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Phasor field waves: a mathematical treatment

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Abstract: Non-line-of-sight imaging can use modulated sources to preserve phase information at the modulation wavelength through a scattering interaction with a diffuser or an optically rough wall that scrambles the optical phase. Reza, et al. formulated the preserved information as a phasor field propagation [Opt. Express (in review)] for time-of-flight imaging of hidden scenes using diffusely scattered light. This paper presents a derivation of the underpinning of phasor field propagation to establish its rigor. The result is an expression for the time-modulated irradiance (phasor field) produced by time-modulated coherent illumination of a phase-scrambling aperture. Speckle effects, representing variance in the phasor field, are also quantified.

1. Introduction

Non-line-of-sight imaging (NLoS), as demonstrated, for example, by Velten et al. [1], attempts to exploit light scattered by a diffuse surface, such as an optically rough wall or diffuser, to image a scene hidden from direct view. Active NLoS techniques exploit structured (temporal, spectral, or spatial) illumination to preserve angular or depth information through the scattering event that defeats traditional imaging techniques, which rely on preservation of the phase of the optical wavefront. This paper is being published as a companion paper to Reza, et al. [2] to clarify and elaborate on their framework for analyzing active NLoS techniques. Reza, et al. [2–4] introduced the idea of phasor propagation to describe the transport of modulated coherent light from a phase-scrambling aperture such as a rough reflecting wall to a sensor plane in the context of time-of-flight measurement for three-dimensional scene reconstruction. Because of the linearity of light superposition, the irradiance at the sensor will oscillate with the same frequency as the light source modulation but with amplitude and phase differences. The description of the resulting irradiance as a complex vector capturing the phase and amplitude with reference to the modulation frequency is referred to as a phasor description. Short laser pulses, as employed in NLoS following Velten et al. [1], represent one example of modulated coherent light, which can be treated as a sum of Fourier component modulations. Frequency mixing to produce modulation for NLoS can also generate modulated coherent light [5]. Dove and Shapiro [6] provided a frequency domain analysis of modulated light transport in the paraxial case and extended the phasor wave description to a richer description related to Wigner distributions, which can incorporate, for instance, the coherence effects of free-space interactions with phase-preserving transmission masks such as solid objects. Their treatment clearly shows how these methods can be extended to the Fourier representation of arbitrary temporal modulation of illumination. They also show a full example of the procedure for NLoS imaging including resolution limits using their extended formulation.

Reza, et al. [2] provide additional background, more detail on applications, partial theoretical justification, important approximations, and promising simulation-based and experimental results supporting the practical validity of both the basic theory and the approximations. This paper attempts to provide stronger theoretical justification and expand the theoretical treatment by primarily addressing two mutually related issues. First, the derivation in Appendix A of [2] assumes the presence of only incoherent effects leading to an irradiance distribution that is the integral of irradiance contributions from all source points. This paper directly incorporates the phase scrambling effects of the diffuse aperture to derive an equivalent result and justify

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the assumption in [2] as well as relating the phasor field scaling constant K_P in [2] to the spatial correlation of the phase scrambling. Second, this paper derives the variance in phasor field measurements, which is due to random coherent superposition of contributions from pairs of aperture points and, therefore, requires a treatment of the aperture that accounts for coherence. Expanding the prior work [2,6] from ensemble-average results over random scatterers to measurement variance due to random variation across such an ensemble provides an important basis for understanding experimental results and generating useful measurements. Given the promise of the phasor field approach and the interest in NLoS, it is important to have a firm theoretical foundation. In what follows, the Huygens-Fresnel integral forms the basis for the time-domain derivation of a physical-optics mathematical description of phasor propagation including non-paraxial and speckle effects.

2. Phasor propagation derivation

Consider a laser source with carrier frequency ω and modulation frequency Ω and an observing sensor with integration time τ, such that $\Omega << 1/\tau << \omega$ (i.e., the sensor can resolve the modulation but not the much faster carrier). In the context of time-of-flight imaging, for c representing the speed of light, $c\tau$ should be small compared to the depth-range of the scene Δr because otherwise the time resolution would not allow meaningful measurement. However, c/Ω should be much less than Δr because otherwise returns from the entire scene would oscillate in synchrony, and the modulation would have no differentiating effect.

