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Executive Summary

The United States maintains a National Defense Stockpile of nonfuel strategic and critical materials, managed by the Department of Defense. Under Section 14 of the Strategic and Critical Materials Stock Piling Act (50 U.S.C. § 98 et seq.), the Secretary of Defense is required to submit biennial reports to Congress concerning what materials the stockpile should contain and in what amounts.

The Institute for Defense Analyses (IDA) performs the analytic work underlying many of the results in the biennial reports. For its analyses, IDA makes extensive use of the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM), a set of modeling tools, procedures, and databases that IDA developed to estimate shortfalls of strategic and critical materials that might occur in a national emergency and to assess the risk of such shortfalls.

This paper documents some recent methodological improvements to RAMF-SM, and it has two purposes: (1) to make people who use the RAMF-SM results aware of these improvements and (2) to provide detailed mathematical descriptions and derivations of the new features to help the people who run RAMF-SM models understand them better.

The primary objective of this work is to increase the realism of the Material Prioritization via Linear Programming (MPLP) process, one of the key components of RAMF-SM. MPLP determines the maximum amount of industrial output that can be produced with the materials that are available in a national emergency. The previous version of MPLP, which is documented in IDA Paper P-33037, was intended to be a proof of concept and, as such, made a number of simplifying assumptions. The methodology developed in the current paper allows for more realistic assumptions and can thus improve MPLP's applicability.

One key improvement involves the mathematical development of a number of new formulas for material consumption ratios (MCRs), which estimate the amounts of materials required to manufacture industrial output. The paper derives formulas for several different kinds of MCRs that take into account a variety of specific conditions that might affect material consumption. Use of these specialized MCRs could increase the realism of RAMF-SM's material demand calculation methods, which are relevant both to MPLP and to RAMF-SM's material shortfall computation procedures.

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Contents

1. Introduction and Background

A. Acronyms

At the outset, the reader should become familiar with the following acronyms, which are used throughout the paper.

RAMF-SM: Risk Assessment and Mitigation Framework for Strategic Materials. RAMF-SM is a set of models, databases, and procedures for determining shortfalls of strategic and critical materials in a national emergency, assessing the risks of such shortfalls, and developing strategies to mitigate the effects of the shortfalls.[1]

MCR: material consumption ratio. An MCR specifies the amount of a given material (measured in mass units, such as tons) that is used to produce a given dollar amount (generally a million or a billion dollars' worth) of output from a particular industry. An MCR is defined for each combination of material and industry under consideration (e.g., tantalum in the electronics industry). The MCR value is zero if the industry does not use the material.[2]

MDCP: Material Demand Computation Program. MDCP is a model that computes the demand for strategic and critical materials, in either a peacetime or a national emergency setting. It is one of the key components of RAMF-SM.[3]

MPLP: Material Prioritization via Linear Programming. This relatively new component of RAMF-SM determines the maximum amount of industrial output that can be produced with the materials that are available in a national emergency. The amount of industrial output that *cannot* be produced is a measure of the risk of the shortfall. If there

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[1] The best introduction to RAMF-SM is James S. Thomason et al., *An Overview of Step 2 of the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM)*, IDA Document D-5432 (Alexandria, VA: Institute for Defense Analyses, March 2015). Step 2, the computation of material shortfalls in a national emergency scenario, is currently the most-exercised step of RAMF-SM.

[2] MCRs are defined and derived mathematically in Eleanor L. Schwartz and James S. Thomason, *Computation of Material Demand in the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM) Process*, IDA Document D-5477 (Alexandria, VA: Institute for Defense Analyses, August 2015).

[3] Documentation of MDCP appears in Eleanor L. Schwartz, *The RAMF-SM Material Demand Computation Program: Documentation and User's Guide,* IDA Paper P-22689 (Alexandria, VA: Institute for Defense Analyses, March 2022).

is a budget for acquiring additional material, MPLP can determine which materials to acquire to maximize the total additional industrial output produced.[4]

B. Objectives of This Paper

The main objective of this paper is to develop methodology that will increase the realism of MPLP.[5] The version of MPLP documented in the Institute for Defense Analyses (IDA) Paper P-33037 was intended to be a proof of concept. As such, the following simplifying assumptions were made in its formulation:

- Only the first year's material demands and supplies were considered, even though the national emergency scenario that RAMF-SM models is generally several years long.
- Defense, civilian, and emergency investment demands were combined, even though RAMF-SM has separate treatments for each category (the term "tier" is often used for category of demand).
- No distinction was made between supply that could satisfy all tiers of demand and supply that could satisfy civilian demand only.
- Market responses of thrift and substitution were not addressed; such responses can reduce material demand.

Chapters 2 and 3 develop methodologies that can address many of the aforementioned issues. Chapter 4 shows how these methodologies can be implemented in MPLP.

This paper's principal methodology develops additional kinds of MCRs. The current version of MDCP can compute separate material demands for each tier and has algorithms to determine the effects of thrift and substitution. However, the current version of MDCP does not use MCRs directly. As a side calculation, it computes a set of "generic" MCRs that do not consider variations by tier and market response. Conversely, MPLP requires using MCRs because it needs to tie material usage to demands in specific industries. The prototype version of MPLP used generic MCRs, but to increase MPLP's realism, MCRs that address the above-noted features are desirable.

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4 MPLP is discussed in Eleanor L. Schwartz and Jerome Bracken, *Material Prioritization via Linear Programming: Proof of Concept and Initial Results,* IDA Paper P-33037 (Alexandria, VA: Institute for Defense Analyses, June 2022).

5 As of the writing of this paper, the new methodologies have not been implemented with actual data. They are expected to be used to support part of the upcoming 2025 Report to Congress on strategic material requirements.

C. History of Material Demand Computation Procedures

1. Material Demand Computation Methods

The underlying principle behind the RAMF-SM material demand computation procedure is to link material usage with industrial output. Two somewhat different methods, used for different groups of materials, had been developed for this purpose. Both methods started by establishing a set of different application areas in which a material was used, and determining the proportions of material used in each application. The key difference between the two methods was whether a defense vs. civilian breakdown of usage in each application was given. For many materials, the Department of Commerce determined the application areas and relative proportions but did not provide information on the defense/civilian split. For other materials, various subject matter experts (SMEs) were asked to specify the application areas and proportions and were explicitly asked to state the fraction of use in each application area that was for defense purposes.

Because of this historical difference in the information gathered, two different algorithms were constructed to compute material demand. The MCR methodology was developed for the materials examined by Commerce; it did not distinguish between defense and civilian industrial output. For the other materials, a so-called "alternative" methodology was constructed. It started by separating historical material consumption in each application into defense usage and civilian usage, based on the fractions that the SMEs provided. Based on the industries associated with the applications, the methodology computed adjustment factors that were applied to the historical consumption amounts to compute estimated future consumption amounts, which were used as the material demand values. Separate adjustment factors were computed for defense usage and civilian usage.

These two distinct algorithms were used for many years, each for its own group of materials. After considerable methodological analysis, the MCR algorithm was determined to be a special case of the alternative algorithm,[6] and a new model, the Material Demand Computation Program (MDCP) was written for RAMF-SM. The new model used the alternative algorithm, which works with application areas rather than directly with industries. As a side calculation, MDCP constructs a set of MCRs for all materials (and outputs them on the _mcr file[7]), simply ignoring any SME-provided defense usage fractions, and computes the material demand that results from applying those MCRs to the demands for industrial output. The result of this side calculation (which is written on the _mco file) will match the main output for those materials for which the defense fraction is

6 IDA Paper P-22689, previously cited, contains a mathematical proof of this result.

7 MDCP distinguishes its various output files by a three-letter code. See Appendix A for a list of these codes and descriptions of the associated output files. That appendix is a reproduction of Table 12 of P-22689.

computed from the economic data, but will in general not match the main output for those materials that have user-specified defense fractions.[8] Overall, the MCR-based output tends to underestimate defense demand.

2. An Unexpected Bonus

MPLP needs a way to compute the material amount associated with a single particular industry—in other words, an MCR. The initial version of MPLP was developed using the MCRs, even though MDCP had supplanted their use. If separate defense MCRs and civilian MCRs could be developed, then both sets could be used with MPLP and could be applied to the corresponding categories of industrial output. This paper develops formulas for these tier-specific MCRs. The paper also shows that applying the defense MCRs to the defense demands for industrial output in the emergency scenario yields the same defense material demand as the MDCP algorithm (and similarly, for civilian MCRs and civilian demand). It is therefore possible to have a material demand computation procedure that both uses MCRs and preserves an explicit defense-civilian split in applications, where appropriate. Going forward, both MPLP and a modified material demand computation procedure can use MCRs—an unexpected bonus finding of this research.

3. Market Responses

One feature of RAMF-SM is the use of market responses. Faced with an impending material shortfall in a national emergency, manufacturers might react in a number of ways. Currently RAMF-SM considers three such market responses:

- Thrift—using less of a material in the production of goods and services
- Substitution—using substitute material(s) for a material that might be in shortfall
- Extra sell—obtaining extra material from certain countries

The first two market responses act to reduce demand for materials, whereas the third tends to increase the available supply. The extra sell market response involves U.S. manufacturers' obtaining preferential access to currently unused capacity. A manufacturer might identify unused foreign material productive capacity and make a special agreement for the producer to produce at or near capacity, with the manufacturer buying the extra production amount. (For details about how the model implements these concepts, see IDA

8 P-22689 extensively discusses this issue.

Papers P-10727 and P-22696.[9]) The extra sell market response is straightforward to implement in MPLP: simply increase the amount of available supply to include material obtained via extra sell.

Because of the way RAMF-SM computes demand for materials,[10] integrating the thrift and substitution modeling into the MPLP framework has not been a straightforward process. The MDCP computer program computes "gross" material demands and then applies adjustments to them for thrift and substitution.[11] The adjustments do not involve the industry demands directly and do not use MCRs. Another issue is that the thrift and substitution market responses are frequently applied to civilian demand only.

Consideration of all these concerns led to the conclusion that a good way to implement thrift and substitution in MPLP might be to adjust the mathematical procedure by which MCRs are derived, to account for thrift and substitution. Chapter 2 outlines these adjustments and develops formulas for the corresponding MCRs.

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9 J. Thomason et al., *IDA Contributions to the Strategic and Critical Materials 2019 Report on Stockpile Requirements, Volume I: Unclassified Contributions*, IDA Paper P-10727 (Alexandria, VA: Institute for Defense Analyses, October 2019);
Eleanor L. Schwartz and James S. Thomason, *The RAMF-SM Stockpile Sizing Module: Updated Documentation and User's Guide*, IDA Paper P-22696 (Alexandria, VA: Institute for Defense Analyses, April 2022).

10 For more information about RAMF-SM's computation of material demand, see IDA Paper P-22689, previously cited.

11 For a description of the process, see IDA Paper P-22689.

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2. A Gallery of Material Consumption Ratios (MCRs)

A. Introduction

This chapter presents mathematical derivations of formulas for several kinds of MCRs that differ from the traditional "generic" MCRs that were used in older versions of the material demand computation module of RAMF-SM. As noted previously, an MCR specifies the amount of a given material (measured in mass units, such as tons) that is used to produce a given dollar amount (generally a million or a billion dollars' worth) of output from a particular industry sector.

The kinds of MCRs considered in this chapter include:

- MCRs that can be used for civilian-related industrial output
- MCRs that can be used for defense-related industrial output
- MCRs that can be used for industrial output necessary for emergency investment
- MCRs that take material substitutability into account
- MCRs that take thrift into account
- MCRs that take both thrift and substitutability into account.

