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# Material Prioritization via Linear Programming (MPLP): Proof of Concept and Initial Results

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# Executive Summary

This Institute for Defense Analyses paper develops several linear programming (LP) formulations that can be used to assess the effects of shortfalls of strategic and critical materials in a national emergency. This analysis proceeds from the basic premise that materials are valuable because they are used to produce essential goods and services. Each industrial sector of the U.S. economy is assumed to need a particular mix of materials in particular proportions to produce its output. In a national emergency, there might not be sufficient materials available (because of increased demand and/or reductions in supply) to produce all the needed goods and services. The basic LP formulation determines how many of these goods and services can be produced if the available materials are allocated to the industrial sectors in an optimal manner, i.e., one that maximizes industrial output. The proportion of goods and services demand that remains unsatisfied is a measure of risk of the material shortfall. Linear programming sensitivity analysis techniques, including examination of the dual variables in the LP, can provide insight into the effect on goods and services production if more of a given material is available.

One of the surprising findings of the paper is the existence of "slack materials." A slack material can be defined as one whose available supply is not fully utilized in the optimal allocation to the industrial sectors, even though the material may be in overall shortfall. It seems paradoxical that some of a shortfall material would not be used to produce goods and services. The reason is that the industry sectors use combinations of materials.

This paper develops several additional LP formulations to explore further the interface between material supplies and industrial production. One of these formulations assumes that there is a certain budget for acquisition of additional materials and determines which materials should be acquired in what amounts, within that budget, to maximize the amount of industrial production possible. A second formulation includes the requirement that a certain minimum percentage of the demand for output in each industrial sector be satisfied. A third formulation expresses demand for goods and services in terms of demand by end users, rather than total industrial output.

The LPs were developed in the course of conducting analyses of the National Defense Stockpile of Strategic and Critical Non-fuel Materials. This stockpile is maintained under law by the Department of Defense, which is required to submit biennial Requirements Reports to Congress concerning what materials the stockpile should contain. This paper presents a

number of illustrative results of the various LP formulations using data from the analytic process underlying the 2021 Requirements Report.

# Contents

# 1. Introduction and Background

This paper presents several linear programming (LP) formulations that can help establish a priority ordering for strategic and critical materials (S&CMs) and can guide government acquisitions of such materials for the National Defense Stockpile (NDS) or for other use in a national emergency.

The Strategic and Critical Materials Stock Piling Act (50 U.S.C. §98 et seq.) provides for the establishment and maintenance of a National Defense Stockpile of strategic and critical non-fuel materials.[1] The Act mandates that the Department of Defense (DOD) submit periodic Requirements Reports to Congress on the status of S&CMs.[2] Much of the analysis underlying those reports has been conducted using the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM). RAMF-SM, which was developed by the Institute for Defense Analyses (IDA), is a suite of procedures, models, and databases that can be used to:

- Assess shortfalls of strategic materials in a national emergency scenario;
- Determine the risks to national security of having such shortfalls; and
- Develop and assess strategies to help reduce those risks.

Appendix A contains a brief description of RAMF-SM.

One key step of RAMF-SM, Step 2, involves assessing material shortfalls in a national emergency scenario. This step encompasses four parts, or substeps:

- Substep 2a. Identify the domestic demand for goods and services (defense and essential civilian) in the scenario.
- Substep 2b. Determine the amounts of S&CMs needed by U.S. firms to manufacture these goods and services (i.e., the demand for S&CMs).
- Substep 2c. Determine the supply (domestic and foreign) of S&CMs available to the United States in the scenario and compare that supply with the demand to determine material shortfalls.

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1 The National Defense Stockpile was established in the World War II era and has been managed by the Department of Defense (DOD) since 1988.

2 The most recent such report as of this writing is Office of Under Secretary of Defense for Acquisition and Sustainment, *Strategic and Critical Materials 2021 Report on Stockpile Requirements* (n.p.: U.S. Department of Defense, February 2021).

- Substep 2d. Model market responses to the shortfalls (see Chapter 5, Section F.1).

The first three substeps are the ones of major concern in this paper. The bulk of Substep 2a is implemented via economic modeling, using forecasting models that consider a set of several hundred industry sectors that together span the U.S. economy.[3] Further adjustments are made to determine the subset of goods and services that are considered essential in the emergency scenario and the additional defense demands arising from the emergency. Substep 2b is often performed using the material consumption ratio (MCR) methodology, as described in Chapter 2, Section C. The available material supplies in Substep 2c are generally lower than in peacetime because of the exigencies of the emergency scenario (e.g., cutoff or diminution of supply from hostile countries).