A scattering aperture (e.g., reflection from a rough wall or transmission through a diffuser) with scattering depth large compared to the carrier wavelength but short compared to the modulation wavelength would randomize the phase of the carrier without affecting the phase of the modulation. The scene would be illuminated by a sum of de-phased carrier contributions, but the amplitude of each de-phased contribution would oscillate at the source modulation frequency with a phase offset determined by the path-length. Due to the coherent but de-phased carrier, the illumination would produce speckle in the scene, but the expected irradiance at any scene point (averaged over an ensemble of random phase scattering surfaces) would oscillate at the modulation frequency with an amplitude equal to the sum of the irradiance contributions from each scattering point (like incoherent illumination). In any particular instantiation of the scatterer, the speckle would oscillate at the modulation frequency.

Irradiance is a non-negative quantity. It would be appropriate to treat the modulation in electric field amplitude where, neglecting phase, a variation $a(t) \sim \cos(½\Omega t)$ leads to irradiance variation $I(t) \sim \cos^2(½\Omega t) = ½[1 + \cos(\Omega t)]$, accounting for a sinusoid and the accompanying constant contribution. If the irradiance at any point oscillates with the modulation frequency, its time dependence can be expressed as a magnitude and phase with respect to the modulation frequency.

Let the electric field at a position $\vec{x}' \equiv (x', y')$ on a scattering aperture with complex transmission function $T\left(\vec{x}'\right)$ be

$$E\left(\vec{x}'\right)\cos\left(\tfrac{1}{2}\Omega t\right)\exp(-j\omega t). \tag{1}$$

From the Huygens-Fresnel integral [7], the irradiance at time t on the sensor plane due to all light incident on the aperture will be

$$I\left(\vec{x}, t\right) = \frac{1}{2\zeta}\left|\iint dA' \frac{E\left(\vec{x}'\right)\exp(-j\omega t)}{j\lambda} T\left(\vec{x}'\right)\frac{\exp(jkr')}{r'}\cos\left[\frac{\Omega}{2}\left(t - \frac{r'}{c}\right)\right]\right|^2, \tag{2}$$

where ζ is the impedance of the surrounding medium, $\vec{x}$ is a point on the sensor, $\vec{x}'$ is a point on the aperture, r' is the distance from $\vec{x}$ to $\vec{x}'$, $\lambda = \omega/c$ is the effective wavelength of the optical

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carrier, $j = \sqrt{-1}$, $k = 2\pi/\lambda$, and $dA' = dx'dy'$. Rearranging and simplifying using trigonometric identities yields

$$I\left(\vec{x}, t\right) = \frac{1}{4\zeta\lambda^2} \iint \iint dA'dA''\{ E\left(\vec{x}'\right) E^*\left(\vec{x}''\right) T\left(\vec{x}'\right) T^*\left(\vec{x}''\right) \frac{e^{jk(r'-r'')}}{r'r''} \left[\cos\frac{\Omega}{2c}(r'-r'') + \cos\Omega\left(t - \frac{r'+r''}{2c}\right)\right]\}, \tag{3}$$

where $\vec{x}''$ is a second point on the aperture with distance r'' from $\vec{x}$. With an aperture transmission function that randomly shuffles the carrier phase by ϕ,

$$T\left(\vec{x}'\right) = T_0 \exp\left[j\phi\left(\vec{x}'\right)\right], \tag{4}$$

$$I\left(\vec{x}, t\right) = \frac{|T_0|^2}{4\zeta\lambda^2} \iint \iint dA'dA'' E\left(\vec{x}'\right) E^*\left(\vec{x}''\right) e^{j\left[\phi\left(\vec{x}'\right)-\phi\left(\vec{x}''\right)\right]} \frac{e^{jk(r'-r'')}}{r'r''} \times \left[\cos\frac{\Omega}{2c}(r'-r'') + \cos\Omega\left(t - \frac{r'+r''}{2c}\right)\right]. \tag{5}$$

Since the phase shifts of the aperture are large compared to the optical carrier wavelength, the result is a net random phase, unless $\vec{x}'$ and $\vec{x}''$ are sufficiently close together to have correlated phase shifts. Note that the instantaneous irradiance in Eq. (5) only varies on the time scale of the modulation, so short-time averaging by a physical sensor on a time scale τ as discussed at the beginning of this section will not alter its measurement.