The underlying principle behind the RAMF-SM material demand computation procedure is to link material usage with industrial output. For each material, a set of application areas (i.e., kinds of industrial production) in which the material is used is established. The application areas are then associated with the industry sectors of the underlying economic model (ILIAD).[12] These concepts underlie both the formation of the MCRs and the computations of the current MDCP algorithm. The reader is encouraged to review IDA publications D-5477 and P-22689 (previously cited). The derivations presented in this chapter assume a familiarity with the ideas elucidated in those papers.

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[12] RAMF-SM makes use of the economic forecasting models LIFT (Long-term Inter-industry Forecasting Tool) and ILIAD (Inter-industry Large-scale Integrated and Dynamic model), which were developed by the Inter-industry Forecasting Project at the University of Maryland (INFORUM). Citations to documentation by Douglas Meade appear in the References section. The current version of ILIAD partitions the U.S. economy into 350 different industry sectors.

This chapter starts by stating some basic notation that will be used throughout. It then presents the formula for generic MCRs, which is derived in IDA Document D-5477. All the algorithms described in this chapter assume that one particular material is under consideration. Accordingly, the notation suppresses all material-related subscripts. MPLP uses combinations of materials, so it is essential for the notation (in Chapter 4 and elsewhere) to identify MCRs by both material and industry.

All the algorithms make use of values in a "reference period"—a recent span of years in which peacetime economic and material consumption data are assumed to be available. The reference period can be different for different materials.

B. Basic Terminology and Generic MCRs

1. Notation

a. Indices

The reference period is indexed as time $t = 0$. The T years of the scenario period are indexed as $t = 1, \ldots, T$, even though the scenario might be posited as occurring some years after the reference period. Let j ($j = 1, \ldots, J$) index application area and i ($i = 1, \ldots, I$) index industry sector. Often, we simply say "industry" rather than "industry sector."

b. Economic Data

Let d_{i0}, c_{i0}, x_{i0}, and m_{i0} denote the average annual defense demand, civilian demand, exports, and imports, respectively, in industry sector i during the reference period under peacetime conditions, all expressed in total requirements terms.[13]

Theoretically, in an equilibrium situation industrial output is the sum of defense demand plus civilian demand plus net exports, where all components are expressed in total requirements terms. IDA Papers D-5477 and P-22689 and the computer programs based on them assume this equivalence. Letting ω_{i0} denote the average annual output of industry sector i during the reference period, then $\omega_{i0} = d_{i0} + c_{i0} + x_{i0} - m_{i0}$.

The traditional MCR computation algorithm, described in IDA Document D-5477, uses the quantities ω_{i0}. However, output can be partitioned into (1) defense demand, and (2) civilian demand plus net exports. That kind of partitioning is the foundation of the algorithms described in this chapter.

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[13] That is, the demands represent total industrial output, including inter-industry flows. For more information on the total requirements concept, see Ronald E. Miller and Peter D. Blair, *Input-Output Analysis: Foundations and Extensions*, 2nd ed. (New York City, NY: Cambridge University Press, 2009).

The processing of the economic data ensures that d_{i0}, c_{i0}, x_{i0}, and m_{i0} are all nonnegative, but the quantity $c_{i0} + x_{i0} - m_{i0}$ (i.e., peacetime civilian demand plus net exports) will occasionally be negative for some i. This condition indicates a possible problem with the underlying economic data, which should be examined, but the computer program simply uses zero instead of the negative value and prints an informative message. In symbols, the quantity

$$q_{i0} = \max\{(c_{i0} + x_{i0} - m_{i0}), 0\}.$$

is used in the subsequent calculations to indicate civilian demand plus net exports.

c. Material Consumption Data and Intermediate Quantities

Let

Γ = average annual amount of consumption of the material in the reference period

φ_j = proportion of consumption used in application area j

$\gamma_j = \Gamma\varphi_j$ = average annual amount of consumption of the material in application area j during the reference period

δ_{ij} = 1 if industry sector i is associated with application area j, 0 if not

S_j = set of industry sectors associated with application area j (i.e., $S_j = \{i|\delta_{ij} = 1\}$).

q_{i0} = average annual civilian demand (including net exports) in the reference period in industry sector i, expressed in total requirements terms, as defined previously

$Q_j = \sum_{i\in S_j} q_{i0}$. This is the total average annual amount of civilian demand of the industries associated with application area j in the reference period. Q_j is also equal to the quantity $\sum_{i=1}^{I} \delta_{ij} q_{i0}$.

d_{i0} = average annual defense demand in industry i in the reference period expressed in total requirements terms, as defined previously

$D_j = \sum_{i\in S_j} d_{i0}$ = total average annual defense demand of the industries associated with application area j in the reference period

$\omega_{i0} = q_{i0} + d_{i0}$ = average annual output of industry sector i during the reference period, as defined previously

$\Omega_j = Q_j + D_j$ = total average annual output of the industries associated with application area j in the reference period

ν_j = proportion of consumption of the material in application area j that is for civilian purposes (this symbol is nu, not v or upsilon). This quantity might be input or might be computed from the underlying industry demands. If not explicitly input, it can reasonably be set to $Q_j/(Q_j + D_j)$.

$\mu_j = 1 - \nu_j$ = proportion of consumption of the material in application area j that is for defense purposes

2. The "Generic" MCR

IDA Document D-5477 derives a formula for a "generic" MCR as

$$\rho_i^{(\text{generic})} = \sum_{j=1}^{J} \gamma_j \frac{\delta_{ij}}{\Omega_j}.$$

At the time that document was written, that quantity was simply *the* MCR; there were no variations. The mathematical derivation of the formula for the generic MCR can be modified in a number of ways to yield the MCRs discussed in the following sections.

C. Derivation of the Civilian and Defense MCR Formulas

This section derives MCRs that specify the amount of civilian material demand per million dollars of output of a given industry sector and similarly for defense material demand. Section C.3 proves a surprising result: if the civilian MCRs are multiplied by the civilian demands, and the defense MCRs are multiplied by the defense demands, the resultant overall civilian and defense material demands are identical to those computed by MDCP.

1. Algebraic Derivation

The average annual amount of consumption (of the given material) in application area j that is for civilian purposes in the reference period is $\nu_j\gamma_j = \nu_j\Gamma\varphi_j$. The overall average annual amount of material consumption for civilian purposes in the reference period is then $\Gamma^{(\text{civ})} = \sum_{j=1}^{J} \nu_j \gamma_j$. The corresponding quantities for defense consumption are $\mu_j\gamma_j$ and $\Gamma^{(\text{def})} = \sum_{j=1}^{J} \mu_j \gamma_j$.

The following exposition parallels IDA Document D-5477. Let $\psi_{ij}^{(\text{civ})} = q_{i0}/Q_j$ if industry sector i is associated with application area j, and 0 if not. The quantity $\psi_{ij}^{(\text{civ})}$ represents the fraction of civilian consumption in application area j that is associated with industry sector i. So, $\psi_{ij}^{(\text{civ})} = \delta_{ij}q_{i0}/Q_j$, and for each given j, $\sum_{i=1}^{I} \psi_{ij}^{(\text{civ})} = 1$. The amount of material consumption in application area j can be apportioned among the industry sectors associated with that application area. Assume that this is done in proportion to the civilian demands of those sectors (i.e., the q_{i0}). That is, the amount of material consumed for civilian purposes in application area j that is associated with industry sector i can be estimated by $\psi_{ij}^{(\text{civ})}\nu_j\gamma_j$. This value is zero for sectors not associated with application area j.

The total amount of material (call it $Z_i^{(\text{civ})}$) associated with industry sector i for civilian purposes is then computed by summing over application area j the amount of material consumed in application area j that is associated with industry i.

In symbols,

$$Z_i^{(\mathrm{civ})} = \sum_{j=1}^{J} \nu_j \gamma_j \, \psi_{ij}^{(\mathrm{civ})}.$$

That is,

$$Z_i^{(\mathrm{civ})} = \sum_{j=1}^{J} \frac{\nu_j \gamma_j \, \delta_{ij} q_{i0}}{Q_j}$$

or

$$Z_i^{(\mathrm{civ})} = q_{i0} \sum_{j=1}^{J} \nu_j \gamma_j \frac{\delta_{ij}}{Q_j}.$$

This is an amount of material consumption for civilian purposes that is associated with industry sector i. The civilian material consumption ratio is the ratio of this amount to the civilian total requirements demand (plus net exports) in industry sector i. But this latter quantity is simply q_{i0}. Thus, the civilian MCR (call it $\rho_i^{(\mathrm{civ})}$) is defined by

$$\rho_i^{(\mathrm{civ})} = \frac{Z_i^{(\mathrm{civ})}}{q_{i0}} = \sum_{j=1}^{J} \nu_j \gamma_j \, \frac{\delta_{ij}}{Q_j}.$$

A completely analogous derivation can be performed with the defense-related quantities, resulting in a defense MCR

$$\rho_i^{(\mathrm{def})} = \sum_{j=1}^{J} \mu_j \gamma_j \, \frac{\delta_{ij}}{D_j}.$$

What if no civilian vs. defense fractions are explicitly input? In that case, a reasonable value for the civilian usage ratio can be computed as $\nu_j = Q_j/(Q_j + D_j)$ for each application area j. Then $\mu_j = 1 - \nu_j = D_j/(Q_j + D_j)$, and both the civilian and defense MCRs have the same value, which is equal to the traditional MCR value. (Some care needs to be taken for pathological cases where Q_j and/or D_j are zero. However, this situation often indicates anomalies in the underlying economic data, which should be reviewed.)

2. Equivalence to the Material Demand Computation Program (MDCP) Algorithm

The MCRs are formed from historical data on material consumption and industrial output. The traditional way of computing material demand in the RAMF-SM emergency scenario was to multiply the MCRs by the demands for industrial output in the emergency scenario, and sum over industry sector. As noted earlier, the traditional algorithm computed one MCR (for each relevant material-industry combination) and applied it to both civilian and defense industrial output. The current MDCP algorithm replaces the use of MCRs by an adjustment factor methodology that can consider explicit civilian and defense usage fractions.

However, if the *civilian* MCRs, computed as described in the previous section, are multiplied by the *civilian* demands (including net exports) for industrial output in the scenario, the resulting material amount is the same as that computed by the current MDCP algorithm. A similar result holds for the defense MCRs and defense demands. A proof of this result follows.

