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# Executive Summary

To sustain the essential civilian economy and provide for the national defense during a national emergency scenario, the United States Department of Defense (U.S. DoD) maintains a stockpile of strategic and critical materials that are key to the functionality of the national industrial base. A biennial report provided to the U.S. Congress by the Office of the Secretary of Defense (OSD)[1] guides the selection of the types and quantities of materials in the national defense stockpile. The National Defense Stockpile (NDS) report includes an extensive aggregation of analytical products and data sets. A primary component of the report is the estimates of potential gross material shortfalls, or the differences between material demands and supplies, under the assumptions and prescriptions of the Base Case planning scenario.[2] Mandated by Congress, this planning scenario includes a variety of detailed military and economic assumptions that are applied in modeling the consumption, supply, and shortfall quantities for a set of strategic and critical materials.

The law (i.e., the National Defense Stock Piling Act) requiring the NDS report also mandates the examination of a variety of excursion or parametric sensitivity studies of the Base Case as part of the report. In compliance with this aspect of the law and as an analysis supplement, some previous NDS reports, including the *2015 NDS Requirements Report*, included a bounded uncertainty analysis of U.S. economic forecasts to augment the excursion studies. To promote consistency with other government analyses, the economic modeling documented by the NDS report is based on the forecasts of the Council of Economic Advisers of the President of the United States. Due to this, there is a potential analytical sensitivity of the NDS report's conclusions to the accuracy of the macroeconomic forecast delivered by the CEA.

To explore this sensitivity issue, the baseline macroeconomic forecast that supports Base Case material demand calculations is systematically perturbed and used to recalculate material shortfalls, characterizing the sensitivity of material shortfalls to macroeconomic forecast uncertainty. Ultimately, this process leads to a better understanding of the effect of economic forecasting errors, which are an equivalent metric of forecast uncertainty, on

[1] The National Defense Stock Piling Act mandates the National Defense Stockpile; see 50 U.S. Code 98, "War And National Defense – sec. 98b, 'National Defense Stockpile,' " 2009 ed.

[2] The Base Case is a national security emergency planning scenario agreed upon by the U.S. DoD for use as a set of policy and technical assumptions in DoD analyses.

the material shortfall estimates contained in the *Analyses for the 2015 National Defense Stockpile Requirements Report to Congress on Strategic and Critical Materials*.[3] While many approaches to this analysis are reasonable, a specific method for examining uncertainty was consistently implemented in past NDS reports and thus reused for the *2015 NDS Requirements Report*. The goal of this paper is to illuminate and quantify the sensitivity of the NDS recommendations to macroeconomic forecasting errors, and to describe the method used to determine the sensitivity.

3 James S. Thomason et al., *Analyses for the 2015 National Defense Stockpile Requirements Report to Congress on Strategic and Critical Materials; Volume 1: Material Assessments and Associated Analyses*, IDA Paper P-5190, Draft Final (Alexandria, VA: Institute for Defense Analyses, August 2015).

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# 1. Introduction

Every two years, the Secretary of Defense (SECDEF) is required to deliver a biennial report on the status and needs of the National Defense Stockpile (NDS) to the United States (U.S.) Congress. The NDS report is mandated by the Strategic Materials Stock Piling Act. SECDEF has designated the Defense Logistics Agency (DLA) Strategic Materials as the lead agency tasked with preparing the NDS report for the SECDEF's approval. In addition to producing the biennial NDS report, DLA Strategic Materials is responsible for procuring, maintaining, and managing the NDS. To produce the NDS report, DLA Strategic Materials asks the Institute for Defense Analyses (IDA) to conduct core analyses and research on the requirements recommendations in the NDS report. This paper documents a portion of those analyses conducted by IDA for DLA Strategic Materials.

An important consideration in the biennial preparation of the NDS report is the sensitivity of the report's findings to policy, strategic, military, and economic assumptions made in the report's analysis. These assumptions stem from a wide range of data sets, policies, and directives furnished by several government agencies and private organizations. Ultimately, the report delivers recommendations for the appropriate composition and quantity of the nation's strategic defense stockpile of critical materials. Logically, the forecasted economic demand and output of the United States over the period analyzed in the report is a major driver of these recommendations. The nominal set of assumptions, scenario definitions, and data that provide the foundation for the NDS report are referred to as the *Base Case*. A substantial component of the Base Case consists of the macroeconomic forecasts for the U.S. economy over the future years to which the Base Case analytical scenario applies. These forecasts are manifested in the output of the macroeconomic simulations employed in the NDS analysis, but they also drive the macroeconomic simulations themselves.

To promote consistency with other government analyses, the NDS report leverages the forecasts of the Council of Economic Advisers (CEA) of the President of the United States. Twice a year CEA, together with the Office of Management and Budget (OMB) and the U.S. Department of Treasury, develops macroeconomic projections of the U.S. economy. These projections reflect the Administration's economic agenda and are published as a summary with the annual *Economic Report of the President*.[4] They also coincide with the production of estimates for the *Budget of the United States Government* and the *Mid-Session Review of the Budget*.[5] The CEA provides IDA with a comprehensive set of forecasts for key macroeconomic indicators compiled on a National Income and Product Accounts (NIPA) basis. While other trusted sources of macroeconomic forecast data are available from both private and alternative federal institutions, the NIPA breakdown of the CEA forecasts is consistent with the president's budget and other government analyses and is sufficiently detailed.[6] Thus, CEA forecasts are uniquely instrumental in IDA's analysis.

One issue arising from this process is the accuracy of the macroeconomic forecast delivered by CEA. Every fiscal year, OMB publishes a volume of analytical perspectives supplementing the *Budget of the United States Government*.[7] This volume provides analysis on the historical error trends of various government and private sector forecasts. According to these analyses, the CEA macroeconomic forecasts historically exhibit a similar level of accuracy to the forecasts and an average of private sector, blue chip forecasts. Non-negligible errors exist between the actual and predicted macroeconomic status of the U.S. economy. The goal of this IDA paper is to illuminate and quantify the sensitivity of the NDS recommendations to these forecasting errors.

### Methodology

Several potential avenues for examining NDS sensitivities to macroeconomic model uncertainty exist along a continuum of technical complexity. For example, if in-depth statistical data characterizing model errors were available, Monte Carlo methods would be a natural fit. Although rigorous and well proven, these methods are technically complex, time consuming, computationally intensive, and potentially excessive for the purposes of the NDS work. Contrasting these methods, bounding analysis offers a more suitable first-order approach for developing insights into the variance of stockpile recommendations as a function of macroeconomic model uncertainty. In bounding analysis, the maximum and minimum bounds for macroeconomic model parameters are established based on the available data quantifying historical uncertainty. Then, using these bounds, alternative model results are generated to represent the envelope of likely possibilities.

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[4] Barrack Hussein Obama II, *Economic Report of the President* (Washington, DC: U.S. Government Printing Office, 2015), https://www.whitehouse.gov/administration/eop/cea/economic-report-of-the-President.

[5] Office of the Management and Budget (OMB), *2015 Budget of the United States Government*, (Washington, DC: U.S. Government Printing Office, 2015), https://www.whitehouse.gov/omb/budget/Overview.

[6] In this paper, Appendix A offers a brief introduction to NIPA.