Let the phasor field be defined as the modulated measurement

$$P_\Omega\left(\vec{x}\right) \equiv \frac{\Omega}{4\pi} \int_0^{4\pi/\Omega} dt \exp(j\Omega t) I\left(\vec{x}, t\right), \tag{6}$$

which, with Eq. (5), becomes

$$P_\Omega\left(\vec{x}\right) = \frac{\Omega}{4\pi} \frac{|T_0|^2}{8\zeta\lambda^2} \iint \iint dA'dA'' \frac{E\left(\vec{x}'\right) E^*\left(\vec{x}''\right)}{r'r''} e^{j\left[\phi\left(\vec{x}'\right)-\phi\left(\vec{x}''\right)\right]} e^{jk(r'-r'')} \int_0^{4\pi/\Omega} dt\, e^{j\Omega t} \times \left[e^{j\Omega\left(t-\frac{r'+r''}{2c}\right)} + e^{-j\Omega\left(t-\frac{r'+r''}{2c}\right)}\right], \tag{7}$$

where the first bracketed cosine term in Eq. (5), which is time invariant, vanishes on modulated measurement, and the second bracketed term in Eq. (5) is expanded in the complex exponential representation of cosine. The first bracketed term in the innermost integrand of Eq. (7) vanishes on integration over a full modulation period, leaving only the second term, which can be explicitly integrated leading to

$$P_\Omega\left(\vec{x}\right) = \frac{|T_0|^2}{8\zeta\lambda^2} \iint \iint dA'dA'' \frac{E\left(\vec{x}'\right) E^*\left(\vec{x}''\right)}{r'r''} e^{j\left[\phi\left(\vec{x}'\right)-\phi\left(\vec{x}''\right)\right]} e^{jk(r'-r'')} e^{j\Omega\left(\frac{r'+r''}{2c}\right)}. \tag{8}$$

The expected value of the phasor field over an ensemble of random scattering realizations is

$$\left\langle P_\Omega\left(\vec{x}\right)\right\rangle = \frac{|T_0|^2}{8\zeta\lambda^2} \iint \iint dA'dA'' \frac{E\left(\vec{x}'\right) E^*\left(\vec{x}''\right)}{r'r''} \left\langle e^{j\left[\phi\left(\vec{x}'\right)-\phi\left(\vec{x}''\right)\right]}\right\rangle e^{jk(r'-r'')} e^{j\Omega\left(\frac{r'+r''}{2c}\right)}, \tag{9}$$

The transmission function phase scrambling is necessarily correlated over the optical wavelength scale. However, it is assumed both that the correlation is only locally supported, in the sense that it vanishes at some small spacing of aperture points, and that it depends only on the spacing of the points,

$$\left\langle e^{j\left[\Delta\phi\left(\overrightarrow{x}'\right)-\Delta\phi\left(\overrightarrow{x}''\right)\right]}\right\rangle = f\left(\left\|\overrightarrow{x}'' - \overrightarrow{x}'\right\|\right) = f\left(\left\|\Delta\overrightarrow{x}'\right\|\right). \tag{10}$$

Finally, it is assumed that the correlation area is sufficiently compact that within the correlation area the slow varying portions of the integrand in Eq. (9) remain nearly constant,

$$f\left(\left\|\Delta\overrightarrow{x}'\right\|\right) \neq 0 \Rightarrow \frac{E\left(\overrightarrow{x}'\right)}{|r'|}e^{j\Omega\frac{r'}{2c}} \approx \frac{E\left(\overrightarrow{x}' + \Delta\overrightarrow{x}'\right)}{|r' + \Delta r'|}e^{j\Omega\frac{r'+\Delta r'}{2c}}. \tag{11}$$