3. Mathematical Proof of Equivalence

The reader's understanding of this proof will be enhanced by first reviewing Chapter 4, Sections B and C, of IDA Paper P-22689. The MDCP algorithm computes the overall civilian demand for a given material in year *t* under emergency scenario conditions as

$$C_t^{\text{tot}} = \sum_{j=1}^{J} C'_{jt} \, ,$$

which is the sum over all application areas *j* of the quantities C'_{jt} (the material demands in application area *j*). The C'_{jt} are defined by

$$C'_{jt} = (1-\mu_j)\gamma_j(V'_{jt}/V_{j0}).$$

The civilian fraction $(1-\mu_j)$ is identical to ν_j, as defined previously. The quantity V_{j0} is the same as Q_j defined previously, denoting the average annual civilian demand (including net exports) in the industries associated with application area *j* in the reference period under peacetime conditions. The quantity V'_{jt} represents the civilian demand in the industries associated with application area *j* in year *t* of the scenario under emergency conditions, and is given by

$$V'_{jt} = \sum_{i=1}^{I} \delta_{ij} q'_{it} \, ,$$

where q'_{it} is the civilian demand (plus net exports) in industry *i* in year *t* of the scenario period. The total demand is then given by

$$C_t^{\text{tot}} = \sum_{j=1}^{J} C'_{jt} = \sum_{j=1}^{J} \nu_j \gamma_j V'_{jt}/Q_j \, .$$

Substituting for V'_{jt} and then interchanging the order of summation, we obtain

$$C_t^{\text{tot}} = \sum_{j=1}^{J} \nu_j \gamma_j V_{jt}'/Q_j = \sum_{j=1}^{J} \frac{\nu_j \gamma_j}{Q_j} \left(\sum_{i=1}^{I} \delta_{ij} q_{it}' \right) = \sum_{i=1}^{I} q_{it}' \left(\sum_{j=1}^{J} \nu_j \gamma_j \delta_{ij}/Q_j \right).$$

The inner sum on j is recognizable as the formula for the civilian MCR $\rho_i^{(\text{civ})}$ derived in the previous section. The formula for the total civilian material demand then becomes

$$C_t^{\text{tot}} = \sum_{i=1}^{I} \rho_i^{(\text{civ})} q_{it}' ,$$

which is the formula for calculating material demand via MCRs.

The proof for defense demands and MCRs is identical, *mutatis mutandis*.

D. Emergency Investment MCRs

The RAMF-SM modeling computes some amount of demand for industrial output in the emergency scenario that is for emergency investment purposes. The material demand needed to produce this industrial output must be accounted for. In the traditional MCR algorithm, the MCRs were multiplied by the emergency investment industrial output demand. The current MDCP algorithm presents some challenges to model emergency investment demand, since the peacetime demand for emergency investment is zero and thus a scenario-to-peacetime ratio cannot be formed.

If the defense and civilian MCRs are the same for all industries, as will occur if the defense fractions for each application area are determined from the economic data, then that common MCR value can also be used for emergency investment demand. If the defense and civilian MCR values are different, it would seem reasonable to use some weighted (convex) combination of the defense and civilian MCRs, with the same weight for all industries. But what should that weight be?

Chapter 4 of IDA Paper P-22689 presents a formula for emergency investment material demand in MDCP that makes use of an arbitrary parameter θ, between 0 and 1, that represents the balance between defense and civilian demand. It can be shown, by a proof similar to the one in Section C, that for any given θ, the formula in P-22689 yields the same emergency investment material demand as produced using the MCR given by $\theta\rho_i^{(\text{def})} + (1-\theta)\rho_i^{(\text{civ})}$. Currently, MDCP assumes that $\theta = 0$, i.e., only the civilian multiplier is used for the emergency investment material demand calculation. This is equivalent to using the civilian MCRs for the MCR-based emergency investment calculation.

A computer program was written to compute the civilian, defense, and emergency investment MCRs by the formulas given above. A second computer program applied them to the demands for industrial output in the 2023 Requirements Report Base Case. The results matched the MDCP results, subject to acceptable rounding error.

E. Substitution MCRs

For MPLP to have the capability to model the market responses of thrift and substitution, it would be desirable to have modified MCRs that would take them into account. (As noted earlier, MPLP uses MCRs, whereas MDCP models the thrift and substitution market responses via adjustments that do not make use of industrial output directly.) This section deals with addressing the substitution response. As discussed below, the MCRs can be modified to take substitution into account. If substitution is possible, the MCR values will be lower, reflecting less use of the "main" material to produce industrial output, because a substitute material is being used instead of some of the main material. As of now, the substitution market response has been modeled for civilian demand only. Analogous formulas could be developed for substitution of defense demand, if appropriate.

The extent of substitutability might evolve over the course of the scenario. For example, it might take manufacturers some time to retool to use the substitute material, so substitutability might be limited in the early scenario years but increase with time. The MDCP program can accept different substitutability factors for each scenario year, and the construction of the MCRs should take this variability into account. Accordingly, let β_{jt} be the reduction factor due to substitutability in application area j in year t of the scenario. The reduction factor is interpreted as a multiplier of the demand. For example, if 30 percent of the use of the main material in application area j in year t can be performed by substitute materials, then β_{jt} should be set to 0.7.

The derivation of the substitution MCRs is a straightforward extension of the procedure for the civilian MCRs, as explained in Section C.1. Let all notation be as in Section C, and let β_{jt} be the reduction factor in application area j in year t due to substitutability. Note that the reduction factor depends on the scenario year, but the consumption amounts and economic data remain the historical peacetime data.

The derivation results in T separate sets of MCRs, each given set to be applied to industrial demand in the corresponding year of the scenario. For each scenario year t in turn, the derivation proceeds as presented in Section C.1, except the amount of civilian material consumption of the given material in application area j is set to $\beta_{jt}\nu_j\gamma_j$ rather than $\nu_j\gamma_j$ because fraction $(1 - \beta_{jt})$ of the consumption is being met by substitute materials. The derivation results in the substitution MCR for scenario year t given by:

$$\rho_{it}^{(\text{civ,sub})} = \sum_{j=1}^{J} \beta_{jt}\nu_j\gamma_j \frac{\delta_{ij}}{Q_j}.$$

F. MCRs for the Thrift Market Response

1. Rationale Behind the Thrift MCRs

Material consumption fluctuates from year to year, and so does industrial output. One might think that the ratio of material consumption to industrial output would be more stable, but it, too, varies from year to year; if the ratio is low, more industrial output can be produced with a given amount of material. This disparity in the consumption-to-output ratio provides the rationale behind the modeling of the thrift market response in RAMF-SM.

The standard material demand computation algorithm uses average annual consumption and industrial output values over the years of the reference period. Many of the quantities postulated in Section B were defined this way. The thrift modeling computes a "quantity of interest" for each separate year in the reference period and then uses the minimum one. The term "quantity of interest" is deliberately ambiguous. It might refer to an amount of demand, an MCR, or something else.

2. Current Version of MDCP

The current version of MDCP can model the thrift market response, but its way of doing so does not involve MCRs.[14] Rather, it proceeds application area by application area. For a given application area, for each year of the reference period in turn, an amount of overall material demand—encompassing all tiers—is computed by the standard MDCP algorithm under the assumption that the reference period consists of that year alone. The "minimizing year" for that application area is defined as the reference period year for which this overall amount is smallest. The demands in the various tiers for that application area in the minimizing year are then used in all further calculations.

3. Formulas for Thrift MCRs

This section develops formulas for several different kinds of MCRs that model the thrift market response in conjunction with other assumptions. Some basic notation is presented below to enhance reader's understanding of the formulas for thrift MCRs.

Theorems such as those developed in Sections C and E do not appear to hold for the thrift MCRs. That is, if the thrift MCRs are applied to the scenario demands, the resultant overall demands are not mathematically guaranteed to equal the demands computed by the MDCP thrift algorithm. However, in the tests done so far, the results are close: see Chapter 3.

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[14] As a *side calculation*, MDCP computes a set of thrift MCRs and the material demands that would result if the thrift MCRs were applied to the scenario demands for industrial output. See Chapter 3.

a. Definitions and Notation for Relevant Quantities

Assume the reference period is R years long; let its years be indexed by $r = 1, \ldots, R$, and let r be some specific year of the reference period. The list of definitions below parallels the list in Section B.1, but quantities have been changed as appropriate to add an index on r. Instead of average annual demand in the reference period, the formulas use demand in the specific year r. Some of the quantities do *not* vary by r but stay the same as in the regular, nonthrift MCR computation.

Γ_r = amount of consumption of the material in year r of the reference period

φ_j = proportion of consumption used in application area j (does not vary with r)

$\gamma_{jr} = \Gamma_r \varphi_j$, = amount of consumption of the material in application area j in year r of the reference period

δ_{ij} = 1 if industry sector i is associated with application area j, and 0 if not

S_j = set of industry sectors associated with application area j (i.e., $S_j = \{i | \delta_{ij} = 1\}$)

q_{ir} = civilian demand (including net exports) in industry sector i in year r of the reference period

$Q_{jr} = \sum_{i \in S_j} q_{ir}$ = total amount of civilian demand of the industries associated with application area j in year r of the reference period. It is clear that Q_{jr} is also equal to the quantity $\sum_{i=1}^{I} \delta_{ij} q_{ir}$.

d_{ir} = defense demand in industry i in year r of the reference period

$D_{jr} = \sum_{i \in S_j} d_{ir}$ = total defense demand of the industries associated with application area j in year r of the reference period

$\omega_{ir} = d_{ir} + q_{ir}$ = output of industry sector i during the year r of the reference period

$\Omega_{jr} = Q_{jr} + D_{jr}$ = total industrial output (i.e., total requirements demand), which equals defense plus civilian plus net exports, of the industries associated with application area j in year r of the reference period

ν_{jr} = proportion of consumption of the material in application area j in year r of the reference period that is for civilian purposes (this symbol is nu, not v or upsilon). This quantity might be explicitly input or might be computed from the underlying industry demands. If explicitly input, it is assumed not to vary with r. If not explicitly input, it can reasonably be set to $Q_{jr}/(Q_{jr} + D_{jr})$, which can vary with r.

$\mu_{jr} = 1 - \nu_{jr}$ = proportion of consumption of the material in application area j in year r of the reference period that is for defense purposes.

b. Construction of the MCRs

Suppose that the reference period (for the particular material being considered) happens to consist of a single year. The formulas for the various MCRs derived in IDA publications D-5477 and P-22689 and earlier in this paper are still valid. Consider a set of MCRs for a given industry sector *i*, the elements of the set corresponding to cases where the reference period consists of specific year *r*. The set has *R* elements. The thrift MCR can then be defined as the minimum value in the set (i.e., the minimum over *r* of the MCRs computed where the reference period is the single year *r*).