The result of the diminished supply in Substep 2c is that there are not enough materials available to manufacture all of the essential demand for goods and services. That is, there are usually shortfalls of some materials. The determination and reporting of these shortfalls is one of the major tasks of the Requirements Reports. The question arises, however, what proportion of the essential demand *can* be manufactured, given that the available materials are used as efficiently as possible? This issue was explored in the 2015 Requirements Report using linear programming to determine an allocation of the material supplies to the various industry sectors.[4] The portion of demand that remains unsatisfied can be considered to be a measure of the risk of the material shortfall to U.S. national security.

Suppose one wanted to procure additional S&CMs (e.g., for the National Defense Stockpile) to reduce that risk. Which materials should be procured and in what amounts? This paper addresses these questions via a linear programming approach similar to the one used in the 2015 Requirements Report. The underlying philosophy is that materials are valuable because they are used to manufacture essential goods and services.

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[3] Currently, RAMF-SM makes use of two economic models developed by the Inter-industry Forecasting Project at the University of Maryland (INFORUM). The models are named LIFT (acronym for long-term inter-industry forecasting tool) and ILIAD (acronym for inter-industry large-scale integrated and dynamic model). References to documentation of these models are as follows:
Douglas S. Meade, *The INFORUM LIFT Model*, Technical Documentation (College Park, MD: Inter-industry Forecasting Project, University of Maryland, November 3, 2017), https://www.researchgate.net/publication/266049640_The_LIFT_Model. Douglas S. Meade, et al, *ILIAD* (College Park, MD: Inter-industry Forecasting Project, University of Maryland, December 2011).

[4] See “Shortfall Consequences Assessments,” by D. Sean Barnett and Jerome Bracken. This text appears as Appendix 20 of Thomason, James S., et al., *Analyses for the 2015 National Defense Stockpile Requirements Report to Congress on Strategic and Critical Materials*, Vol. I, Material Assessments and Associated Analyses, IDA Paper P-5190 (Alexandria, VA: Institute for Defense Analyses, August 2015).

Traditionally, RAMF-SM first determines demands for essential goods and services and then computes the amounts of materials necessary to manufacture these goods and services. Here, one starts with a set of available material supplies and tries to determine how many essential goods and services can be manufactured with them. In effect, this is looking at the same problem from two different angles. However, the traditional RAMF-SM Step 2 analysis has generally examined materials separately, trying to see what factors would decrease demand or increase supply for an individual material. When trying to determine the industrial production that is possible with a given set of material supplies, it is necessary to look at materials in combination, since each industry sector generally uses a number of different materials to manufacture its products.

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# 2. Linear Programming Formulations

## A. Introduction

As noted above, the basic concept is that materials are valuable because they are used to produce goods and services. If there are materials in shortfall, then there is not enough material on hand to produce all the necessary goods and services. We are interested in how much can be produced with the materials that are on hand, and the additional production that would be possible if more materials were available. The problem is not straightforward because producing a good (or service) generally involves a set, or vector, of different materials in certain proportions; these vectors generally differ among goods.

## B. Demands for Goods and Services

The Material Prioritization using Linear Programming (MPLP) process starts with estimates of demands for goods and services for a number of industry sectors that together span the U.S. economy. (The Inter-industry Large-scale Integrated and Dynamic (ILIAD) economic model specifies 352 sectors.) One can distinguish "final" demands, which comprise goods and services used by end-users such as households and the Federal Government, from inter-industry demands, which are goods and services used by other industries to produce their products. Inter-industry demands are also called intermediate demands. Total requirements demands are the sum of final demands and inter-industry demands. Economic input-output theory has developed methodology by which total requirements demands can be computed from the final demands. In particular, a vector of final demands (by industry sector) can be multiplied by a Leontief inverse matrix to produce a corresponding vector of total requirements demands. For more information on input-output theory, see the paper by Ao[5] and the book by Miller and Blair.[6] The book by Gass develops input-output theory in the context of linear programming.[7] In the analyses for the Requirements Reports, final demands in the scenario are determined by economic

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5 Wallice Ao, Justin M. Lloyd, Amrit K. Romana, and Eleanor L. Schwartz, *Methods in Macroeconomic Forecasting Uncertainty Analysis: An Assessment of the 2015 National Defense Stockpile Requirements Report*, IDA Paper P-5310 (Alexandria, VA: Institute for Defense Analyses, March 2016).