[7] OMB, *2015 Budget*, https://www.whitehouse.gov/omb/budget/Analytical_Perspectives.

For the NDS report methodology, at least two variations on the bounding analysis are feasible:

- In an open-loop approach, the baseline economic model outputs are modified to capture modeling uncertainty bounds. This method is attractive to the extent that it does not require re-running the entire macroeconomic modeling and simulation suite; for this reason alone, it introduces a number of simplifying assumptions.
- Alternatively, using a closed-loop approach involves modifying model inputs and completely rerunning the macroeconomic simulations to produce true parameterized representations of maximum and minimum model bounds. While this method may present greater accuracy, it is also more time consuming and computationally intensive.

This paper discusses both analytical techniques and offers some derivations to relate the techniques to implicit economic theoretical assumptions. Furthermore, this paper compares and discusses the available data sources to support various uncertainty studies, such as the bounding analyses described previously. Finally, the specific analysis[8] conducted for the *2015 NDS Requirements Report* is presented, along with the resulting changes in major NDS report findings as an outcome of the sensitivity study.

Chapter 2 provides an overview of the various possible technical approaches to modeling uncertainty in the macroeconomic analysis of the NDS report, along with a discussion of their relative merits. Chapter 2 also offers a general overview of the structure and theoretical basis for the economic data used in this analysis. Further, chapter 2 explains and discusses the specific uncertainty methodology employed in the *2015 NDS Requirements Report*. Chapter 3 includes the results of this uncertainty analysis and compares to the data generated using the Base Case. Conclusions and recommendations for future analysis protocols are presented in Chapter 4. These focus on (1) the results of IDA's investigation of the legacy method of modeling macroeconomic forecast uncertainty for the *2015 NDS Requirements Report*, and (2) the finding that the worst-case historic forecast uncertainties can lead to significant differences in material shortfall results.

Appendix A provides a summary of the theory of input/output accounting, and Appendix B is a primer on the national income and product accounts. Appendix C consists of a theoretical discussion and proofs that are the foundation of the analysis techniques proposed in this paper. Appendix D is an overview of the data sources used to support the uncertainty analysis results of chapter 3. Appendix E contains a list of figures and tables,

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[8] James S. Thomason et al., *Analyses for the 2015 National Defense Stockpile Requirements Report to Congress on Strategic and Critical Materials; Volume 1: Material Assessments and Associated Analyses*, IDA Paper P-5190, Draft Final (Alexandria, VA: Institute for Defense Analyses, August 2015).

while Appendix F contains a list of references. Appendix G lists the abbreviations used within this paper.

# 2. Economic Uncertainty Analysis

IDA employs a suite of macroeconomic models, the Long-term Inter-industry Forecasting Tool (LIFT) and Inter-industry Large-scale Integrated and Dynamic Model (ILIAD), to perform the economics analysis of the NDS study. Developed by the Interindustry Forecasting Project at the University of Maryland (INFORUM), these models enable the systematic decomposition of aggregate macroeconomic metrics into industry-level categories. IDA researchers calibrate the models to be consistent with the detailed NIPA breakdown of the CEA's economic forecasts, and then apply them to compute the corresponding projections of industry-level final output requirements. These output requirements are used in subsequent calculations of raw material demands that drive the NDS recommendations, and specifically, the types and amounts of the strategic and critical materials needed to be stockpiled for national defense purposes. This chapter explains the methodology developed to study the sensitivity of NDS recommendations to the uncertainty inherent in the CEA's economic forecasts.

## A. Model Overview

The input/output (I/O) framework of the LIFT and ILIAD models is instrumental to the economic analyses of the NDS study in two ways:

- the I/O framework is a systematic method to study the interdependencies between all industrial sectors in the economy; and
- the I/O framework provides a direct link between the industry-level activities and aggregate macroeconomic indicators, represented by the National Income Identity (NII).

A detailed technical overview of Leontief I/O modeling is provided in Appendix B. This section provides a more specific discussion of the role of I/O models and the NII in the NDS analysis.

Let $i$ be the industry index and $n$ denote the total number of industrial sectors in the economy. The most recent version of ILIAD models the U.S. economy across 360 industrial sectors and therefore $n = 360$. Let $y_i$ denote the value of the total output of industry $i$. Each industry's output, $y_i$, can be either bought by other industrial sectors of the economy for the production of other goods, or consumed by end-users.[9] Let $z_{i,j}$ denote

[9] In I/O analysis, values are recorded typically in monetary terms for consistency.

intermediate demand representing the purchase of industry $i$'s output by industry $j$ that is used in the production of industry $j$'s output. Let $f_i$ denote final demand representing the purchase of industry $i$'s output by the end-users. Therefore, the total output of industry $i$ can be represented by

$$y_i = z_{i,1} + \cdots + z_{i,j} + \cdots + z_{i,n} + f_i = \sum_{j=1}^{n} z_{i,j} + f_i. \quad (2.1)$$

The *final aggregate demand* for industry $i$'s production, $f_i$, includes purchases by U.S. households, businesses, the government, and foreign entities. Final demand is defined by the NII as the sum of consumer spending (*C*), investment spending (*I*), government spending (*G*), and net exports (*NX*). Net exports (*NX*) are equal to exports (*EX*) minus imports (*IM*). From the perspective of the demand side, one can represent $f_i$ as follows:

$$f_i = f_i^C + f_i^I + f_i^G + f_i^{NX} \quad (2.2)$$

where $f_i^C$, $f_i^I$, $f_i^G$, and $f_i^{NX}$ denote the final demand of industry $i$'s products from households, businesses, the government, and foreign entities respectively.

Summing final demands across all $n$ industries yields the NII:

$$\sum_{i=1}^{n} f_i = \sum_{i=1}^{n} f_i^C + \sum_{i=1}^{n} f_i^I + \sum_{i=1}^{n} f_i^G + \sum_{i=1}^{n} f_i^{NX} = \\ C + I + G + NX = GD2P. \quad (2.3)$$

Therefore, a direct link between industry-level activities and the aggregate economy is established via the above derivation and summarized by

$$GDP = \sum_{i=1.}^{n} f_i \quad (2.4)$$

To support the objectives of the NDS analysis, final demand for each industry is categorized differently than in the NII, although in a related way. In particular, for the purpose of NDS analysis, final demand for each industry consists of civilian, defense, export, and import demands. Government spending (*G*) in NII is subcategorized as the sum of federal defense spending (*DEF*), federal nondefense spending (*NDEF*), and state and local spending (*SL*) spending:

$$f_i^G = f_i^{DEF} + f_i^{NDEF} + f_i^{SL} \quad (2.5)$$

and net export (*NX*) is equal to export (*EX*) minus import (*IM*):

$$f_i^{NX} = f_i^{EX} - f_i^{IM}. \quad (2.6)$$

This yields to an expanded version of equation (2.2):

$$f_i = f_i^C + f_i^I + \left(f_i^{DEF} + f_i^{NDEF} + f_i^{SL}\right) + (f_i^{EX} - f_i^{IM}). \tag{2.7}$$

Let civilian demand for any industry $i$ be defined as follows:

$$f_i^{CIV} = f_i^C + f_i^I + f_i^{NDEF} + f_i^{SL}. \tag{2.8}$$

Total final demand for any industry $i$ is then the sum of civilian (CIV), defense (DEF), and export demand (EX) for industry $i$ minus import demand (IM):

$$f_i = f_i^{CIV} + f_i^{DEF} + f_i^{EX} - f_i^{IM} \ . \tag{2.9}$$

Industry-specific forecasts of final demands and their corresponding demand components on a NIPA account basis[10] are computed by calibrating inputs to the INFORUM models to match the CEA macroeconomic projections. In particular, the final demand values $f_i^{CIV}$, $f_i^{DEF}$, $f_i^{EX}$, and $f_i^{IM}$ are of first-order importance as they serve as inputs to subsequent calculations for output requirements and material demands.