For example, the generic MCR, as derived in IDA Document D-5477, is computed by the formula

$$\rho_i = \sum_{j=1}^{J} \gamma_j \frac{\delta_{ij}}{\Omega_j}.$$

Consider the formula

$$\rho_{ir}'^{(\text{thrift})} = \sum_{j=1}^{J} \gamma_{jr} \frac{\delta_{ij}}{\Omega_{jr}}.$$

The left-hand side of this expression can be considered a potential thrift MCR or "proto-thrift MCR." The right-hand side is equal to what the "regular" MCR, computed in accordance with the derivation in IDA Document D-5477, would be if the reference period consisted of the single year *r*. As *r* varies, the value of $\rho_{ir}'^{(\text{thrift})}$ will most likely vary, since different years of the reference period generally have different amounts of consumption and economic output. One can define the thrift MCR as the minimum value over the years of the reference period:

$$\rho_i^{(\text{thrift})} = \min_{r \in \{1,\ldots,R\}} \{\rho_{ir}'^{(\text{thrift})}\}.$$

This value represents the lowest possible value for the amount of material consumed per dollar of output in industry sector *i*. The modeling regards this minimal value as a surrogate for the most efficient or "thrifty" production. As a side calculation, the current version of MDCP computes the thrift MCRs (and outputs them on the _mtf file) and computes the demand that would result if the thrift MCRs were applied to the scenario demands for industrial output (_mto file).[15]

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[15] See Appendix A for an explanation of the MDCP output file codes.

c. Civilian and Defense Thrift MCRs

A similar construction can be performed for the civilian and defense MCRs derived earlier in this paper to obtain thrift variants. Namely, one can define a "proto-civilian-thrift" MCR:

$$\rho_{ir}^{\prime(\text{civ,thrift})} = \sum_{j=1}^{J} \nu_{jr}\gamma_{jr} \frac{\delta_{ij}}{Q_{jr}}.$$

This quantity represents the amount of material that is required to manufacture a given dollar amount of civilian-related output of industry sector *i*, given that the reference period consists of the single year *r* only. The civilian thrift MCR—call it $\rho_i^{(\text{civ, thrift})}$—can then be defined by

$$\rho_i^{(\text{civ,thrift})} = \min_{r\in\{1,\ldots,R\}} \{\rho_{ir}^{\prime(\text{civ,thrift})}\}.$$

A completely analogous derivation can be performed with the defense-related quantities, resulting in a proto-defense-thrift MCR

$$\rho_{ir}^{\prime(\text{def,thrift})} = \sum_{j=1}^{J} \mu_{jr}\gamma_{jr} \frac{\delta_{ij}}{D_{jr}}$$

and a defense thrift MCR

$$\rho_i^{(\text{def,thrift})} = \min_{r\in\{1,\ldots,R\}} \{\rho_{ir}^{\prime(\text{def,thrift})}\}.$$

For emergency investment demand, the civilian thrift MCRs can be used. The 2023 Requirements Report Base Case specifies that thrift can be applied to emergency investment demand in the third and fourth scenario years.

d. MCRs for Thrift and Substitution

The formulas for the MCRs developed for substitution, as described in Section E, can be modified to model thrift and substitution in the same manner as the other MCRs. As noted in Section E, the extent of substitutability might evolve over the course of the scenario. As before, let β_{jt} be the reduction factor due to substitutability in application area *j* in year *t* of the scenario. Do not confuse the two different subscripts: *t* represents year of the scenario and indexes the substitutability factor only; *r* represents year of the reference period and is used to index historical economic and material consumption information.

As of now, the substitution response is modeled for civilian demand only. Following the presentation in Section E, we can posit the civilian-proto-thrift-substitution MCR for scenario year *t* as

$$\rho_{irt}^{\prime(\text{civ,thrift,sub})} = \sum_{j=1}^{J} \beta_{jt} \nu_{jr} \gamma_{jr} \frac{\delta_{ij}}{Q_{jr}}$$

and the civilian-thrift-substitution MCR

$$\rho_{it}^{(\text{civ,thrift,sub})} = \min_{r \in \{1,\ldots,R\}} \{\rho_{irt}^{\prime(\text{civ,thrift,sub})}\}.$$

For each industry sector i, there are T different such MCRs, one for each year of the scenario period. Analogous formulas could be developed for MCRs for defense demands with thrift and substitution.

G. An Issue for Further Work: The Role of Imports

The derivation of the MCRs—and indeed the entire MDCP algorithm—rests on (domestic) industrial output's being well approximated by (domestic) defense demand plus (domestic) civilian demand plus exports minus imports, where all components are expressed in total requirements terms. Using the terminology defined in Section C.1, $\omega_{i0} = d_{i0} + c_{i0} + x_{i0} - m_{i0}$. The MCR computation algorithms described in this paper proceed by separating output into defense-related output and civilian-related output. For the purposes of computing separate defense and civilian MCRs, the output is partitioned into

$$\omega_{i0} = d_{i0} + (c_{i0} + x_{i0} - m_{i0}).$$

Currently, all of the net exports are lumped with the civilian output. One would expect, however, that some imports and exports are defense-related. Net exports should thus be partitioned between defense-related and civilian-related, with each group added to its corresponding domestic demand. The problem has been that data to do this have not been available. As mentioned in Chapter 2, RAMF-SM uses two economic models developed by the Inter-Industry Forecasting Project at the University of Maryland (INFORUM) to forecast demand for goods and services. These models generate values for imports and exports by industry sector but do not separate them into defense-related vs. civilian-related.[16] Some kind of external attempt could be made to establish such separation. For example, arms sales could be considered defense-related exports, whereas imports in a sector could be allocated in proportion to the domestic demand.

[16] Jacklyn Kambic, personal communication, December 15, 2022.

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3. Material Demands and MCR Testing

MCRs can be used to compute material demand; traditionally (before the development of MDCP), they were the main way to do that. As noted, for a given material, the MCR ρ_{im} denotes the amount of material m required to make a certain dollar amount of output of industry sector i. Multiplying the MCR by the demand for output of industry sector i yields the demand for material m by that sector. Summing over the industry sectors yields a total amount of demand for material m. Separate calculations are performed for each demand tier. For example, if the following notation has been defined

- d'_{it} = defense demand for the output of industry sector i in scenario year t under certain specified conditions
- v'_{it} = emergency investment demand for the output of industry sector i in year t under certain specified conditions
- q'_{it} = civilian demand (plus net exports) for the output of industry sector i in scenario year t under certain specified conditions

then the total demand for material m in scenario year t is computed as

$$\sum_{i=1}^{I}\left(\rho_{itm}^{(\text{conditions})}d'_{it} + \rho_{itm}^{(\text{conditions})}v'_{it} + \rho_{itm}^{(\text{conditions})}q'_{it}\right),$$

where the "conditions" superscript is a stand-in for the various specifications as to tier or market response that the MCR might model. (The conditions might be different for different tiers.) The subscript t on the MCR ρ will be used only for the substitution market response. Traditionally, a single MCR was used for all different tiers, and no market responses were modeled.

This analysis examined the material demand files that resulted from applying various different sets of MCRs to the 2023 Requirements Report Base Case industrial demand data. According to the mathematical derivations in Chapter 2 and elsewhere, some of these files should match or partially match the files that MDCP generated for the Base Case. For the other instances, it was desired to see how far apart the MCR-generated demands were from the MDCP demands.

Accordingly, a computer program was written to generate several different MCR sets using the formulas from Chapter 2 with the Base Case economic and material-related data, as follows:

1. Traditional, "generic" MCRs, which can apply to all tiers of demand (Chapter 2, Section B)
2. MCRs explicitly to compute defense demand, without market responses (Chapter 2, Section C)
3. MCRs explicitly to compute civilian demand, without market responses (Chapter 2, Section C)
4. Thrift MCRs for combined civilian and defense tiers (Chapter 2, Section F.3.b). Intended to compute demand under thrift without separating the tiers
5. MCRs to compute defense demand under the thrift market response (Chapter 2, Section F.3.c)
6. MCRs to compute civilian demand under the thrift market response (Chapter 2, Section F.3.c)
7. MCRs to compute civilian demand under the substitution market response (not thrift). Separate values computed for each scenario year. (Chapter 2, Section E)
8. MCRs to compute civilian demand under the substitution and thrift market responses. Separate values computed for each scenario year. (Chapter 2, Section F.3.d)

In what follows, the various MCR sets will be identified by their numbers in the above list.

As side calculations, MDCP itself currently generates MCR sets corresponding to items 1 and 4, above. The underlying algebra implies that the item 1 file should be identical to the MDCP-generated _mcr file and that item 4 should be identical to the MDCP-generated _mtf file. Comparison of the actual files reveals agreement consistent with acceptable rounding error.

The eight MCR sets were used to generate six different material demand files, using the MCR sets listed above, the MCR algorithm, and the industry demand data of the 2023 Requirements Report Base Case emergency scenario. Table 1 lists these files and indicates which MCR set was used for each tier of demand.

Table 1. Test Material Demand Files and Contributory MCRs

File	Characterization	MCR set (see above) used, by tier			Compare to MDCP file
		Defense	Emg. Inv.	Civilian	
O1	Traditional	1	1	1	_mco file (constructed with generic MCRs)
O2	Separate tier treatment	2	3	3	_out file (main output file)
O3	Thrift	4	4	4	_mto file (constructed with thrift MCRs throughout)
O4	Thrift with split tiers	5	6	6	_tdo file (thrift output file without profile)
O5	Substitution for civilian demand	2	3	7	civilian portion of _sbo file
O6	Thrift and substitution for civilian demand	5	6	8	civilian portion of _tso file

Table 1 indicates a correspondence between the demand files and certain output files generated by the current version of MDCP. The proofs developed in Chapter 2 imply that algebraically, many of these demand files should be identical to their corresponding MDCP file, as is the case for files O1, O2, O3, and O5. Some of the actual values do match exactly; most of them differ because of rounding error, which is acceptable given the level of precision of the computer programs. The percentage differences are less than 0.01 percent.

As noted earlier, the MCR thrift algorithm is not algebraically equivalent to the current thrift algorithm of MDCP, so file O4 is not guaranteed to match the MDCP thrift file. The differences are slight; most are less than 0.01 percent, and the largest is about 1.6 percent. Similarly, the values in the civilian portion of file O6 are close to the civilian portion of the MDCP-generated demand file with thrift and substitution.

An additional note: the MDCP computer program accepts a “thrift profile” that can specify the particular years and demand tiers for which thrift will be applied. Material demands under thrift are computed for each combination of tier and scenario year but will appear in the output file for only those combinations the user selects via the profile. (Other tier-year combinations will use the nonthrift demands.) The use of thrift MCRs can be tailored similarly (i.e., the thrift MCRs can be used only for selected tier-year combinations). In the 2023 Requirements Report Base Case, thrift was applied to civilian demand in all scenario years but to defense and emergency investment demand only in the third and fourth years.

Conclusion: More testing should be done, but these initial results indicate that the use of separate defense and civilian MCRs is an acceptable alternative to the current MDCP methodology. Moreover, the thrift and substitution market responses can reasonably be implemented via specially-tailored MCRs. These tier-specific MCRs can be used in both a revised MDCP program and in modified formulations of MPLP that account for defense and civilian demands separately.

4. Extensions to Material Prioritization via Linear Programming (MPLP)

A. Introduction

1. Background and Chapter Plan

As stated at the beginning of this paper, the version of MPLP documented in IDA Paper P-33037 was intended to be a prototype, a proof of concept. As such, a number of simplifying assumptions were made in its formulation, including:

- Defense, civilian, and emergency investment demands were added together, even though RAMF-SM has separate treatments for each category.
- No distinction was made between supply that could satisfy all tiers of demand and supply that could satisfy civilian demand only.
- There was no treatment of the market responses of thrift and substitution, which can act to lessen material demand.
- Only the first-year material demands and supplies were considered, even though the national emergency scenario that RAMF-SM generally models is several years long.

The specialized MCRs developed in the preceding chapters can be used to address the first three limitations, as discussed in Section B, which develops a formulation in which each tier of demand has a separate associated linear programming problem. Modeling of a multiyear scenario is discussed in Section C. The modeling in Section C does not separate demands by tier, and it uses generic MCRs. The formulations of Sections B and C can be regarded as interim steps to the formulation derived in Sections D and E, which takes into account both the tier separation of Section B and the multiyear scenario of Section C. The Section D/E formulation is expected to become the main MPLP formulation going forward. Some concerns and issues must be addressed when integrating the tier separation modeling with the multiyear modeling; they are discussed in Sections D and E. Section F describes an alternative formulation that uses weights for the various tiers.

2. A Special Note: Lower Bounds on Production

IDA Paper P-33037 (Chapter 4) noted the following characteristic of the prototype MPLP formulation:

> Optimal solutions to the material prioritization linear programming formulation frequently specify that *no* production be performed in some subset of the industry sectors under consideration. This result is consistent with optimal LP solutions' often having a number of variables with zero values. From a real-world perspective, however, it is unrealistic for a given industry to stop producing. One might wish to specify a minimum production requirement for every sector. This requirement can be implemented in the LP framework via either explicit constraints or a variable substitution technique.