6 Ronald E. Miller and Peter D. Blair, *Input-Output Analysis: Foundations and Extensions*, Second edition (New York City, NY: Cambridge University Press, 2009).

7 Saul I. Gass, *Linear Programming: Methods and Applications*, Third edition (New York City, NY: McGraw-Hill, 1969).

forecasting models (with adjustments for scenario-specific characteristics); they are then multiplied by a Leontief inverse matrix to yield total requirements demands.

Some of the demands for goods and services, both final and inter-industry, are met by imports.[8] The rest must be met by U.S. industry. The term industrial demand is used to denote the demand for the output of (U.S.) industry, in other words, industrial demand is the total requirements goods and services demands minus imports. Materials are frequently used to produce items such as parts or components, which are sold to other manufacturers. That is, material usage by industry is not tied only to final demand, but frequently to inter-industry demand as well. Material needs by U.S. manufacturers are a function of industrial demand. Industry demands are measured in millions of constant-year dollars. Material amounts are measured in mass units (such as tons); different materials can have different units of measure.

In an equilibrium situation, such as peacetime, it is assumed that sufficient materials are available to produce all the demand. In a national emergency scenario, fewer imports of goods, services, and materials might be available, possibly leading to insufficient material on hand to satisfy all the industrial demand. The problem explored in this paper concerns how to use the available material as efficiently as possible.

## C. Material Consumption Ratios and Linear Programming

One of the major data sets used by RAMF-SM is the set of material consumption ratios (MCRs), which associate material usage with the economic output of specific industry sectors. For each combination of material and industry sector, the MCR gives the number of mass units of that material required to make a certain "reference amount" in constant-year dollars of the output of that industry sector. This paper uses a million dollars as the reference amount; the units are unimportant as long as all values are set consistently.[9] The MCRs provide a linkage between material use and economic activity. A particular industry sector can utilize several different materials, and a given material might be used by many industry sectors. The MCRs were originally developed based on data collected by the Department of Commerce (DOC) regarding material usage by specific industries. The MCR computation methodology has been expanded to be able to process information from other agencies and subject matter experts.

The MCRs developed for use by RAMF-SM play a pivotal role in the LP formulation for material prioritization. Each industry sector has its own "recipe" of associated material requirements, as generated via the MCRs. Many industry sectors can demand a given

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8 The economic models that forecast demand also forecast imports.

9 RAMF-SM measures industrial output in millions of constant-year dollars. In some databases the MCRs are expressed in terms of material quantity per *billion* dollars of industrial output; the appropriate conversion can be performed as necessary.

material. Not enough materials are available to manufacture all the essential industrial output. To determine the maximal amount that *can* be manufactured, one must evaluate many possible combinations of goods and services and determine whether the material amounts required to manufacture a given combination exceed the available material supplies. Linear programming is a good tool for doing this. The LP algorithm moves from combination to combination in a logical, efficient way.

It is worth noting that in the data used for the 2021 Requirements Report (RR21), only about 31% of the total demand for goods and services (expressed in dollar terms) is in sectors that require the use of materials. Of the 352 sectors considered by the economic model, only 185 are associated with any material use (i.e., have MCRs). The process of developing the MCRs involves identifying the applications a material is used in (e.g., electronics) and then associating each application with a set of industry sectors. Some industry sectors simply are not associated. In general, service industries do not use materials. Sectors without MCRs do not enter into the MPLP analysis.

## D. Notation and Assumptions

Define the following notation:

$I$ = the total number of industry sectors considered.

$i$ = index for industry sector ($i = 1,\ldots, I$).

$M$ = total number of materials considered.

$m$ = index for material ($m = 1,\ldots, M$).

$D_i$ = the demand for industrial output from industry sector $i$ (in the case of interest), measured in millions of constant-year dollars.

$\rho_{im}$ = the material consumption ratio for industry sector $i$ for material $m$, measured in mass units of material $m$ needed to produce a million dollars of output from industry sector $i$.

$S_m$ = supply of material $m$ available after the various decrements of the emergency scenario are applied (measured in mass units of material $m$).

To produce a million dollars of output of industry sector $i$ requires a vector $(\rho_{i1}, \rho_{i2},\ldots, \rho_{iM})$ of the $M$ materials. Linearity is assumed, so to produce $x$ million dollars of output of industry sector $i$ requires a vector $(\rho_{i1}x, \rho_{i2}x,\ldots, \rho_{iM}x)$ of the $M$ materials.