Finally, let $CIV$, $DEF$, $EX$, and $IM$ denote the *total spending* in civilian, defense, export, and import sectors across all $i$ industries. Employing equations (2.4) and (2.8), the gross domestic product (GDP) is now represented by the following:

$$\begin{aligned} GDP &= \sum_{i=1}^{n} f_i = \sum_{i=1}^{n}\left(f_i^{CIV} + f_i^{DEF} + f_i^{EX} - f_i^{IM}\right) \\ &= \sum_{i=1}^{n} f_i^{CIV} + \sum_{i=1}^{n} f_i^{DEF} + \sum_{i=1}^{n} f_i^{EX} - \sum_{i=1}^{n} f_i^{IM} \\ GDP &= CIV + DEF + EX - I. \end{aligned} \tag{2.10}$$

[10] The NIPA account breakdown of the GDP (as described in Appendix B) offers a higher level of detail than the NII.

## B. The Forecast Uncertainty Assessment Methodology

To capture the CEA's forecast uncertainty, IDA researchers leverage the official statistics provided in the *Analytical Perspectives* volume with the *Budget of the United States Government*.[11] *Analytical Perspectives* reports historical forecast errors of CEA's prediction on the growth rate of real GDP. The following section details a systematic procedure to translate the measure of forecast uncertainty from the aggregate (*GDP*) level to the industry level. This approach focuses on quantifying the implications on NDS recommendations when the economy experiences a higher (or lower) than expected growth. The analytical procedure consists of a series of steps.

- First, a new equilibrium *GDP* corresponding to the high (or low) economic growth case is computed.
- Second, a scale factor is calculated as a function of the baseline (original) equilibrium *GDP*, new equilibrium *GDP*, and defense spending, which is assumed to be held at the baseline level because of policy constraints.[12]
- Next, this scale factor is used to calculate civilian, export, and import demands for each industry under the new equilibrium.
- Finally, output requirements and material demands corresponding to the new equilibrium *GDP* are computed.

To compute the new equilibrium *GDP* corresponding to the high (or low) economic growth case, first note that, with the baseline CEA macroeconomic projections, the following baseline values have been produced by the economic simulations via the INFORUM models for each year $t$ of the scenario period:

$GDP_t$: U.S. *GDP* in year $t$;

$f_{i,t}^{CIV}$: civilian demand for the products of industry $i$ in year $t$;

$f_{i,t}^{DEF}$: defense demand for the products of industry $i$ in year $t$;

$f_{i,t}^{EX}$: exports of the products of industry $i$ in year $t$; and

$f_{i,t}^{IM}$: imports of the products of industry $i$ in year $t$.

[11] Office of the Management and Budget (OMB), *2015 Budget of the United States Government* (Washington, DC: U.S. Government Printing Office, 2015), https://www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode=BUDGET.

[12] Appendix C contains the proof of the validity of the scale factor and formalizes the necessary economic assumptions.

Let $g_t$ denote the growth rate of real *GDP* implied by the baseline CEA forecast in year *t*. Thus,

$$g_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}. \tag{2.11}$$

Equivalently,

$$GDP_t = GDP_{t-1}(1 + g_t). \tag{2.12}$$

The measure employed to characterize forecast uncertainty of CEA's baseline economic projections is the *root mean square error (RMSE)* (in percentage terms) of the average annual growth rates for real *GDP*, documented in the *Analytical Perspectives* volume with the *Budget of the United States Government.*[13] Let $r$ denote this measure and call it the uncertainty parameter. Given the baseline forecast of $GDP_t$, a set of possible values of actual *GDP* in year $t$ is implied by a parametrized value of $r$. The maximum and minimum values in this parameterization are the equilibrium *GDP* in the cases in which the economy grows at a rate $r\%$ respectively higher or lower than expected.

To calculate the new equilibrium *GDP*, first consider the high economic growth case. Suppose from year $t-1$ to $t$, the real *GDP* grows at $r\%$ higher than the baseline CEA projections. Let $GDP_t^+$ represent the equilibrium *GDP* in such a case. Then,

$$GDP_t^+ = GDP_{t-1} * (1 + g_t + r\%). \tag{2.13}$$

Similarly, for the low economic growth case, the new equilibrium *GDP* is computed by

$$GDP_t^- = GDP_{t-1} * (1 + g_t - r\%). \tag{2.14}$$

The above equations can be used recursively when the analysis of interest spans multiple years. For notational simplicity, let $GDP_t'$ be alternative equilibrium *GDP* in year $t$ representing either $GDP_t^+$ or $GDP_t^-$ for the high or low economic growth case.

Next, recall that the *GDP* can be represented as the sum of *total spending* in civilian, defense, export, and import sectors by

$$GDP_t = CIV_t + DEF_t + EX_t - IM_t \tag{2.15}$$

and each of the right-hand side value can be calculated by summing $f_{i,t}^{CIV}$, $f_{i,t}^{DEF}$, $f_{i,t}^{EX}$ and $f_{i,t}^{IM}$, respectively, across all $n$ industries.

[13] Appendix D has additional information on the *Analytical Perspectives* data and provides an explanation for the measure employed.

Suppose due to forecast error, the economy is at a different equilibrium and the new equilibrium gross domestic product is $GDP_t'$. One can calculate how total spending in each sectors varies under the following conditions:

- The defense spending is held at the baseline level because of policy constraints.
- Each of the other sectors of the economy, namely, the civilian and the international trade (export and import) sectors, takes up the same fraction of the non-defense economy in both the baseline economy (with $GDP_t$) and the new economy (with $GDP_t'$).

To be specific, consider civilian spending $CIV_t$. In the baseline economy, the civilian sector takes up a fraction $\frac{CIV_t}{GDP_t - DEF_t}$ of the *non-defense* economy. At the new equilibrium, the *non-defense* economy is represented by $GDP_t' - DEF_t$ since defense spending is held at the baseline level. Therefore, the adjusted total civilian spending is calculated by

$$CIV_t' = \left(\frac{CIV_t}{GDP_t - DEF_t}\right) * (GDP_t' - DEF_t) = CIV_t * \left(\frac{GDP_t' - DEF_t}{GDP_t - DEF_t}\right). \tag{2.16}$$

Adjusted export and import spending can be calculated using the same equation. Therefore, for all sectors of the *non-defense economy*, the scale factor applied to obtain the respective adjusted total spending is

$$\gamma_t \equiv \frac{GDP_t' - DEF_t}{GDP_t - DEF_t}. \tag{2.17}$$

Finally, the scale factor $\gamma_t$ is transitive from the aggregate level to the industry level under some commonly used economic assumptions.[14] For the NDS report, the new equilibrium industry-by-industry final civilian, export, and import demands are calculated by multiplying the baseline final demand values by the scale factor and the by-industry defense demands remain the same. In short,

$$f_{i,t}^{h\,\prime} = \gamma_t f_{i,t}^h \tag{2.18}$$

and

$$f_{i,t}^{DEF\,\prime} = f_{i,t}^{DEF} \tag{2.19}$$

for $h \in \{civ, ex, im\}$.