Minimum production requirements (i.e., lower-bound constraints on industrial production amounts) can be implemented in all the LP formulations presented or derived in this chapter. The lower bound could conceivably vary by demand tier and industry sector. For example, defense demands could have higher minimum production requirements than the other tiers. It is unclear, however, how the lower bound might vary with industry sector. IDA Paper P-33037 developed formulas that could be used if a minimum (fixed) percentage of the demanded output in each industry sector were required to be produced. This kind of constraint can be applied to the formulations in this chapter, perhaps with defense demands having a higher percentage requirement. If it is provided, sponsor guidance on lower bounds could be followed.

The presence of lower-bound constraints might result in the LP having no feasible solution. This situation will occur if the material supplies available in the scenario plus whatever additional supplies can be obtained within the acquisition budget are less than the amounts of material necessary to manufacture the minimum production amount. The acquisition budget would then need to be increased. The linear programming software package will identify if a given LP case has no feasible solution.

B. Separate Analyses for Different Tiers of Demand

The material shortfall computation procedure of RAMF-SM (Substep 2c) is performed mainly by the Stockpile Sizing Module (SSM). The SSM gives demands a strict priority order for satisfaction: defense, then emergency investment, then civilian.[17] This priority ordering makes it possible for MPLP to be run in a sequential fashion to address demand in the various tiers. The procedure can use the specialized MCRs developed in the previous chapters, and it employs certain intermediate data from the RAMF-SM models,

[17] This priority ordering has been part of the specifications of the SSM ever since that model was first developed.

as described below. The presentation in this section assumes that only the supplies and demands in the first scenario year are under consideration. Section D discusses some issues involved in applying the analysis for separate tiers to a multiyear scenario.

1. Data Available from Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM)

RAMF-SM, in its various intermediate output files, provides a wealth of detailed data on demand and supply for goods, services, and materials. Among these data are the available amounts (by material and scenario year) of the two categories of material supply: "totally usable" supply, which can be used to satisfy demand in any tier; and "civilian-usable-only" supply, which can satisfy only civilian demand. The RAMF-SM output also shows goods and services demands by industry sector and scenario year for each category of demand (net exports are added to civilian demand).[18] These data are used in the modeling of the tier priorities.

2. Defense Tier

The first step of the procedure is to use only the defense demands for goods and services (i.e., defense-related industrial output), the defense MCRs (with or without thrift, as desired), and the available totally usable supply amounts. An acquisition budget for additional materials can be included as desired. It is important to include all the materials with shortfalls in any tier, not just those materials with defense shortfalls; the amounts of all materials that are used to manufacture defense-related industrial output affect the amounts of material available to manufacture output for the other tiers. The MPLP linear program will compute the maximum amount of defense-related goods and services that can be produced and the amounts of material used in producing them, including additional material acquisitions as specified.

3. Emergency Investment Tier

RAMF-SM's treatment of emergency investment demand is in some sense a hybrid. Only the totally usable material supply is considered capable of satisfying it, but the civilian MCRs are generally used to compute it. Emergency investment demands, for both goods/services and materials, tend to be minimal.

MPLP can be set up to compute the amount of emergency investment goods and services demand that can be satisfied. First, the totally usable material supply that was actually used to satisfy defense demand is subtracted from the initially available amount of totally usable supply, and the result is used as the supply available to meet emergency

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[18] Demand for goods and services, when expressed in total requirements terms, is essentially the same as demand for industrial output.

investment demand. The value of any additional material that was acquired to meet defense demand is subtracted from the acquisition budget, and the resultant budget amount is used. The goods and services demands for emergency investment are used, with the civilian MCRs. Even though most emergency investment shortfalls are zero, all the materials with shortfalls in some tier—not just those with emergency investment shortfalls—should be included in the analysis. MPLP will determine the maximum amount of emergency investment demand for goods and services that can be produced with the available materials and additional acquisition within the budget.

4. Civilian Tier

The totally usable supply that was not used to meet demand from the other tiers is added to the civilian-usable-only supply, and the result is used. The acquisition budget is decremented by the value of any additional material that was procured to satisfy emergency investment demand. The civilian MCRs are used, with the thrift and substitution adjustments as desired. The civilian goods and services demands (plus net exports) are used. MPLP is then run to determine the maximum amount of civilian demand for goods and services that can be satisfied by the available materials, with additional acquisition as appropriate.

C. Extending MPLP to Model a Multiyear Scenario

1. Introduction

This section develops a linear programming methodology that can extend the MPLP formulation to a multiyear national emergency scenario. Although the national emergency scenarios for the Requirements Reports to Congress extend over a 4-year period, the current version of MPLP, which was developed as a prototype and proof of concept, does not consider material demand and supply amounts that vary year by year. In discussing the MPLP results for the 2021 Requirements Report (RR21), IDA Paper P-33037 noted:

> The RR21 emergency scenario is several years long, but only the supplies, demands, and shortfalls in the first year were considered. Most shortfalls occur then. Larger supplies of materials are often available later in the scenario—too late to offset first-year demands.

In the RR21 emergency scenario, most shortfalls did indeed occur in the first year. However, in the 2023 Requirements Report study, a number of materials also had shortfalls in subsequent scenario years. In dollar terms, only about half the total material shortfall occurred in the first year, for reasons including a more stringent rule for usability of supply from dominator countries. Applying MPLP to only the first-year shortfalls underestimates the severity of the material shortfalls' effects on goods and services production. Extending the MPLP methodology to cover a multiyear emergency scenario would better address this problem.

2. A Multiyear Linear Programming (LP) Formulation

The objective of the LP is to allocate the available material supplies to the various industries so that the maximum amount of goods and services can be produced over the scenario period.

a. Indexes and Constants

Most of the following notation is similar to that in IDA Paper P-33037. However, the terminology differs from that of Chapters 2 and 3.

I = the total number of industry sectors considered

i = index for industry sector ($i = 1, \ldots, I$)

M = total number of materials considered

m = index for material ($m = 1, \ldots, M$)

T = total number of time periods (years) in the scenario

t = index for time period ($t = 1, \ldots, T$)

D_{it} = the demand for industrial output from industry sector i in time period t, measured in millions of constant-year dollars

ρ_{im} = the material consumption ratio for industry sector i for material m, measured in mass units of material m needed to produce a million dollars of output from industry sector i. (This section assumes that a "generic" MCR is being used, which is the same for all demand tiers and time periods.)

s_{mt} = supply of material m that becomes available in time period t after the various decrements of the emergency scenario are applied (measured in mass units of material m). This quantity represents material that was not previously available and does not include newly procured material or material in the National Defense Stockpile (NDS).

N_m = amount of material m in the NDS at the start of the scenario. This quantity can be set to zero if one desires not to include NDS material. This amount does not include additional procurement as specified by the decision variables z_m (see below).

θ_m = market price of material m, $m = 1, \ldots, M$ (\$M per mass unit)

B = total budget for procurement of additional material (\$M)

b. Decision Variables and Discussion

The major decision variables in the LP are the amounts of output x_{it} that industry sector *i* is to produce in time period *t*. The objective function is to maximize the total output (i.e., the sum of the x_{it}). For each time period, the total amount of material *m* used in producing output in time period *t* equals the sum over industry sector of the x_{it} multiplied by the MCR ρ_{im}. That total amount cannot exceed the available supply of material *m* available in time period *t*.[19]

Suppose there is a budget *B* for the procurement of additional materials. Let the decision variable z_m represent the amount of material *m* to procure. For now, assume that the procurements are in place well before the beginning of the emergency scenario and can be used to produce goods and services in any scenario year. This assumption will be relaxed later. The total cost of the material procured (using the prices θ_m) cannot exceed the budget amount. Let us keep track of these procurements independently of what material might be in the NDS.

Assume that material not used in a given time period can be carried over for use in subsequent time periods. Let σ_{mt} denote the amount of material *m* that is available in time period *t* (and thus can be used by the industries to manufacture goods and services in time period *t*). For the first time period, $\sigma_{m1} = N_m + s_{m1} + z_m$. The amount of material *m* actually used to produce goods and services in time period *t* equals $\sum_{i=1}^{I} \rho_{im} x_{it}$. For time periods $t > 1$, the amount of material *m* available, σ_{mt}, is given by the recursion formula

$$\sigma_{mt} = \sigma_{m,t-1} - \sum_{i=1}^{I} \rho_{im} x_{i,t-1} + s_{mt} \ .$$

The quantities σ_{mt} are regarded as (intermediate) decision variables in the linear programming formulation. For each time period, the amount of material *m* used for goods and services production is constrained to not exceed σ_{mt}, for each *m*.

c. Linear Programming Formulation

Putting together the information in the preceding sections yields the linear programming formulation:

19 To produce a million dollars of output of industry sector *i* requires a vector $(\rho_{i1}, \rho_{i2}, \ldots, \rho_{iM})$ of the *M* materials. Linearity is assumed, so to produce *x* million dollars of output of industry sector *i* requires a vector $(\rho_{i1}x, \rho_{i2}x, \ldots, \rho_{iM}x)$ of the *M* materials.

maximize $\sum_{t=1}^{T} \sum_{i=1}^{I} x_{it}$,

subject to

$\sum_{i=1}^{I} \rho_{im} x_{it} \leq \sigma_{mt} \quad m = 1,\ldots, M,\ t = 1, \ldots, T$

$\sum_{m=1}^{M} \theta_m z_m \leq B$

$\sigma_{m1} = N_m + z_m + s_{m1} \quad m = 1, \ldots, M$

$\sigma_{mt} = \sigma_{m,t-1} - \sum_{i=1}^{I} \rho_{im} x_{i,t-1} + s_{mt} \quad m = 1, \ldots, M,\ t = 2, \ldots, T$

$0 \leq x_{it} \leq D_{it} \quad i = 1, \ldots, I,\ t = 1, \ldots, T$

$z_m \geq 0 \quad m = 1, \ldots, M$

$\sigma_{mt} \geq 0 \quad m = 1, \ldots, M,\ t = 1, \ldots, T.$

As in the initial MPLP formulation, the upper bounding constraints $x_{it} \leq D_{it}$ serve to make the problem equivalent to minimizing the total unsatisfied industrial demand. Without such upper bounds, the LP solution might specify that some industries make far in excess of what is needed from them while other industrial demands go unsatisfied. The ratio $(\Sigma\Sigma x_{it})/(\Sigma\Sigma D_{it})$ is a measure of the overall proportion of industrial demand satisfied; that ratio can be reported as the main output statistic of the modeling.

3. Problem Size

Because most of the variables and constraints vary by time period, the problem size is considerably larger than the "basic" MPLP formulation in IDA Paper P-33037. Given *I*, *M*, and *T* as defined above, the total number of decision variables is ($IT + M + MT$), the total number of constraints (not including upper bounding constraints) is $1 + 2MT$, and the number of upper bounding constraints is *IT*. Table 2 shows these numbers for several illustrative values of *I*, *M*, and *T*.