Utilizing these assumptions, Eq. (9) reduces to

$$\left\langle P_\Omega\left(\overrightarrow{x}\right)\right\rangle = \frac{|T_0|^2}{8\zeta\lambda^2}\iint dA'\frac{\left|E\left(\overrightarrow{x}'\right)\right|^2}{r'^2}e^{j\Omega\frac{r'}{c}}\iint d\Delta x' d\Delta y' f\left(\left\|\Delta\overrightarrow{x}'\right\|\right)e^{\pm jk\Delta r'}, \tag{12}$$

where the $\pm$ comes from switching $\overrightarrow{x}'$ with $\overrightarrow{x}''$. Taking half the sum of the two forms leads to

$$\left\langle P_\Omega\left(\overrightarrow{x}\right)\right\rangle = \frac{|T_0|^2}{8\zeta}\iint dA'\frac{\left|E\left(\overrightarrow{x}'\right)\right|^2}{r'^2}e^{j\Omega\frac{r'}{c}}\left[\frac{1}{\lambda^2}\iint d\Delta x' d\Delta y' f\left(\left\|\Delta\overrightarrow{x}'\right\|\right)\cos k\Delta r'\right]. \tag{13}$$

This is the desired phasor field propagation law corresponding to that in [2] with the bracketed term in Eq. (13) equal to twice the constant K_P from [2].

In the limit of fine-grained scattering, $f\left(\left\|\Delta\overrightarrow{x}'\right\|\right) \approx 1$ in a region of size $\sim\lambda^2$ and zero elsewhere. In the paraxial region more than a few optical wavelengths from the wall, the cosine term is also nearly one, so the entire bracketed term is approximately one. Then the expected value of the phasor field is

$$\left\langle P_\Omega\left(\overrightarrow{x}\right)\right\rangle = \frac{|T_0|^2}{8\zeta}\iint dA'\frac{\left|E\left(\overrightarrow{x}'\right)\right|^2}{r'^2}e^{j\Omega\frac{r'}{c}}, \tag{14}$$

which is the integral of the incoherently combined irradiance contribution from each aperture location with its associated modulation phase. Equation (14) is the exact analog of Eq. (5) in Reza, et al. [2] and Eq. (15) in Dove and Shapiro [6] for a given modulation frequency. For ease of comparison, note that, from Eqs. (1) and (4), the phasor field just after the scatterer is $|T_0|^2\left|E\left(\overrightarrow{x}'\right)\right|^2 \Big/ 8\zeta$. Reza, et al. [2] arrived at this result by assuming incoherent irradiance summation. Dove and Shapiro [6] arrived at this result for the paraxial case in the fine-grained scattering limit with a frequency-domain analysis equivalent to the time-domain analysis shown here and a Gaussian impulse approximation to reduce the bracketed term in Eq. (13) above. As the phase scrambling becomes coarser, the phasor phase contribution remains unchanged, but its magnitude rises in proportion to the area of nonzero correlation (due to the expanding area of support of the integral in the bracketed term of Eq. (13), which is strictly real). The region of paraxial approximation also shrinks to keep the cosine factor near one over the larger integration area. Outside the paraxial region, the bracketed term becomes a slow-varying obliquity factor. If phase scrambling is correlated over a region of size $M\lambda$ x $M\lambda$, the bracketed term is approximately $M^2\sin(\pi\sin\theta)/(\pi\sin\theta)$, where $\sin\theta = \left\|\overrightarrow{x} - \overrightarrow{x}'\right\|/r'$.

This formulation works equally well with a variable magnitude transmission function and with the full obliquity term in the Fresnel-Kirchoff diffraction formula, both of which can be considered as slow-varying contributions to $E\left(\overrightarrow{x}'\right)$.