[14] Appendix C provides theoretical foundation for this transitivity.

This procedure is applied to each year of interest and can be generalized recursively to conduct sensitivity analysis of case-specific scenarios that span multiple years. For example, the *2015 NDS Requirements Report* explores the maximum and minimum economic growth cases. The maximum (and minimum) economic growth case is one in which the economy grows at a higher (lower) rate than expected throughout the scenario period. The corresponding equilibrium results for output requirements and material demands are computed to provide the widest possible bounds that characterize the "envelop" of likely possibilities. Chapter 3 presents the shortfall values of the maximum and minimum economic growth cases. Similarly, the procedure can be applied to analyze other sensitivity cases such as one in which the economy experience higher than expected growth in some year and lower growth in other years.

## C. Approaches to Alternative Uncertainty Assessments

By definition, *homothetic preferences* imply that in the same period consumers with different incomes but facing the same prices will demand goods in the same proportions. In terms of IDA's uncertainty analysis, this means that if *GDP* increases by a given percentage, final demand in each sector (and therefore material requirements) will increase by the same percentage. This assumption greatly simplifies the economic uncertainty analysis. Without this assumption, however, it is still possible for IDA researchers to forecast material requirements in the case of higher or lower than predicted *GDP* growth. Two alternative methods, an open-loop approach and a closed-loop approach, are explored in this section.

To understand the differences between the previously described uncertainty analyses and methods that dispense with the homothetic preference assumption, we contrast the procedural architecture of the two approaches. Figure 1 illustrates the general process for the uncertainty analysis described in section 2.B, while Figure 2 illustrates the alternative process that is necessary when homothetic preference assumptions are dropped.

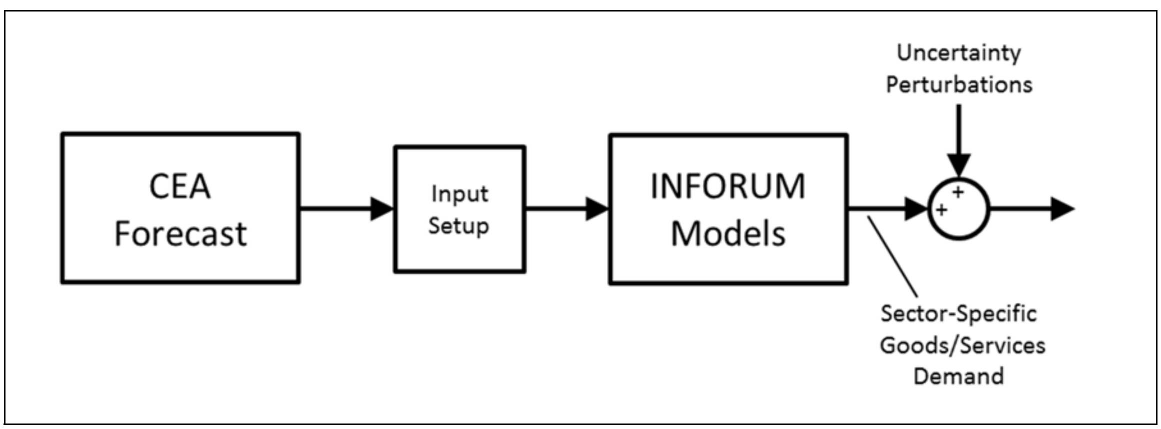


**Figure 1. Open-Loop Uncertainty Model Structure**

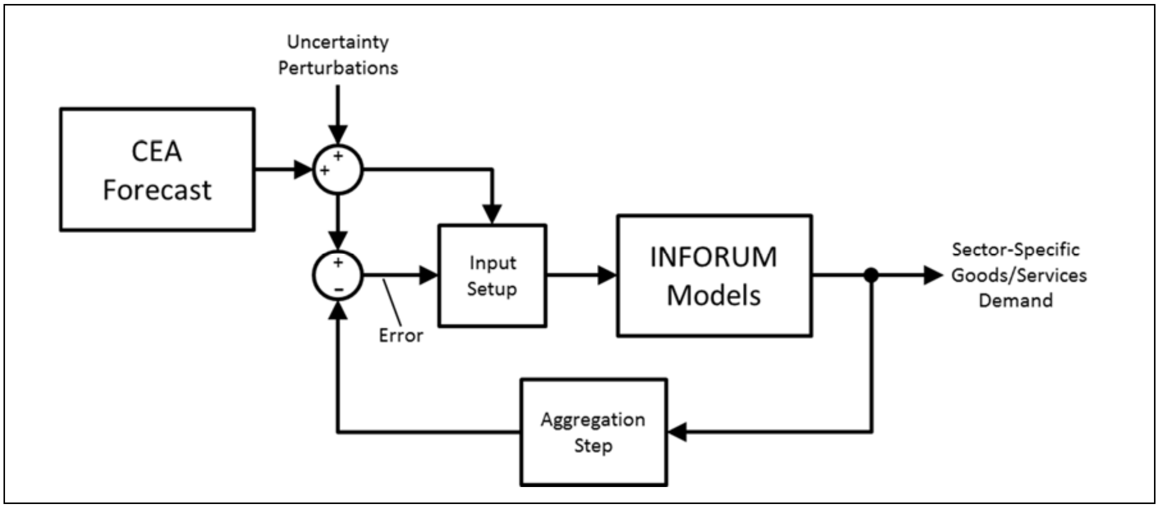


**Figure 2. Closed-loop Uncertainty Model Structure**

The open-loop uncertainty analysis perturbs the output of the INFORUM macroeconomic models after they converge to a general equilibrium solution. A valid manipulation because of homothetic preference assumptions, the output is perturbed to characterize uncertainty bounds at the industry sector, as opposed to macroeconomic, level. Using this approach, the relative relationships between industry-level goods and services demand are preserved from the simulation solution for the baseline economy and proportionally scaled to reflect a larger or smaller total economic growth rate.

In the alternative closed-loop approach depicted in Figure 2, uncertainty bounds or perturbations are applied at the macroeconomic level and prior to simulation in the INFORUM models. A feedback loop of model outputs is closed to compare them with the desired perturbed macroeconomic variables. The error signal formed from this comparison is used to adjust simulation parameters, and a new general equilibrium solution is recomputed. Repeating this process until *convergence*, a set of sector level demands are produced that reflect major macroeconomic perturbations, but that also reflect a new economic equilibrium between industry sectors as simulated by the INFORUM models.

While a closed-loop approach might offer greater modeling rigor and theoretical accuracy than an open-loop approach, the computational and analytical effort required for such a method is significantly increased. Multiple simulation iterations, involving repeated model parameter adjustments, are required. To some extent, this process may be automated through customized programs or mathematical software packages, but the final algorithm still requires much more computational power than the more straightforward calculations discussed in chapter 2. With these caveats, several variations on the alternative closed-loop approach are possible, with two specific possibilities of interest outlined in the next two sections.