Table 2. Problem Size Parameters for the MPLP Multiyear Formulation

Inputs		**Example 1**	**Example 2**	**Example 3**
I = Number of industry sectors		200	150	30
M = Number of materials		50	36	10
T = Number of time periods (years)		4	4	4
Problem Size Parameters				
Number of decision variables	*IT* + *M* + *MT*	1,050	780	170
Number of regular constraints	1 + 2*MT*	401	289	81
Number of upper bounding constraints	*IT*	800	600	120

Example 1 might be a typical problem size. Example 2 is from the 2021 Requirements Report. Example 3 illustrates the abridgment and truncation necessary to get the problem to fit into Excel Solver's limits of 200 decision variables and 100 constraints, not counting upper bounding constraints. Increasing the number of years in the scenario and including more industry sectors will increase the number of variables and constraints even more. To analyze the multiyear problem adequately requires linear programming software with a capacity larger than Solver's.

4. A Possible Extension

The formulation in Section 2 has assumed that all of the extra procurement (variables z_m) is considered advance planning and will be in place before the start of the emergency scenario. However, the starting time of the scenario is random and might occur while the procurement is in progress. How can one model a situation where not all of the extra procurement might be in place when the scenario begins?

One could approach this problem by doing a preliminary run of the Section 2 formulation to identify the particular materials and amounts to be procured (z_m), within the given budget. These amounts could then be partitioned judgmentally (i.e., with human judgment) by year of expected arrival, where "expected" considers the start date of the scenario. That is, the user could partition the computed value z_m into a set of values $\{z_{mt}, t=1,\ldots, T\}$ which would then be treated as constants. Only the amount z_{mt} would be available as extra procurement at the start of time period t. The equations for the available material amounts σ_{mt} would be redefined as follows:

$$\sigma_{m1} = N_m + z_{m1} + s_{m1}.$$

$$\sigma_{mt} = \sigma_{m,t-1} - \sum_{i=1}^{I} \rho_{im} x_{it-1} + s_{mt} + z_{mt}\ ,\ t = 2, \ldots, T.$$

A modified LP would then be run, using the above equations, treating the z_{mt} as constants, and setting B to 0. The amount of goods and services that could be manufactured might decrease from the preliminary problem, due to the delays in arrival of the procured material.

D. Integrating the Separate Tier and Multiple Year Extensions

On first blush, it would seem straightforward to integrate the two extensions to MPLP—the separate treatment of different demand tiers and the consideration of a multiyear scenario—into a single procedure. One could simply run the multiyear scenario LP using only defense demands, defense MCRs, and totally usable supply, then run it for emergency investment demand, then for civilian demand. The problem comes when determining the available supply for the subsequent tier analysis.

Consider the first "pass" of the procedure, in which defense demands are treated: the multiyear MPLP formulation can be set up using the defense goods and services demands, the defense MCRs, and the available totally usable supply. The defense demand in a given year could be satisfied by supply that originates in that year or any prior year. The LP algorithm will identify the amounts of output in each industry sector to be manufactured in each year. From these values, the amounts of each material *used* in a given year can be derived, but it is not clear in which year the material supply *originated*. Indeed, there might be many possibilities. For the next pass of the procedure, one needs to know the amounts of materials available for that pass. These are the materials, by year, that were *not* used to satisfy defense demand. That is, the amounts that *were* used to satisfy defense demand need to be subtracted from the initial supply—and the year of origination of those amounts is unclear. To allow maximum flexibility in the next pass, it would be better to have the resultant available supply amounts originate as early as possible.

Given the initial amounts of defense-usable supply and the amounts of such supply that were used to satisfy defense demand, an algorithm has been developed to determine an assignment pattern in which the material left unassigned occurs as early in the scenario as possible. Section E presents the details of this algorithm. It is not a linear programming problem and can be implemented on a spreadsheet or via a computer program.

The material left unassigned after the first pass becomes the available supply for the second pass, which sets up the multiyear MPLP to compare this supply with the emergency investment demands for goods and services and thus determine the maximum amount of those demands that can be satisfied. The algorithm can then be run again to determine the earliest possible years of origination for the supply left unassigned. (With the current data, it is almost always the case that all of the emergency investment demand occurs in the first scenario year, so all of the supply originating in the subsequent years remains unassigned.)

For each scenario year, the available civilian-usable only material supply is added to the material left unassigned after the second pass. The sum is used as the supply input to the third pass of the procedure, which sets up the multiyear MPLP to compare that supply against the civilian demand for goods and services (using the civilian MCRs) and to determine the maximum amount of those goods and services that can be manufactured.

Between passes, the acquisition budget must also be updated appropriately. Section E presents a detailed description of the three-pass procedure, including the supply adjustment algorithm.

E. Details of Separate Tier/Multiple Year Procedure

1. Introduction

Section D laid out in broad terms a procedure for addressing the problem of determining the maximum amount of industrial output that could be manufactured in a multiyear scenario with (possibly) different MCRs for each demand tier. The procedure involves a sequence of "passes," each of which considers a different demand tier. This section provides greater detail about the procedure.

The procedure's overall structure is as follows:

1. Set acquisition budget and supply available for pass 1.
2. Run MPLP for pass 1 (defense demands and MCRs).
3. Determine acquisition budget and supply available for pass 2.
4. Run MPLP for pass 2 (emergency investment demands and MCRs[20]).
5. Determine acquisition budget and supply available for pass 3.
6. Run MPLP for pass 3 (civilian demands and MCRs).

In the steps above, "MPLP" refers to the multiyear linear programming formulation discussed in Section C.2. In this version, any extra procurement is assumed to be in place at the beginning of the scenario and is assumed to be capable of satisfying the corresponding type of demand. The notation in that section is used here, with a few modifications.

Prior to invoking this procedure, an underlying SSM run will have identified the material shortfalls in each demand tier. For a given tier, an acquisition budget greater than the shortfall amount for that tier will result in complete satisfaction of the industrial demands in that tier. In such cases, the linear program is not well-defined, and the MPLP run will not be able to identify a unique set of acquisition amounts. To address this issue, an interim value for the budget is set to the tier shortfall amount. MPLP still needs to be run so that the remaining supply available for the next pass can be determined.

As discussed in Section D, there might be many ways of determining the supply available for a subsequent pass, even given that the MPLP run for that pass identified the maximal amount of industrial output that could be produced. One would like to have the remaining material supply, which then becomes available for use in the next pass, occur as early in the scenario as possible. Section E.2 presents this algorithm as a self-contained module. Section E.3 describes the full procedure, discussing each pass in turn.

2. The Earliest Remaining Supply Algorithm

This section assumes that one particular material and one demand tier are under consideration.

Assume the scenario is T years long. Define the following variables.

---

[20] As discussed in Chapter 2, Section D, it is often very reasonable to use the civilian MCRs for computing emergency investment material requirements.

q_τ = the amount of supply of the given material (measured in mass units) that originates in scenario year τ ($\tau = 1, \ldots, T$)[21]

u_t = the demand for the material (for use in industrial production) in scenario year t ($t = 1, \ldots, T$)

$v_{\tau t}$ = the amount of supply originating in year τ that is used to satisfy demand in year t (defined for pairs (τ, t) where $\tau \leq t$).

This algorithm assumes that material is not perishable, so that material not used in a given year can be carried forward for use in subsequent years. However, material originating in a given year cannot be used to satisfy demand in prior years. The values for q_τ and u_t are inputs to the algorithm, but $v_{\tau t}$ is not. The quantity u_t is the amount of material that is used to produce goods and services in scenario year t. This amount might consist of a mixture of material amounts originating in any of the years prior to and including t, and there might be many such mixtures.

The constraints of the underlying linear programming problem guarantee that there is at least one feasible flow pattern $\{v_{\tau t}\}$ such that all demand is satisfied (i.e., $\sum_{\tau=1}^{t} v_{\tau t} = u_t$ for all t). In general, there are many such feasible flow patterns; the LP solution will not specify a particular one. We seek the one where the most flow occurs at the latest possible time, implying that the remaining supply, which is not assigned to meet demand and is therefore available for future passes, occurs as early as possible, allowing for maximum flexibility in the next pass.

The determination of flow pattern $\{v_{\tau t}\}$ that represents the "latest-occurring mixture" is an optimization problem, but the problem is simple enough that linear programming is not needed to solve it. The algorithm is encapsulated in the following pseudocode, shown in Figure 1. This pseudocode uses two additional variables:

r_τ = material originating in year τ that has not been assigned to satisfy demand.

w_t = demand in year t that has not yet had supply assigned to satisfy it.

[21] Unfortunately, the alphabet has only 26 letters. This is not the same q as is used for civilian demand plus net exports in Chapter 2.

```
initialize r_τ to q_τ for τ = 1, …, T.
initialize w_t to u_t for t = 1, …, T.
for t = T down to 1 by –1
        for τ = t down to 1 by –1
            set v_τt = min{ r_τ , w_t }
            decrement r_τ by v_τt
            decrement w_t by v_τt
        end of loop on τ
end of loop on t
```

Figure 1. Pseudocode for Earliest Remaining Supply Algorithm

The final values of w_t will be zero, since there is guaranteed to be a feasible flow pattern. The final values of r_τ represent the supply remaining after all supply-to-demand assignments have been made. The indexing of the loops ensures that the latest-originating supplies are assigned first and that no more supply is assigned than is necessary to meet demand. This pseudocode can be implemented on a spreadsheet or in any programming language.

When this algorithm is implemented as part of the multipass procedure, values will be computed for each material m: the notation used will be $q_{\tau m}$, u_{tm}, $r_{\tau m}$, and w_{tm}.

3. The Multipass Procedure

a. Preliminaries

As stated in Section E.1, the overall structure of the multipass procedure is as follows:

1. Set acquisition budget and supply available for pass 1
2. Run MPLP for pass 1 (defense demands and MCRs)
3. Determine acquisition budget and supply available for pass 2
4. Run MPLP for pass 2 (emergency investment demands and MCRs)
5. Determine acquisition budget and supply available for pass 3
6. Run MPLP for pass 3 (civilian demands and MCRs).

The linear programming problem referred to as "MPLP" above has the following basic structure.

maximize $\sum_{t=1}^{T} \sum_{i=1}^{I} x_{it}$

subject to

$\sum_{i=1}^{I} \rho_{imt} x_{it} \leq \sigma_{mt}$ $\quad m = 1,\ldots, M, t = 1, \ldots, T$

$\sum_{m=1}^{M} \theta_m z_m \leq B$

$\sigma_{m1} = N_m + z_m + s_{m1}$ $\quad m = 1, \ldots, M$

$\sigma_{mt} = \sigma_{m,t-1} - \sum_{i=1}^{I} \rho_{i,m,t-1} x_{i,t-1} + s_{mt}$ $\quad m = 1, \ldots, M, t = 2,\ldots, T$

$0 \leq x_{it} \leq D_{it}$ $\quad i=1, \ldots, I, t = 1,\ldots, T$

$z_m \geq 0$ $\quad m = 1, \ldots, M$

$\sigma_{mt} \geq 0$ $\quad m = 1, \ldots, M, t = 1,\ldots, T.$

This LP is very similar to the "generic" multiyear linear programming problem as articulated in Section C.2, which the reader is encouraged to review, except a time index has been added to the MCRs to account for possibly different market responses in the various scenario years. A variation of this LP is run on each pass of the algorithm, with the input parameters reset as appropriate for that pass. The initial supply values s_{mt} and the acquisition budget B are reset based on the results of previous passes. The MCRs are those developed for the demand tier associated with that pass; they can include the thrift and substitution market responses as desired. The decision variables are x_{it}, the amounts of industrial output to produce, and z_m, the material acquisition amounts. The optimal values of the x_{it} represent the industrial output to produce in the demand tier associated with that pass. If the acquisition budget does not exceed the shortfall for the tier being considered, then the optimal additional acquisition amounts z_m for that pass will be used in that pass and are not available for further passes.