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3. Speckle

The above results relate to the expected value of the phasor field. There will also be some "speckle" variation, which would diminish with broad-band illumination. Because the phasor field is complex-valued rather than strictly real and positive like irradiance, the variance of the measurement over the ensemble of random scattering apertures does not conform to the standard notion of speckle. The phasor field variance is given by

$$\begin{aligned}\sigma^2_{P\Omega} \equiv \left\langle \left|P_\Omega\left(\vec{x}\right)\right|^2\right\rangle - \left|\left\langle P_\Omega\left(\vec{x}\right)\right\rangle\right|^2 = \frac{|T_0|^4}{64\zeta^2\lambda^4}\iint\iint\iint\iint dA_1 dA_2 dA_3 dA_4 \{\\ E\left(\vec{x}_1\right)E^*\left(\vec{x}_2\right)E^*\left(\vec{x}_3\right)E\left(\vec{x}_4\right)C\left(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4\right)\frac{e^{jk(r_1-r_2-r_3+r_4)}}{r_1r_2r_3r_4}e^{\frac{j\Omega}{2c}(r_1+r_2-r_3-r_4)}\}\end{aligned} \tag{15}$$

from Eqs. (8) and (9), where

$$\begin{aligned}C\left(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4\right) = &\underbrace{\left\langle e^{j\left(\phi\left(\vec{x}_1\right)-\phi\left(\vec{x}_2\right)-\phi\left(\vec{x}_3\right)+\phi\left(\vec{x}_4\right)\right)}\right\rangle}_{a}\\ &-\underbrace{\left\langle e^{j\left(\phi\left(\vec{x}_1\right)-\phi\left(\vec{x}_2\right)\right)}\right\rangle\left\langle e^{j\left(-\phi\left(\vec{x}_3\right)+\phi\left(\vec{x}_4\right)\right)}\right\rangle}_{b}.\end{aligned} \tag{16}$$

In Eq. (16), a and b will be zero unless the phases in the exponentials deterministically cancel, which only occurs for aperture points in sufficiently close proximity (one "spot") to have correlated phase shifts. This leaves three distinct regions of integration where a or b may be nonzero. Adopting the notation $\Delta\vec{x}_{ij} = \vec{x}_i - \vec{x}_j$, the regions are

1. $\left\|\Delta\vec{x}_{12}\right\|$ small, $\left\|\Delta\vec{x}_{34}\right\|$ small, but $\left\|\Delta\vec{x}_{13}\right\|$ large: $a = b \neq 0$;
2. $\left\|\Delta\vec{x}_{13}\right\|$ small, $\left\|\Delta\vec{x}_{24}\right\|$ small, and $\left\|\Delta\vec{x}_{12}\right\|$ small: $a \neq 0$, $b \neq 0$;
3. $\left\|\Delta\vec{x}_{13}\right\|$ small, $\left\|\Delta\vec{x}_{24}\right\|$ small, but $\left\|\Delta\vec{x}_{12}\right\|$ large: $a \neq 0$, $b = 0$.

Region 1 represents the product of the independent deterministic phasor field contributions from each of two spots on the aperture for every pair of distinct spots. Region 2 represents the magnitude squared deterministic contribution from every single spot. Region 3 represents the magnitude squared of the random phasor contributions from coherent optical interference of every pair of distinct spots. In region 1, a and b identically cancel. In region 3, a is of order unity and b is zero. In region 2, a and b are both of order unity and nearly cancel, and region 2 is also smaller than region 3 by a factor proportional to λ^2/A. Therefore, the contribution of region 2 can be approximately neglected. In region 3, the cross-correlation term reduces to

$$\left\langle e^{j\left(\phi\left(\vec{x}_1\right)-\phi\left(\vec{x}_3\right)\right)}\right\rangle\left\langle e^{j\left(-\phi\left(\vec{x}_2\right)+\phi\left(\vec{x}_4\right)\right)}\right\rangle. \tag{17}$$

Only region 3 remains to contribute to Eq. (15), leaving

$$\sigma_{P\Omega}^2 = \frac{|T_0|^4}{64\zeta^2\lambda^4} \iint_A dA_1 \iint_{\Delta \vec{x}_1} dA_3 \iint_{A-\Delta\vec{x}_1} dA_2 \iint_{\Delta\vec{x}_2} dA_4 \Big\{ E\left(\vec{x}_1\right) E^*\left(\vec{x}_2\right) E^*\left(\vec{x}_3\right) E\left(\vec{x}_4\right) f\left(\left\|\Delta\vec{x}_{13}\right\|\right) f\left(\left\|\Delta\vec{x}_{24}\right\|\right) \frac{e^{jk(r_1-r_2-r_3+r_4)}}{r_1 r_2 r_3 r_4} e^{\frac{j\Omega}{2c}(r_1+r_2-r_3-r_4)} \Big\}, \tag{18}$$