### 1. Alternative Approach No. 1: Uniform Scaling of NIPA Components

In IDA's analysis, the major NIPA components of *GDP* that are extracted from the CEA's forecast are (1) personal consumption expenditures, (2) gross private domestic investment, (3) net exports, and (4) government investment and consumption expenditures. One alternative to scaling output *GDP* growth rates by their historical error would be to scale *GDP* and NIPA components prior to their input as INFORUM model parameters. In this situation, government investment and consumption expenditures would not be changed. This is because of the assumption that the government budget is not altered when the economy experiences higher or lower growth than expected.

When using a closed-loop architecture, the INFORUM models would be run for each bounding case of higher or lower than expected growth. Each run, as in the Base Case, would start by executing LIFT. In doing so, analysts would calibrate government spending to the baseline budget and adjust the other three main *GDP* components to their new values as LIFT parameters. LIFT would then map the economy to 110 sectors. As in the Base Case, LIFT outputs would be input to ILIAD, which would further disaggregate the economy into 360 sectors. The outputs from ILIAD would be used to derive material requirements using the same material consumption ratios as done before.

This approach does not assume that each sector grows at the same rate. It has the advantage of recalculating output in each sector relative to either more or less growth in each major macroeconomic component. However, this approach does assume that as *GDP* increases, personal consumption expenditures, gross private domestic investment, and net exports grow at the same rate. This is a rather large assumption that is likely false. Alternative Approach #2 aims to work around this premise.

### 2. Alternative Approach No. 2: Proportional Scaling of NIPA Components

Another variation on the closed-loop approach is scaling the major NIPA components by their individual historical errors, rather than using overall *GDP* growth errors. This may be the more accurate of the two approaches. First, one would need to look at the last six years of CEA forecasts and BEA reports to compare predicted measures with realized outcomes. As an example, for personal consumption expenditures, gross private domestic investment, and net exports totals, a six-year average RMSE could be calculated. The values of these three components would then be increased and decreased by their historical annual RMSE. Assumptions about the defense budget still hold so government investment and consumption expenditures would not be changed. Next, the INFORUM suite of models would be run twice more for each bounding case of higher or lower than expected growth fixing government spending to the baseline projections and the other three main *GDP* components to their new values. The outputs from ILIAD would be used to derive material requirements using the same material consumption ratios as done before.

This approach is superior to the previous alternative in that it makes fewer assumptions about the growth of major components of *GDP*. However, in addition to this approach being more time consuming, data on the major components of *GDP*'s predicted and realized outcomes for the last six years is not currently available to IDA researchers.

# 3. 2015 NDS Uncertainty Analysis Results

The *2015 NDS Requirements Report* projects future requirements for strategic and critical materials based, in part, on a long-range forecast of the U.S. economy, and is thus constrained by the uncertainty inherent in the forecast. The Base Case utilizes a macroeconomic forecast published and used by economists in the Federal Government. To obtain an estimate of the maximum and minimum bounds of the sensitivity of materials requirements estimates to macroeconomic forecast uncertainty, IDA researchers analyzed two sensitivity cases with higher and lower macroeconomic growth than the Base Case. One of these cases assumes that the annual economic growth rate is 1.1% higher than the Base Case forecast growth rate, while the other case assumes that the growth rate is 1.1% lower than in the Base Case.

The changes in growth were applied starting in 2014 and extending throughout the scenario period (2017–2020) using the calculation procedure described in section 2.B. Growth in U.S. output could naturally be construed as also affecting the U.S. supply of materials, but that possibility is not accounted for in the IDA analysis: U.S. material supply remains the same as in the Base Case. The results are in Table 1 where shortfalls represent totals over the

**Table 1. Sensitivity Cases – Economic Growth**

| SSM Run No. | Description | No. of Materials with Shortfall | Value of Shortfall ($M) | Percent of Base Case Shortfall Value |
|---|---|---|---|---|
| 1 | Base Case | 23 | 2,638 | 100 |
| 150 | Higher Economic Growth (1.1% higher per year) | 26 | 3,117 | 118 |
| 151 | Lower Economic Growth (1.1% lower per year) | 21 | 2,212 | 84 |

It is evident that a seemingly minor change in the growth rate has a significant effect on the shortfall of the strategic material supply. For the 68 materials examined, the total shortfall value in the higher economic growth case is 18% higher than for the Base Case. The number of materials with shortfalls rises. In the lower economic growth case, the total shortfall is 16% lower than the Base Case. Note: These changes are overwhelmingly in the civilian shortfall. The defense shortfall total exhibits small changes.

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# 4. Conclusion

As part of its role in preparing a draft NDS report in collaboration with DLA Strategic Materials, IDA conducts a set of nominal and perturbed macroeconomic studies to generate detailed industrial demand data for use in the report's analytical products. A major component of these studies is the examination of the sensitivity of the macroeconomic simulation results to uncertainties and errors in the macroeconomic forecasts upon which the simulations are based. IDA bases its nominal analyses on annual CEA forecasts of major U.S. macroeconomic variables provided on a NIPA basis. A historic assessment of the accuracy of government and private forecasts provided by the Congressional Budget Office (CBO) was applied for sensitivity studies. The statistical and economic data available from the CBO and the CEA was described and summarized to convey the empirical framework of this study. A variety of reasonable approaches to performing uncertainty analyses based on the error data are conceivable and this paper outlined some of those methods. If very detailed, industry and sector level forecast uncertainty statistics were available, rigorous Monte Carlo simulation approaches could be made available. Such simulation techniques were not discussed in depth, however, partially because of lack of data to support them. Alternatively, error-bounding type analyses can be used and these were discussed in varying levels of detail.

Generally, the error bounding sensitivity analyses considered in this paper are possible to categorize as open-loop and closed-loop methods. In an open-loop approach, the outputs of the nominal macroeconomic simulations are systematically scaled and manipulated in post-processing steps to characterize variability. In closed-loop approaches, uncertainty data is applied to alter simulation inputs, and the entirety of the macroeconomic simulation process is fully repeated using these perturbations. This paper focused on the open-loop approaches due to their relative simplicity, speed, and computational efficiency. Even using open-loop methods, however, several variations are available. These options were described, in some cases theoretically analyzed, and discussed. Section 2.B detailed the specific method employed for the *2015 NDS Requirements Report*, with the results of the uncertainty analysis compared to the nominal analysis results in chapter 3.

The simplicity, computational speed, and limited supporting data requirements of open-loop bounding uncertainty analyses make them attractive when compared to more intensive closed-loop and Monte Carlo techniques. The closed-loop bounding approaches, however, imply a set of statistical and economic assumptions that may be considered restrictive or limit the ultimate accuracy of the analytical conclusions that are based on them. Assuming the availability of the required uncertainty data, one goal of future research is to develop a better understanding of the differences in the results of closed-loop analytical methods compared to alternative open-loop techniques. For instance, a tractable hybrid uncertainty model might seek to vary top-line macroeconomic variables in a bounding analysis at the input of the macroeconomic simulation using a closed-loop approach. The material shortfall calculated from this model could then be compared to the post-processing results detailed in this IDA paper. This is an area of future interest for IDA researchers and of importance to the sponsor, DLA Strategic Materials. If additional analysis reveals that significant differences exist between the shortfall predictions of various uncertainty modeling approaches, the formalized approach approved by the sponsor for uncertainty analysis might be reconsidered.