Before the work with MPLP commences, an SSM run will have been performed. The SSM identifies overall shortfalls (sum over all materials) $F^{(def)}$, $F^{(ei)}$ and $F^{(civ)}$ for each demand tier, expressed in dollar terms. It also computes amounts of available supply (after all the decrements of the emergency scenario have been applied) by material and year, separated by use category. Let $S_{mt}^{(tot)}$ denote the amount of totally usable supply of material m that becomes available in year t; let $S_{mt}^{(civ)}$ denote the amount of civilian-usable-only supply of material m that becomes available in year t. (These supply amounts do not include new acquisitions that might be specified by the MPLP runs.) Let B_0 be the amount of acquisition budget initially specified.

b. Pass 1: Defense Demands

The first step of this pass is to set up the inputs for the MPLP run. The acquisition budget for this pass is set as $B_1 = \min\{B_0, F^{(def)}\}$. The available supplies s_{mt} of material m in year t are set to the available (post-decrement) totally usable supplies $S_{mt}^{(tot)}$, as computed by the SSM run. For the first year, supply from the NDS, N_m, can be included if desired. The industry demands D_{it} are the *defense* demand for goods and services in industry sector i in year t, and appropriate defense MCRs are used (see Chapter 2, Sections C.1 and F.3). The linear program is then run with these inputs.

The solution of the LP yields optimal values $x_{it}^{(\text{def})}$ of the defense-related industrial output to be manufactured. The amount of supply used on this pass is given by

$$u_{tm} = \sum_{i=1}^{I} x_{it}^{(\text{def})} \rho_{imt}^{(\text{def})}$$

for each combination of material and year. The LP also finds values z_{m1} of optimal new acquisitions, where the subscript 1 indicates this is pass 1. The values z_{m1} will probably be zero for many materials m. Let C_1 denote the total cost of these new acquisitions: $C_1 = \sum \theta_m z_{m1}$.

To find the supply available for pass 2, we use the earliest remaining supply algorithm (Section E.2). For year 1, the supply input to the algorithm is set as $q_{1m} = N_m + z_{m1} + s_{m1}$. For subsequent years τ, $q_{\tau m}$ is set to $s_{m\tau}$. The material in the NDS, N_m, is included only in year 1 and only for pass 1. On subsequent passes, the appropriate portion of it will be included in the $q_{\tau m}$ and $r_{\tau m}$.

Given the pass 1 available supplies $q_{\tau m}$ and the pass 1 material amounts used u_{tm}, the earliest remaining supply algorithm computes remaining supplies $r_{\tau m}$, which are ready for pass 2.

c. Pass 2: Emergency Investment Demands

As noted earlier, emergency investment demands and shortfalls tend to be minimal. Nonetheless, this pass is necessary to determine the material supplies used to satisfy emergency investment demands and the remaining material supplies that are available to satisfy civilian demands.

The inputs for the linear program are set as follows. The acquisition budget is set to $B_2 = \min\{B_0 - C_1, F^{(\text{ei})}\}$. The first term is the amount of the original budget that was not used to procure material to satisfy defense demands. The supply amounts $s_{m\tau}$ input to the LP are the values $r_{\tau m}$ computed in pass 1. These amounts are all totally usable supply. As noted above, material in the NDS is accounted for in the values $r_{\tau m}$ and is therefore not explicitly included. The industry demands D_{it} are the *emergency investment* demands for goods and services in industry sector i in year t. (Generally, emergency investment demands occur in the first year only.) As explained in Chapter 2, Section D, it is usually appropriate to use the civilian MCRs for emergency investment, with the thrift market response applied in certain years, as desired. The linear program is then run with these inputs.

The solution of the pass 2 LP yields optimal values $x_{it}^{(\text{ei})}$ of the emergency-investment-related industrial output to be manufactured. The amount of supply used on this pass is given by

$$u_{tm} = \sum_{i=1}^{I} x_{it}^{(\text{ei})} \rho_{imt}^{(\text{civ})}$$

for each combination of material and year. The LP also finds values z_{m2} of optimal new acquisitions (subscript 2 for pass 2). Let C_2 denote the total cost of these acquisitions: $C_2 = \sum \theta_m z_{m2}$.

To find the unassigned supply, which will be available for pass 3, we use the earliest remaining supply algorithm. For year 1, the supply input to the algorithm is set as $q_{1m} = z_{m2} + s_{m1}$. For subsequent years τ, $q_{\tau m}$ is set to $s_{m\tau}$. Given the supplies $q_{\tau m}$ and the material amounts used u_{tm}, the earliest remaining supply algorithm computes remaining supplies $r_{\tau m}$, which are ready for pass 3.

d. Pass 3: Civilian Demands

Even though the civilian tier is treated last, it is worth noting that the vast majority of industrial and material demand occurs in it.

The inputs for the linear program are set as follows. The acquisition budget is set to $B_3 = \min\{B_0 - C_1 - C_2, F^{(\text{civ})}\}$. The first term is the amount of the original budget that was not used to procure material to satisfy defense or emergency investment demands. The supply amounts s_{mt} input to the LP are the values r_{tm} computed in pass 2 plus the civilian-usable-only supply amounts $S_{mt}^{(\text{civ})}$. As noted previously, material in the NDS is accounted for in the values r_{tm} and is therefore not explicitly included. The industry demands D_{it} are the *civilian* demands for goods and services in industry sector i in year t, and the civilian MCRs are used, with thrift and/or substitution in the appropriate years, as desired (see Chapter 2). The pass 3 linear program is then run with these inputs. The solution of this LP yields optimal values $x_{it}^{(\text{civ})}$ of the civilian-related industrial output to be manufactured. Since this is the last pass of the procedure, no further supply calculations are necessary. The total industrial output produced is

$$\textstyle\sum_{t=1}^{T} \sum_{i=1}^{I} \left(x_{it}^{(\text{def})} + x_{it}^{(\text{ei})} + x_{it}^{(\text{civ})} \right).$$

The ratio of this amount to the overall industrial demand

$$\sum_{t=1}^{T} \sum_{i=1}^{I} \left(D_{it}^{(\text{def})} + D_{it}^{(\text{ei})} + D_{it}^{(\text{civ})} \right)$$

is a measure of effectiveness of the supply pattern; one minus this ratio is a measure of risk associated with the emergency scenario.

F. An Alternative Formulation: A Weighed Objective Function

This section presents an alternative linear programming formulation that considers a multiyear scenario and treats the different demand categories (tiers) separately but is less cumbersome than the sequential treatment of the tiers described in Sections D and E. Instead of treating the three demand tiers in a sequential, strict priority fashion, this formulation maximizes a weighted sum of the industrial output produced, where the weights vary by tier. For example, defense-related production can be weighted higher than the other tiers. These weights are judgmental. As in Sections D and E, a distinction is made between totally usable supply (which can satisfy demand in all categories) and civilian-usable-only supply, which can satisfy civilian demand only.

1. Indexes and Constants

Most of the following notation is similar but not identical to that of the preceding sections:

I = the total number of industry sectors considered.

i = index for industry sector (i = 1, …, I).

M = total number of materials considered.

m = index for material (m = 1, …, M).

T = total number of time periods (years) in the scenario.

t = index for time period (t = 1, …, T).

k = index for demand tier (category): 1 = defense; 2 = emergency investment; 3 = civilian.

D_{ikt} = the demand for industrial output from industry sector i in time period t in tier k, measured in millions of constant-year dollars.

l_{ikt} = lower bound on the amount of industrial output from industry sector i in time period t in tier k to be produced (measured in millions of constant-year dollars). This quantity can be set to zero if desired.

ρ_{imkt} = the material consumption ratio for industry sector i for material m, measured in mass units of material m needed to produce a million dollars of output from industry sector i for use in tier k in time period t. The MCRs can include adjustments for the market responses of thrift and substitution, as appropriate. These adjustments might be different for different time periods (hence the extra subscript t).

N_m = amount of material m in the NDS at the start of the scenario. This amount can be set to zero if it is desired not to include NDS material. This amount does not include additional procurement as specified by the decision variables z_m (see Section F.2 below); it is assumed to be available at the start of the scenario and to be capable of satisfying demand in any tier.

s_{mt1} = totally usable supply of material *m* that becomes available in the emergency scenario in time period *t*, given the various decrements of the scenario (measured in mass units of material *m*). This variable represents material that was not previously available and does not include additional procured material or material in the NDS.

s_{mt2} = civilian-usable-only supply of material *m* that becomes available in the emergency scenario in time period *t*, given the various decrements of the scenario (measured in mass units of material *m*). This variable represents material that was not previously available and does not include additional procured material or material in the NDS.

θ_m = market price of material *m*, $m = 1, \ldots, M$ ($M per mass unit).

B = total budget for procurement of additional material ($M).

w_k = weighting factor for industrial output in tier *k* (used in the objective function). For now, let it vary by tier only; it could conceivably also vary by industry sector and time period. These weights are judgmental inputs.

2. Decision Variables and Discussion

First we present and briefly define the decision variables; further discussion appears in Section F.4.

x_{ikt} = the amount of industrial output in industry sector *i* that is to be produced in time period *t* for use in tier *k* ($M).

z_m = the extra procurement amount of material *m* (over and above the material that becomes available in the scenario). This material is assumed to be available at the start of the scenario and to be capable of satisfying demand in any tier.

u_{mt11} = the amount of totally usable supply of material *m* that is used to satisfy defense and emergency investment demand in time period *t*.

u_{mt12} = the amount of totally usable supply of material *m* that is used to satisfy civilian demand in time period *t*.

u_{mt2} = the amount of civilian-usable-only supply of material *m* that is used to satisfy (civilian) demand in time period *t*.

σ_{mt1} = the amount of totally usable supply of material *m* that is available in time period *t*.

σ_{mt2} = the amount of civilian-usable-only supply of material *m* that is available in time period *t*.

3. Objective Function

The objective function is to maximize the weighted total industrial output produced, summing over all time periods, industry sectors, and demand tiers, where the tiers are weighted by the input weights w_k, represented as :

$$\text{maximize} \sum_{k=1}^{3} \sum_{t=1}^{T} \sum_{i=1}^{I} w_k \, x_{ikt}.$$

To weight defense demand more heavily, w_1 would be larger than w_2 and w_3, but the relative values are essentially judgmental.

4. Constraints and Discussion

Many of the constraints are the same as in the regular MPLP formulation, with the same rationale.

Each production value x_{ikt} must be less than the demand amount D_{ikt}. As noted elsewhere, this constraint is necessary to avoid a situation in which some industries produce far more than is demanded from them while other industrial demands go unsatisfied. The lower bound constraints $l_{ikt} \leq x_{ikt}$ can also be imposed.

The total additional procurement must be within the acquisition budget: $\sum_{m=1}^{M} \theta_m z_m \leq B$.

The use of two different kinds of supply—totally usable and civilian-usable-only—necessitates introducing some new decision variables. Civilian-usable-only supply can be used to satisfy only civilian demand, whereas totally usable supply can be used to satisfy demand in any tier. We let the linear program decide the amounts of which kinds of supply should be used for which tier by making those amounts decision variables notated u_{mt11}, u_{mt12}, and u_{mt2}, as stated in Section F.2, above.

The decision variables σ_{mt1} and σ_{mt2} represent the amounts of totally usable supply and civilian-usable-only supply, respectively, of material m that are available in time period t. They are defined iteratively for each material m by the recursion formulas

$\sigma_{m11} = N_m + s_{m11} + z_m$ for the first time period.