where A is the whole aperture and $\Delta\vec{x}_i$ indicates a region within the correlation distance of $\vec{x}_i$. Since the correlation area is very small compared to the total aperture, the small exclusion region in the third double integral can be neglected. Taking the slow varying portion out of the inner integrals and following the procedure leading from Eq. (12) to Eq. (13) then leads to

$$\sigma_{P\Omega}^2 = \frac{|T_0|^4}{64\zeta^2} \left| \iint_A dA_1 \frac{\left|E\left(\vec{x}_1\right)\right|^2}{r_1^2} \left[\frac{1}{\lambda^2} \iint_{\Delta\vec{x}_1} dA_3 f\left(\left\|\Delta\vec{x}_{13}\right\|\right) \cos k\Delta r_{13} \right] \right|^2 . \tag{19}$$

In the limit of fine-grained scattering where the correlation length is approximately λ, when the paraxial approximation also holds, the bracketed term is approximately one, and

$$\sigma_{P\Omega}^2 = \frac{|T_0|^4}{64\zeta^2} \left| \iint_A dA_1 \frac{\left|E\left(\vec{x}_1\right)\right|^2}{r_1^2} \right|^2 . \tag{20}$$

4. Discussion and conclusion

The overall measurement (Eq. (8)) can be understood as depicted in Fig. 1 and by analogy to standard objective speckle [7] in the fine-grained scattering paraxial limit as follows. The three exponential terms in Eq. (8) together form a random unit phasor other than in the comparatively small integration region where the random phase contributions of the aperture are correlated. The contribution from the correlated integration region is the expected value given in Eq. (14) because, once the correlation condition is satisfied, the first two exponentials in Eq. (8) approach unity, and the third, the modulation term with a non-vanishing deterministic exponent, is preserved. As with objective speckle, the mean value is given by the incoherent sum of irradiance contributions across the scatterer, which, in this case, include the modulation phase. The remaining regions of integration in Eq. (8) form a random phasor sum, but now the three exponential terms in aggregate form a unit phasor with uniformly distributed random angle. In effect, the first exponential absorbs the other two to form a net random phase. The variance is therefore the speckle variance expected from Eq. (8) without the modulation phase. The variance of objective speckle is equal to its mean, which, without the modulation phase, would be just as in Eq. (20). Note that the variance of the phasor magnitude is larger than the mean square phasor magnitude due to the absence of the deterministic phasor phase contribution. Standard approaches to speckle diversity (including the frequency diversity implicit in pulsed laser approaches) would reduce the relative variance and may explain some of the reported success with phasor field approaches [2].

In extensions of the work of Reza, et al. [2], a firm theoretical foundation may provide greater confidence in the phasor field approach. Estimated speckle variations in phasor field measurements may aid in understanding experimental results and planning useful measurements. The large size of the variance compared with the mean suggests that speckle suppression will be important.

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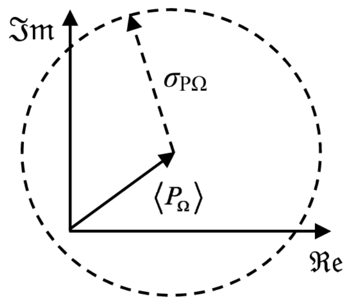

Fig. 1. Illustration of the expected phasor field (solid) with random variation (dashed) larger than the mean

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14. ABSTRACT

Non-line-of-sight imaging can use modulated sources to preserve phase information at the modulation wavelength through a scattering interaction with a diffuser or optically rough wall that scrambles the optical phase. Reza, et al. formulated the preserved information as a phasor field propagation [Opt. Express (in review)] for time-of-flight imaging of hidden scenes using diffusely scattered light. This paper presents a derivation of the underpinning of phasor field propagation to establish its rigor. The result is an expression for the time-modulated irradiance (phasor field) produced by time-modulated coherent illumination of a phase-scrambling aperture. Speckle effects, representing variance in the phasor field, are also quantified.

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