The objective of the LP is to allocate the available material supplies to the various industries so that the maximum amount of goods can be produced. The decision variables in the LP are the amounts of output $x_i$ that industry sector $i$ is to produce. The objective function is to maximize the total output, i.e., the sum of the $x_i$. The total amount of material

$m$ used in producing this output equals the sum over industry sector of the $x_i$ multiplied by the MCR $\rho_{im}$. That total amount cannot exceed the available supply of material $m$.

## E. Production LP Formulation

The notation and concepts of the preceding section can be combined and formalized as a linear programming problem, the "production LP formulation:"

maximize $\sum_{i=1}^{I} x_i$

subject to

$\sum_{i=1}^{I} \rho_{im} x_i \leq S_m \quad m=1,\ldots, M$

$0 \leq x_i \leq D_i \quad i=1,\ldots, I.$

The upper bounding constraints $x_i \leq D_i$ serve to make the problem equivalent to minimizing the total unsatisfied industrial demand. Without such upper bounds, the LP solution might specify that some industries make far in excess of what is needed from them while other industrial demands go unsatisfied. Using the optimal values of the $x_i$, the amount of unsatisfied demand, $D_i - x_i$, and proportion of demand unsatisfied, $(D_i - x_i)/D_i$, can be noted for each industry. These values are measures of the risk of not having enough material available.[10] The ratio $(\Sigma x_i)/(\Sigma D_i)$ is a measure of the overall proportion of industrial demand satisfied; that ratio is reported as the main output statistic of the modeling.

## F. Material Prioritization via Dual Variables

Every linear programming problem has associated with it a set of dual variables, one for each constraint in the LP. The dual variable associated with a given constraint shows the change in the objective function value that would occur if the constraint were relaxed by one unit. In the LP formulation above, the dual variable for the constraint on material $m$ indicates the extra amount of goods and services production (in $M) that would be possible if there were one more mass unit of material $m$ available, everything else being held constant. In a sense, the dual variable represents a fair price for the material; for this reason, dual variables are sometimes called shadow prices. If the market price of a material is less than the associated dual variable, then the material would seem to be a good candidate for acquisition. Materials can be prioritized by taking the ratios of the dual variables to the market prices. (Some adjustment for inflation might need to be made because the goods

[10] See Appendix 20 of James S. Thomason et al., *Analyses for the 2015 National Defense Stockpile Requirements Report*.

and services production dollar amounts are measured in the constant-year dollars used in the economic modeling, while the market prices are in current dollars.)

## G. LP Formulation with Budget Constraint

The dual variables are meaningful only for small changes in one constraint at a time. Accordingly, the question of which materials, in what amounts, to acquire to ameliorate shortfalls might be more appropriately addressed via an LP formulation with a budget constraint. Let all the previous notation still stand, and in addition, define new notation as follows:

$\theta_m$ = market price of material $m$, $m$=1,…, $M$ ($M per mass unit).

$B$ = total amount of money available for procuring materials ($M).

$z_m$ = number of mass units of material $m$ to procure (in addition to the material already on hand). The $z_m$ are decision variables in the LP.

This leads to the "budget LP formulation:"

maximize $\sum_{i=1}^{I} x_i$

subject to

$$\sum_{i=1}^{I} \rho_{im} x_i \leq S_m + z_m \quad m=1,\ldots,M$$

$$\sum_{m=1}^{M} \theta_m z_m \leq B$$

$$0 \leq x_i \leq D_i \quad i=1,\ldots,I$$

$$z_m \geq 0 \quad m=1,\ldots,M .$$

The parameters $\theta_m$ and $B$ can be expressed in current dollars, even if the $x_i$ and $\rho_{im}$ are measured in the constant dollars used in the economic modeling. In this formulation, it might be appropriate to have the available material supplies $S_m$ include the material amounts in the National Defense Stockpile so that all the budget $B$ can be used to procure additional material. The dual variable associated with the budget constraint indicates how much more industrial production would be possible if a million dollars more funding for material procurement were made available.

It is clear that the production formulation is equivalent to the budget formulation with a budget of zero dollars.