# Appendix A
# The Basic Framework of Input/Output Analysis

Leontief input/output (I/O) analysis is a macroeconomic technique that shows the interdependencies of all sectors in an economy. In its most basic form, it is a system of $n$ linear equations with $n$ unknowns. Each equation describes the distribution of an industry's product throughout the economy, to other industries and to itself. This information is easiest to visualize in a transactions table as shown in Table A-1. This table represents a hypothetical economy composed of five industries.

**Table A-1. I/O Transactions Table**

| | | Producers as Consumers | | | | | Final Demand | | | | Total Output |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Industry 1 | Industry 2 | Industry 3 | Industry 4 | Industry 5 | Personal Consumption Expenditures | Gross Private Domestic Investment | Govt. Purchases of Goods and Services | Net Exports of Goods and Services | |
| Producers | Industry 1 | | | | | | | | | | |
| | Industry 2 | | | | | | | | | | |
| | Industry 3 | | | | | | | | | | |
| | Industry 4 | | | | | | | | | | |
| | Industry 5 | | | | | | | | | | |
| Value Added | | | | | | | | | | | |
| Total Outlays | | | | | | | | | | | |

*Source:* Adapted from Ronald F. Miller and Peter D. Blair, *Input/Output Analysis: Foundations and Extensions* (New York: Cambridge University Press, 2009), 3.

The Producers rows describe the distribution of a producer's output throughout the economy. The Producers as Consumers columns describe the composition of inputs required by each industry to produce its output. The row labeled Value Added accounts for other non-industrial inputs to production, such as labor and imports. The columns labeled Final Demand record the sales by each sector to various final markets. The values are typically displayed in monetary terms, rather than physical units, for consistency.

## Setting up I/O Analysis

An I/O model is generally constructed from observed data for a particular economic area in a particular period. The economic activity must be separated into $n$ industries. In the case of the Long-term Inter-industry Forecasting Tool (LIFT), 110 sectors are specified. In the case of the Inter-industry Large-scale Integrated and Dynamic Model (ILIAD), 360 sectors are specified.

If we denote $y_i$ by the total output for industry $i$, then industry $i$'s distribution of its product through sales to other industries and to final demand is represented as

$$y_i = z_{i1} + \cdots + z_{ij} + \cdots + z_{in} + f_i = \sum_{j=1}^{n} z_{ij} + f_i \,. \tag{A.1}$$

There will be an equation like this for all $n$ industries, which will result in the system of linear equations

$$\begin{aligned} y_1 &= z_{11} + \cdots + z_{1i} + \cdots + z_{1n} + f_1 \\ &\vdots \\ y_i &= z_{i1} + \cdots + z_{ii} + \cdots + z_{in} + f_i \\ &\vdots \\ y_n &= z_{n1} + \cdots + z_{ni} + \cdots + z_{nn} + f_n \,. \end{aligned} \tag{A.2}$$

Let

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}, \boldsymbol{Z} = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix}, \boldsymbol{f} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}. \tag{A.3}$$

Also, let $i$ be a column vector of *1*s.

Then equation A.2 can be represented compactly as a matrix equation,

$$\boldsymbol{y} = \boldsymbol{Z}i + \boldsymbol{f}. \tag{A.4}$$

**Production Functions and I/O Analysis**

A fundamental assumption input/output analysis makes is that the inter-industry flows from industry $i$ to industry $j$ depend entirely on the total output of industry $j$ for that same period. That is, for each industry pair, there is a fixed ratio of inputs purchased from one to the output produced by the other. This ratio, $a_{ij}$, is known as a *technical coefficient* and can be represented by

$$a_{ij} = \frac{z_{ij}}{y_j}. \tag{A.5}$$

All of the technical coefficients can be represented in an $n \times n$ matrix:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}. \tag{A.6}$$

Note that since the technical coefficients are constant, the I/O framework implies that the production functions exhibit constant returns to scale.

**Deriving the Leontief Inverse**

Applying Property A.5 to the distribution of products through the economy (equation A.1) shows more detailed relationship between the outputs of all industries:

$$y_i = \sum_{j=1}^{n} a_{ij} y_j + f_i\,. \tag{A.7}$$

Thus, there is a system of $n$ linear equations that can be expanded as

$$\begin{aligned} y_1 &= a_{11}y_1 + \cdots + a_{1i}y_i + \cdots + a_{1n}y_n + f_1 \\ &\vdots \\ y_i &= a_{i1}y_1 + \cdots + a_{ii}y_i + \cdots + a_{in}y_n + f_i \\ &\vdots \\ y_n &= a_{n1}y_1 + \cdots + a_{ni}y_i + \cdots + a_{nn}y_n + f_n. \end{aligned} \tag{A.8}$$

Bringing all $y$ terms to the left, and grouping the $y_1$together in the first equation, the $y_2$ together in the second equation, and so forth for all $n$ equations shows these equations in terms of final demand:

$$\begin{aligned} (1 - a_{11})y_1 - \cdots - a_{1i}y_i - \cdots - a_{1n}y_n &= f_1 \\ &\vdots \\ 1a_{i1}y_1 - \cdots + (1 - a_{ii})y_i - \cdots - a_{in}y_n &= f_i \\ &\vdots \\ -a_{n1}y_1 - \cdots - a_{ni}y_i - \cdots + (1 - a_{nn})y_n &= f_n\,. \end{aligned} \tag{A.9}$$

Let **I** be the $n \times n$ identity matrix so

$$\mathbf{I} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}. \tag{A.10}$$

Then

$$(\mathbf{I} - \mathbf{A}) = \begin{bmatrix} (1 - a_{11}) & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & (1 - a_{22}) & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \cdots \\ -a_{n1} & -a_{n2} & \cdots & (1 - a_{nn}) \end{bmatrix}. \tag{A.11}$$

The complete $n \times n$ system in equation (A.9) can be compactly represented as the matrix equation

$$(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{y} = \boldsymbol{f}. \tag{A.12}$$

As long as $|\mathbf{I} - \mathbf{A}| \neq \mathbf{0}$, the inverse of $(\mathbf{I} - \mathbf{A})$ exists. Then, using standard matrix operations, output can be calculated as

$$\boldsymbol{y} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{f} = \boldsymbol{L}\boldsymbol{f}. \tag{A.13}$$

where $(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{L} = [l_{ij}]$ is known as the Leontief inverse. These equations are

$$\begin{aligned} y_1 &= l_{11}f_1 + \cdots + l_{1i}f_i + \cdots + l_{1n}f_n \\ &\vdots \\ y_i &= l_{i1}f_1 + \cdots + l_{ii}f_i + \cdots + l_{in}f_n \\ &\vdots \\ y_n &= l_{ni}f_i + \cdots + l_{ni}f_i + \cdots + l_{nn}f_n\,. \end{aligned} \tag{A.14}$$

# Appendix B
# National Income and Product Accounts

National Income and Product Accounts (NIPAs) are produced by the Bureau of Economic Analysis. NIPAs consist of seven accounts that differentiate total production and the distribution of earnings from total production into subcategories. Collectively, summary accounts constitute a double-entry system in which an expenditure recorded in one account for one sector is also recorded as a receipt in an account of another sector or in the same sector. This system of integrated double-entry accounts provides a comprehensive measure of economic activity in a consistently defined framework.

