best practices for each of the iterative stages of the defense Portfolio Decision Analysis process.
2. Identify and integrate (where possible) existing tools that support these stages.
3. Address the three key capability gaps identified in Subsection 4.A: namely, treatment of polytomous alternatives, value interactions within portfolios, and treatment of risk and uncertainty in attributes and resources.

4. Formulate the resulting selection problem as an optimization that can be at least approximately solved to identify robust solutions to the underlying defense portfolio selection problems.



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- <sup>1</sup> NASA treats cost and schedule as jointly distributed random variables that can also be coupled with mission success. Under this approach, cost and schedule are rarely considered in isolation, since each is conditional on the other. This reflects NASA's recognition of the underlying complexity of the interactions among resources, schedule, and technological accomplishment.
- <sup>2</sup> For reasons we discuss in section 3.D, we feel that in the specific context of defense portfolio selection the various types of risk and uncertainty should be treated as separate value dimensions, rather than being folded into the other values using probability distributions and expected utility (as in MAUT).
- <sup>3</sup> An example in which interactions have the same functional form would be the case where there is a constant "synergy premium" for selecting additional alternatives from a given set. In that case, the value of selecting a subset of  $M$  elements from a synergistic set would be  $(\text{sum of individual values}) \times (1 + P)^M$ , where  $P$  is the synergy premium.
- <sup>4</sup> If the attributes of failing to implement each alternative are being modeled explicitly, the eighth (empty) subset should also be included as an alternative.

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<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b>	
				<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>	
<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b>					
<b>13. SUPPLEMENTARY NOTES</b>					
<b>14. ABSTRACT</b>					
<b>15. SUBJECT TERMS</b>					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b>
<b>a. REPORT</b>	<b>b. ABSTRACT</b>	<b>c. THIS PAGE</b>			<b>19b. TELEPHONE NUMBER (Include area code)</b>