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# 3. Slack Materials and the Paradox of the Unused Material

## A. Slack Materials

One of the most surprising findings of the LP results is that even if a material's available supply in an emergency scenario is less than the demand for the material (i.e., the material is in shortfall), not all of that available supply is necessarily usable to produce goods and services. There might be quite a few such "slack materials." To express the above thoughts in symbols, use the notation defined previously and let $\{x_i\}$ denote the optimal solution to the production formulation. Material $m$ is in shortfall, meaning that the full demand for material $m$, $\sum_{i=1}^{I} \rho_{im} D_i$, is greater than the material supply $S_m$. However, it is possible that $\sum_{i=1}^{I} \rho_{im} x_i$ is strictly less than $S_m$. The overall dollar amount of material used in the optimal solution is $\sum_{m=1}^{M} \theta_m (\sum_{i=1}^{I} \rho_{im} \, x_i)$. The ratio of this amount to the total dollar amount of available material supply, $\sum_{m=1}^{M} \theta_m S_m$, is a measure of the fraction of available material that is used to produce goods and services, and one minus this ratio is the fraction of available material unused. This latter fraction is reported as one of the statistics of the LP run.

The fact that slack materials exist is perhaps the major conclusion of this exercise. It is not immediately apparent that the full available supply of a shortfall material is not necessarily used to manufacture goods and services. The slack materials tend to be those for which the shortfall is not that severe, i.e. the ratio of available supply to demand is relatively close to one. The supply constraints corresponding to the slack materials have dual variable values of zero.

## B. The Paradox of the Unused Material

It seems paradoxical that there are there slack materials. The phenomenon probably occurs because materials are used in combination rather than singly to manufacture goods. Each industry sector requires a set of materials in specific proportions. One of those materials might be in especially severe shortfall, so only a small amount of industrial output can be produced. There is more than enough of a number of other materials to produce that small amount, even if there is not enough to produce the full amount of demand.

A partial formalization of the above idea is as follows. Suppose there is only one industry sector, and assume that industry uses all the materials in question ($m$=1,…, $M$). Let $\rho_m$ be the MCR for material $m$ (for the one industry) and let $S_m$ be as defined previously. Note that the $S_m$ values for different materials are computed independently of one another.

To produce a million dollars of output of the industry requires a vector $(\rho_1, \rho_2, \ldots, \rho_M)$ of the $M$ materials. By linearity, to produce $x$ million dollars of output of the industry requires a vector $(\rho_1 x, \rho_2 x, \ldots, \rho_M x)$ of the $M$ materials.

The available supply of material 1 is $S_1$. The amount of production possible in the industry is at most $S_1/\rho_1$; otherwise, the amount of material 1 required will exceed the available supply. Similarly considering Material 2, the amount of production in the industry is at most $S_2/\rho_2$, and so forth. All together, these constraints imply that the maximum possible amount of production in the industry, call it $x$ (in millions of dollars), must satisfy

$$x \leq q = \min_m \{S_m/\rho_m\}.$$

Let $m^*$ denote the material such that $S_{m^*}/\rho_{m^*} = q$. (One could call material $m^*$ the tight material.) The production amount $x$ will be at most $q$. For all the other materials $m \neq m^*$, no more than the amount $q\rho_m$ will be used. Since $q \leq S_m/\rho_m$, then $q\rho_m \leq S_m$, and the amount $S_m - q\rho_m$ is unused (i.e., slack, excess).

The actual problem is more complicated because there are many industry sectors, and a material that is excess for one sector might find use in another. However, this argument illustrates how slack in material supply might occur.

## C. The Paradox of the Unused Material—a Whimsical Example

The following notional example might further clarify why the unused material paradox occurs.

Suppose that there is only one essential good under consideration—brownies. Assume that in the emergency scenario, it has been determined that there is a demand for ten pans of brownies. Suppose that the manufacture of brownies requires four materials: flour, sugar, butter, and chocolate. Posit that due to supply reductions in the emergency scenario, there are shortfalls of all these materials. In particular, assume that:

- There is enough flour available to manufacture six pans of brownies;
- Enough sugar available to manufacture five pans;
- Enough butter available to manufacture three pans; and
- Enough chocolate available to manufacture one pan.

With the materials available, how many pans of brownies can be manufactured? The answer is only one pan—limited by the availability of chocolate. Only one pan's worth of flour, sugar, and butter can be used; the rest cannot. Even though there are shortfalls of those three materials, obtaining more of them would not increase the number of pans of brownies that could be manufactured because chocolate is the most limited material.

If more chocolate were obtained, additional amounts of the available other ingredients could be used, without any extra purchase, to increase the number of pans of brownies that could be manufactured. If more than two pans’ worth of (additional) chocolate were obtained, however, no more than three pans of brownies could be manufactured because butter would then become the most limited material.