The production side of Account 1 (Domestic Income and Product Account), is particularly useful to the analysis in the *2015 NDS Requirements Report*. An example of this account from the 2005 NIPAs is displayed in Table B-1.

**Table B-1. Account 1. Domestic Income and Product Account (Billions of Dollars)**

| | |
|---|---|
| Personal Consumption Expenditures | 8742.4 |
| Durable Goods | 1033.1 |
| Nondurable Goods | 2539.3 |
| Services | 5170.0 |
| Gross Private Domestic Investment | 2057.4 |
| Fixed Investment | 2036.2 |
| Nonresidential | 1265.7 |
| Structures | 338.6 |
| Equipment and Software | 927.1 |
| Residential | 770.4 |
| Change in Private Inventories | 21.3 |
| Net Exports | -716.7 |
| Exports | 1301.1 |
| Imports | 2019.9 |
| Govt. Consumption Expenditures and Gross Investment | 2372.8 |
| Federal | 878.3 |
| National Defense | 589.3 |
| Nondefense | 289.0 |
| State and Local | 1494.4 |
| **Gross Domestic Product** | **12455.8** |

*Source:* U.S. Bureau of Economic Analysis, *Measuring the Economy: A Primer on GDP and the National Income Product Accounts* (Washington, DC: 2014), 9.

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# Appendix C
# The Scale Factor: Assumptions and Proofs

This appendix provides the theoretical foundations for the methodology proposed previously in chapter 2. Specifically, we show the validity of the scale factor and formalize the necessary economic assumptions such that the scale factor is transitive from the aggregate level to the industry level.

There are $n > 0$ industries in the economy. In the *2015 NDS Requirements Report*, $n = 360$. By the definition of the National Income Identity, the gross domestic product (GDP) equals the sum of final demands across all industries: [15]

$$GDP = \sum_{i=1}^{n} f_i \,. \tag{C.1}$$

As previously discussed, final demand for each industry is subcategorized by civilian, defense, export, and import demands:

$$f_i = f_i^{CIV} + f_i^{DEF} + f_i^{EX} - f_i^{IM}. \tag{C.2}$$

Note that, while all other subcategories are summed, imports are subtracted to compute final demand. To simplify notation, for the rest of this appendix, imports will be regarded as a negative value. Thus, final demand can be the summation of all the subcategories mentioned above.

One can view the aggregate civilian, defense, import, and export demands as representative consumers in the economy. To simplify notation, let $k = \{civ, def, ex, im\}$ be the identifier of the representative consumers. Let $w_k > 0$ denote the expenditure (i.e., budget) constraint for each representative consumer. For example, $w_{DEF}$ represents the defense budget, or equivalently, the constraint on defense spending.

The output of each industry $i$ has a price $p_i > 0$, for $i = 1, 2, \dots, n$. Let $\boldsymbol{p} = (p_1, p_2, \dots, p_n)^T \in \boldsymbol{R}_+^n$ denote the price vector. Assume that the prices are constant under small perturbations in the economy. Given the price vector $\boldsymbol{p}$ and its own expenditure constraint $w_k$, each representative consumer $k$ acts as a rational decision maker and chooses an optimal demand allocation characterized by vector $\boldsymbol{z}_k(\boldsymbol{p}, w_k) =$

[15] The analysis is the same for any year *t*. Therefore, the time subscript *t* is suppressed for notational simplicity.

$(z_{k,1}(\boldsymbol{p}, w_k), z_{k,2}(\boldsymbol{p}, w_k), \dots, z_{k,n}(\boldsymbol{p}, w_k))^T \in \boldsymbol{R}_+^n$ to maximize its utility. The final demand of industry $i$'s output by representative consumer $k$ can be written as

$$f_i^k(\boldsymbol{p}, w_k) = p_i * z_{k,i}(\boldsymbol{p}, w_k). \tag{C.3}$$

Recall that C$IV$, $DEF$, $EX$, and $IM$ denote the total spending in civilian, defense, export, and import sectors across all $i$ industries. With continuous utility functions and locally non-satiated preferences, each representative consumer's equilibrium total spending equals to its budget constraint. For example, $w_{def} = DEF$, and in the aggregate,

$$\sum_k w_k = CIV + DEF + EX - IM = GDP. \tag{C.4}$$

Chapter 2 shows that for each *non-defense* sector of the economy, civilian, export, and import, the new equilibrium spending under a new equilibrium can be calculated by multiplying the old equilibrium by sector total spending by the scale factor $\gamma$. For example, let $GDP$ denote the *original equilibrium gross domestic product* and let $GDP'$ denote the *new equilibrium gross domestic product*. Let $CIV'$ denote the new civilian equilibrium spending, which can be calculated by

$$CIV' = \gamma * CIV \tag{C. 5}$$

where

$$\gamma = \frac{GDP' - DEF}{GDP - DEF}.$$

For the scale factor to be transitive from the aggregate level to the industry level, the following assumption is made on each representative consumer's preferences.

**Assumption 1**

Each representative consumer $k$'s (civilian, defense, import, and export) preference "$\succsim$" on $\boldsymbol{R}_+^n$ is rational, monotone, continuous, and *homothetic*, that is, if $l \sim m$, for some commodity vectors $l, m \in \boldsymbol{R}_+^n$, then $\alpha l \sim \alpha m$, for any $\alpha > 0$.

Cobb-Douglas utility functions and Leontief utility functions are examples of utility functions reflecting homothetic preferences, which are widely used in economic analyses.

**Lemma 1**

Under assumption 1, each representative consumer $k$ admits a continuous utility function that is homogeneous of degree 1, that is, $u_k(\alpha m) = \alpha u_k(m)$, for any $m \in \boldsymbol{R}_+^n$. Moreover, each consumer's Walrasian demand[16] $\boldsymbol{z}_k(\boldsymbol{p}, w_k)$ takes the form of $\boldsymbol{z}_k(\boldsymbol{p}, w_k) =$

[16] A Walrasian demand is the optimal solution to the utility maximization problem, given one's budget constraint.

$\boldsymbol{B}_k(\boldsymbol{p})w_k$, in which $\boldsymbol{B}_k(\boldsymbol{p}) = (B_{k,1}(\boldsymbol{p}), B_{k,2}(\boldsymbol{p}), \dots, B_{k,n}(\boldsymbol{p}))^T$ is a positive continuous function in $\boldsymbol{p}$.[17]

Now, let $h \in \{civ, ex, im\}$ be the identifier of the representative consumers of the *non-defense* sectors of the economy (i.e., civilian, export, import). To show that equation (2.18) holds, recall that each representative consumer's equilibrium total spending equals to its budget constraint. Therefore, for each representative consumer $h$, its budget constraint in the new equilibrium is $w_{h'} = \gamma * w_h$. By lemma 1, representative consumer *h*'s optimal demand for good *i* is

$$z_{h,i}(\boldsymbol{p}, w_{h'}) = z_{h,i}(\boldsymbol{p}, \gamma * w_h) = \gamma * z_{h,i}(\boldsymbol{p}, w_h). \tag{C.6}$$

Therefore, in the new equilibrium, the final demand of industry $i$'s output by representative consumer $h$ can be written as

$$f_i^{h'} = p_i * z_{h,i}(\boldsymbol{p}, w_{h'}) = p_i * \left(\gamma * z_{h,i}(\boldsymbol{p}, w_h)\right). \tag{C.7}$$

Rearranging yields the following equation (2.18)

$$f_i^{h'} = \gamma * \left(p_i * z_{h,i}(\boldsymbol{p}, w_h)\right) = \gamma * f_i^h. \tag{C.8}$$

[17] Andreu Mas-Colell, Jerry Green, and Michael D. Whinston, *Microeconomic Theory* (New York: Oxford Printing Press, 1995), chapters 3 and 4.