$\sigma_{mt1} = \sigma_{m,t-1,1} - (u_{m,t-1,11} + u_{m,t-1,12}) + s_{mt1}, \quad t = 2, \ldots, T.$

$\sigma_{m12} = s_{m12}$ for the first time period.

$\sigma_{mt2} = \sigma_{m,t-1,2} - u_{m,t-1,2} + s_{mt2}, \quad t = 2, \ldots, T.$

These formulas become constraints in the linear programming problem.

For each kind of supply, the amounts of supply used cannot exceed the amount available (i.e., for each m and t, $(u_{mt11} + u_{mt12}) \leq \sigma_{mt1}$ and $u_{mt2} \leq \sigma_{mt2}$).

The amounts of supply used to satisfy demands are in fact the sums of the demands, taking the appropriate kinds of supply and tiers of demand into account. There are thus the equality constraints:

$$\sum_{i=1}^{I} \rho_{im1t} x_{i1t} + \sum_{i=1}^{I} \rho_{im2t} x_{i2t} = u_{mt11} \qquad m = 1, \ldots, M,\ t = 1, \ldots, T$$

for defense and emergency investment demand; and

$$\sum_{i=1}^{I} \rho_{im3t} x_{i3t} = u_{mt12} + u_{mt2} \qquad m = 1, \ldots, M,\ t = 1, \ldots, T,$$

for civilian demand.

All decision variables must be nonnegative.

5. Linear Programming Formulation

Putting together the information in the proceeding sections yields the linear programming formulation:

$$\text{maximize} \sum_{k=1}^{3}\sum_{t=1}^{T}\sum_{i=1}^{I} w_k\, x_{ikt}$$

subject to

$\sum_{m=1}^{M} \theta_m z_m \leq B$

$\sum_{i=1}^{I} \rho_{im1t} x_{i1t} + \sum_{i=1}^{I} \rho_{im2t} x_{i2t} = u_{mt11}$ $\quad m = 1, \ldots, M$, $t = 1,\ldots, T$

$\sum_{i=1}^{I} \rho_{im3t} x_{i3t} = u_{mt12} + u_{mt2}$ $\quad m = 1, \ldots, M$, $t = 1, \ldots, T$

$u_{mt11} + u_{mt12} \leq \sigma_{mt1}$ $\quad m = 1, \ldots, M$, $t = 1, \ldots, T$

$u_{mt2} \leq \sigma_{mt2}$ $\quad m = 1, \ldots, M$, $t = 1, \ldots, T$

$\sigma_{m11} = N_m + s_{m11} + z_m$ $\quad m = 1, \ldots, M$

$\sigma_{mt1} = \sigma_{m,t-1,1} - (u_{m,t-1,11} + u_{m,t-1,12}) + s_{mt1},$ $\quad m = 1, \ldots, M, t = 2, \ldots,T$

$\sigma_{m12} = s_{m12}$ $\quad m = 1, \ldots, M$

$\sigma_{mt2} = \sigma_{m,t-1,2} - u_{m,t-1,2} + s_{mt2},$ $\quad m = 1, \ldots, M, t = 2, \ldots, T$

$l_{ikt} \leq x_{ikt} \leq D_{ikt}$ $\quad i = 1, \ldots, I, t = 1, \ldots, T, k = 1, \ldots, 3$

$z_m \geq 0$ $\quad m = 1, \ldots, M$

$\sigma_{mt1} \geq 0$ $\quad m = 1, \ldots, M, t = 1, \ldots, T$

$\sigma_{mt2} \geq 0$ $\quad m = 1, \ldots, M, t = 1, \ldots, T$

$u_{mt11} \geq 0$ $\quad m = 1, \ldots, M, t = 1, \ldots, T$

$u_{mt12} \geq 0$ $\quad m = 1,\ldots, M, t = 1, \ldots, T$

$u_{mt2} \geq 0$ $\quad m = 1,\ldots, M, t = 1, \ldots, T$

6. Problem Size

Given I, M, and T, the number of decision variables and the number of constraints are shown in Table 3 and Table 4.

Table 3. Number of Decision Variables in the Weighted Tier LP

Type of Variable	Number of Variables	Typical Example (I = 200, M = 50, T = 4)
x	$3IT$	2,400
z	M	50
σ	$2MT$	400
u	$3MT$	600
Total	$3IT + M + 5MT$	3,450

Table 4. Number of Constraints in the Weighted Tier LP

Type of Constraint	Number of Constraints	Typical Example (I = 200, M = 50, T = 4)
Budget	1	1
Material use definition	$2MT$	400
Material availability	$2MT$	400
Flow conservation	$2MT$	400
Upper bounding	$3IT$	2,400
Lower bounding	$3IT$	2,400
Nonnegativity	$M + 5MT$	1,050
Total not including bounding and nonnegativity	$6MT + 1$	1,201
Bounding and nonnegativity	$6IT + M + 5MT$	5,850
Total	$6IT + M + 11MT + 1$	7,051

7. An Additional Remark

This formulation assumes that all extra procurement amounts $\{z_m\}$ are available at the outset of the scenario. The delayed availability modeling discussed in Section C.4 could be applied to the above formulation as well.

5. Conclusions

Strategic and critical materials are valuable because they are used to produce essential goods and services. This premise underlies both the traditional RAMF-SM modeling and the MPLP modeling. The difference between the two is a matter of emphasis. The traditional RAMF-SM modeling looks at the material requirements needed to manufacture a particular set of goods and services and determines whether the material supply available in a posited emergency scenario is sufficient to cover those material requirements. If not, the modeling determines the amounts of shortfall for each material under consideration. In contrast, MPLP determines the maximum amount of goods and services that *can* be manufactured with the materials that *are* available in the emergency scenario. MPLP does implicitly compute material requirements, but they are not the focus of the analysis: goods and services demands are.

The initial version of MPLP, which was documented in IDA Paper P-33037, was intended as a proof of concept and did not incorporate all the features of RAMF-SM. This paper is an attempt to integrate more of those features into the MPLP modeling. Some of the specific improvement are as follows.

A. Improvements to the Material Consumption Ratios

MPLP is oriented around the use of MCRs. This paper has developed formulas for MCRs that take several additional features into account, including:

- Separate consideration of the three categories of demand—defense, emergency investment, and civilian—including separate material consumption ratios for each category;
- Consideration of the market responses of thrift and substitution, which can act to lessen the amounts of material needed to produce a certain set of goods and services.

This paper produced an unexpected finding: the material demand computation procedure of the traditional RAMF-SM modeling can be reformulated to explicitly use MCRs throughout. Historically, MCRs were used for some materials but were not considered appropriate for materials with intensive defense-related uses. The development of separate defense and civilian MCRs, as derived in this paper, enables this reformulation. In addition, the MCRs that take into account thrift and substitution can be used within the RAMF-SM material demand computation module.

B. Expanded MPLP Formulations

This paper expanded the formulations of the prototype MPLP formulation discussed in IDA Paper P-33037. These new formulations improve the relevance of MPLP to the problems that RAMF-SM models. These new formulations include:

- Separate treatment of supply that can satisfy all categories of demand vs. supply that can satisfy civilian demand only
- Sequential satisfaction of defense demand, then emergency investment demand, and then civilian demand, using the same priority ordering as in the traditional RAMF-SM modeling
- Treatment of a multiyear scenario in which material not used in one year can be carried forward for use in subsequent years
- Integration of the sequential satisfaction and multiyear scenario methodologies
- An alternative multiyear formulation that maximizes a weighted combination of the industrial output in the different demand categories, rather than a sequential treatment.

C. Future Directions

One possible future direction is the separation of import and export data into defense-related and civilian-related, as discussed in Chapter 2, Section G. In the absence of formal data, one could experiment with some reasonable approaches. This kind of separate treatment could conceivably have significant effects on both the computed material demands and the MPLP results.

A near-term project is to implement all the formulations of Chapter 4 with actual linear programming software. Software with a capacity greater than Excel Solver's has been procured for that purpose. It will be especially interesting to compare the results of the strict priority tier treatment (Chapter 4, Sections D and E) with the weighted tier formulation (Chapter 4, Section F).

A possible extension to the formulations in Chapter 4 is to model a situation in which diminishing marginal returns occur as industrial production increases. This would lead to a nonlinear programming problem, but one with a concave objective function. Such problems are fairly easy for commercial optimization software to solve.

Appendix A.
MDCP Output File Codes

The following table appeared as Table 12 of IDA Paper P-22689, the documentation of the Material Demand Computation Program. As explained there, each MDCP file is characterized by a 3-letter code. The table presents the codes and explanation of the output files' contents.

Table A-1. MDCP Output Files and Codes

Output File Code	Description and Comments
out	Main output file—material demands by material, tier, and year. Suitable for input to the SSM
tfo	Material demands with thrift only, using given profile. Suitable for input to the SSM
sbo	Material demands with substitution only, using given profile. Suitable for input to the SSM
tso	Material demands with thrift and substitution, using given profiles. Suitable for input to the SSM
mss	Informative message file
rpt	Detailed report by application; shows steps of computation
ind	Material demand by tier, year, and industry (see Chapter 2, Section B.4)
in2	Detailed working file: demands by application, tier, year, and industry
app	Material demand by year, tier, and application
ap2	Material demand by tier and application
emb	Shows the embedded demand increment by industry and material; see Chapter 2, Section E.3
mcr	MCRs
mco	Material demands computed via MCR method
mlf	Military fractions for each application
mtf	Thrift MCRs
mto	Output file of demands computed with thrift MCRs via the MCR method without thrift profile by year and tier. Useful for diagnostic purposes
tdo	Demands with thrift only totals by year and tier without profile. Useful for diagnostic purposes

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Appendix B.
Illustrations

Figures

Tables

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Appendix C.
References

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Appendix D.
Abbreviations

IDA	Institute for Defense Analyses
ILIAD	Inter-industry Large-scale Integrated and Dynamic Model
INFORUM	Inter-industry Forecasting Project at the University of Maryland
LIFT	Long-term Inter-Industry Forecasting Tool
LP	Linear Programming; Linear Programming Problem
MCR	Material Consumption Ratio
MDCP	Material Demand Computation Program
MPLP	Material Prioritization via Linear Programming
NDS	National Defense Stockpile
RAMF-SM	Risk Assessment and Mitigation Framework for Strategic Materials
RR21	2021 Requirements Report
SME	Subject Matter Expert
SSM	Stockpile Sizing Module

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14. ABSTRACT

This paper documents some recent methodological improvements to the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM) process, which the Institute for Defense Analyses uses to conduct studies of the National Defense Stockpile of Strategic and Critical Non-fuel Materials. The paper provides mathematical development of a number of new formulas for material consumption ratios (MCRs), which estimate the amounts of material required to manufacture industrial output. The formulas develop MCRs that take into account several specific conditions that might affect material consumption. Use of these specialized MCRs has the potential to increase the realism of RAMF-SM's material demand computation procedures. The paper also describes some enhancements to the Material Prioritization via Linear Programming (MPLP) algorithm, which determines how much industrial output can be produced with the limited supplies of materials available in a national emergency. Some of the new enhancements to MPLP make use of the specialized MCRs.

15. SUBJECT TERMS

Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM); National Defense Stockpile; material consumption ratios (MCRs); Material Prioritization via Linear Programming (MPLP); Strategic Non-fuel Materials; Critical Non-fuel Materials

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