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# Appendix D
# Analytical Perspectives Forecast Data

Every fiscal year, the Office of Management and Budget publishes a volume of *Analytical Perspectives* along with the *Budget of the United States Government*. *Analytical Perspectives* provides statistics on the accuracy of the economic projections by the current Administration, the Congressional Budget Office (CBO), and Blue Chip Consensus—an average of about 50 private-sector economic forecasts. The statistics include forecast data for two-year average annual growth rates and six-year average annual growth rates for real gross domestic product (GDP). These averages come from year-to-year growth rates dating back to 1982. For each of these actual and predicted average annual growth rates, a *mean error (ME)*, *mean absolute error (MAE)*, and *root mean square error (RMSE)* are calculated. The calculations for each of these errors are displayed in the following equations:

$$ME = \frac{\sum_{t=1}^{n} g_t - g_t^*}{n} \tag{D.1}$$

$$MAE = \frac{\sum_{t=1}^{n} |g_t - g_t^*|}{n} \tag{D.2}$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (g_t - g_t^*)^2}{n}} \tag{D.3}$$

where

$t$ = index for year,

$g_t^*$ = the predicted growth rate of real *GDP* for year *t*,

$g_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$, the actual growth rate of real *GDP* for year *t*, and

$n$ = the number of years in the sample.

The six-year growth rate errors were used in the analysis rather than the two-year rate errors because six-year horizons include business cycle effects. Table D-1 is a compilation of the six-year average annual real *GDP* growth errors reported in the 2011 to 2015 volumes of *Analytical Perspectives*.

**Table D-1. Forecast Errors**

| | Six-Year Average Annual Real *GDP* Growth | Admin. | CBO | Blue Chip |
|---|---|---|---|---|
| **2011** | | | | |
| | ME | -0.1 | -0.4 | -0.5 |
| | MAE | 0.7 | 0.6 | 0.7 |
| | RMSE | 0.8 | 0.8 | 0.8 |
| **2012** | | | | |
| | ME | -0.0 | -0.3 | -0.3 |
| | MAE | 0.8 | 0.7 | 0.7 |
| | RMSE | 0.9 | 0.9 | 0.9 |
| **2013** | | | | |
| | ME | 0.1 | -0.2 | -0.2 |
| | MAE | 0.8 | 0.8 | 0.8 |
| | RMSE | 1.0 | 1.0 | 1.0 |
| **2014** | | | | |
| | ME | 0.2 | -0.1 | -0.1 |
| | MAE | 0.9 | 0.8 | 0.8 |
| | RMSE | 1.1 | 1.1 | 1.1 |
| **2015** | | | | |
| | ME | 0.2 | -0.1 | -0.1 |
| | MAE | 0.9 | 0.9 | 0.9 |
| | RMSE | 1.1 | 1.2 | 1.1 |

S*ource*: Reported in the 2011 to 2015 volumes of *Analytical Perspectives*.

*Note:* For each listed year, the forecast errors are calculated based on data from January 1982 to the release of the budget for that fiscal year.

While the errors for the CBO and blue chips are also interesting for economists, the Administration's data is equivalent to the data of Council of Economic Advisors, whose forecast was used in the Institute for Defense Analyses macroeconomic modeling for the NDS. Thus, only the Administration's error trends are utilized in the uncertainty analysis.

The ME calculation is the more simple and seems the more desirable, given that it is lower than MAE and RMSE. However, MAE has the advantage of measuring accuracy by considering average magnitude, regardless of the direction of error. RMSE also considers magnitude regardless of direction. In addition, since errors are squared before they are averaged, heavier weight is given to large errors. This error measure is greater than the previous two, but because it measures both accuracy and variability, RMSE is the focus of economic uncertainty analysis.

According to the 2014 and 2015 *Analytical Perspectives* volumes, the RMSE of the Administration's prediction for six-year average annual *GDP* growth is 1.1. This implies that the Administration's prediction has recently been off by about 1.1%. Therefore, in the following bounding uncertainty analysis, two extreme cases are considered: the case where annual *GDP* growth is consistently 1.1% higher than predicted and the case where annual

*GDP* growth is consistently 1.1% lower than predicted. In other words, 1.1% is the uncertainty parameter evaluated in the *2015 NDS Requirements Report*.

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# Appendix E
# Illustrations

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# Appendix F
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# Appendix G
# Abbreviations

| | |
|---|---|
| CBO | Congressional Budget Office |
| CEA | Council of Economic Advisers |
| CIV | civilian |
| DEF | defense |
| DLA SM | Defense Logistics Agency Strategic Materials |
| DoD | Department of Defense |
| EX | export |
| GDP | gross domestic product |
| I/O | input/output |
| IDA | Institute for Defense Analyses |
| ILIAD | Inter-industry Large-scale Integrated and Dynamic Model |
| IM | import |
| INFORUM | Interindustry Forecasting Project at the University of Maryland (University Park) |
| LIFT | Long-term Inter-industry Forecasting Tool |
| M | million |
| MAE | mean absolute error |
| ME | mean error |
| NDS | National Defense Stockpile |
| NII | National Income Identity |
| NIPA | National Income and Product Accounts |
| NX | net export |
| OMB | Office of Management and Budget |
| OSD | Office of the Secretary of Defense |
| RMS | root mean square |
| RMSE | root mean square error |
| SECDEF | Secretary of Defense |
| U.S. | United States |

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**14. ABSTRACT**

To sustain the essential civilian economy and provide for the national defense during a national emergency scenario, the United States (U.S.) Department of Defense maintains a stockpile of strategic materials that are critical to the national industrial base. The types and quantities of materials selected for the national defense stockpile is guided by a biennial report to the U.S. Congress, the National Defense Stockpile (NDS) report. The law requiring the NDS report also mandates parametric sensitivity studies of the primary planning scenario analyzed in the report. Addressing this mandate, some previous NDS reports comprise a bounded uncertainty analysis of U.S. economic forecasts as part of the sensitivity studies. In these studies, the baseline macroeconomic forecast that supports material demand calculations is systematically perturbed and used to recalculate stockpile requirements, characterizing the sensitivity of stockpile needs to macroeconomic forecast uncertainty. This paper provides an overview of the possible approaches to modeling uncertainty in the macroeconomic analysis of the NDS report, along with a discussion of their relative merits. The underlying theory, methodology, and results of the uncertainty analysis are explained and discussed. Some conclusions and recommendations for future analysis protocols are also presented

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