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# 4. Meeting a Minimum Percentage of Demand

## A. Introduction

Optimal solutions to the material prioritization linear programming formulation frequently specify that *no* production be performed in some subset of the industry sectors under consideration. This result is consistent with the fact that optimal LP solutions often have a number of variables with zero values. From a real-world perspective, however, it is unrealistic for a given industry to stop producing. One might wish to specify a minimum production requirement for every sector. This requirement can be implemented in the LP framework via either explicit constraints or a variable substitution technique.

This minimum production requirement could, of course, vary by industry sector, but it is not clear exactly how much requirement should be applied to each particular sector. Initially, one can explore cases in which the same percentage of demand in each sector must be satisfied.

To satisfy a given percentage of each sector's demand will require certain amounts of materials as determined by the MCRs, demand amounts, and percentage values. It is possible that for some materials, the minimum material requirement will exceed the available supply. How should such cases be treated?

## B. Mathematical Formalization

Let all notation be as defined previously, and in addition, let $\alpha$ be a parameter (between 0 and 1) representing the minimum fraction (or equivalent percentage) of industrial demand that is to be satisfied in each sector.

Define, for each material $m$,

$$Q_m = \sum_{i=1}^{I} \rho_{im} D_i$$

$Q_m$ is the amount of material $m$ that is needed to satisfy the full amount of demand for industrial output. By the definition of a shortfall material, $Q_m$ exceeds the available supply, $S_m$, for each material. By the linearity of the MCR assumption, to satisfy the proportion $\alpha$ of industry demand in each sector would require $\alpha Q_m$ units of material $m$, for each material. Depending on the material, $\alpha Q_m$ might be less than, equal to, or greater than $S_m$. If there are some materials $m$ for which $\alpha Q_m > S_m$, then it is impossible to satisfy proportion $\alpha$ of industry demand in each sector without acquiring additional material. The quantity $\max\{(\alpha Q_m - S_m), 0\}$ can be called the deficit for material $m$. The total dollar value of the additional material required, call it $R$, is given by the sum of the dollar values of the deficits. That is,

$$R = \sum_{m=1}^{M} \theta_m \max\{(\alpha Q_m - S_m), 0\}.$$

## C. Implementation in the LP Formulation

There are two ways that the imposition of a minimum satisfaction fraction can be implemented in the LP formulation. One way is to impose explicit constraints that the production in each industry sector, $x_i$, be greater than or equal to $\alpha D_i$, for $i$=1,…, $I$. Then if $\alpha Q_m > S_m$ for at least one material $m$, the production formulation problem has no feasible solution. If the budget formulation is used with a budget of less than $R$, then the problem also has no feasible solution. The LP package should be able to identify these cases. If the budget formulation is used with a budget greater than $R$, it seems reasonable (for feasibility) that the LP solution would indicate that purchases of at least the deficit amounts be made for each material with a deficit.

An alternative to specifying explicit constraints is to use a substitution technique, which produces results that are algebraically equivalent to the explicit constraints method. (Appendix B provides a formal proof of this equivalence.) The substitution technique does not increase the number of constraints in the LP, which might be an advantage if the LP software being used has a low limit on the number of constraints.[11] The substitution technique starts by computing the material requirements necessary to produce the minimum industrial amounts specified. These requirement amounts are subtracted from the available material supplies. The decision variables in the LP are the incremental amounts of industrial output to produce in sector $i$, over the minimum, using the remaining material supplies. That is, each $S_m$ is decremented by the amount $\alpha Q_m$ and the resultant value is used as the available material supply in the LP. The value of the new decision variable equals $x_i - \alpha D_i$.

If a material supply becomes negative, it must be assumed that the deficit amount of each material will be purchased for a total of $R$ million dollars. The amount $R$ can be accounted for in a side calculation rather than the LP itself. The available material supplies in the LP formulation will then be nonnegative. Alternatively, the negative values $S_m - \alpha Q_m$ can be input as the available supplies and the budget can be set to a value that is at least $R$. A feasible solution should then specify the appropriate additional material purchases. This was the approach used in the examples in Chapter 5, Section E.

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[11] This is the case with Excel Solver, which has been used to generate the results reported in Chapter 5.

# 5. Results from the 2021 Requirements Report (RR21) Base Case

## A. Data Setup and Assumptions

The MPLP procedure was implemented on certain data from the 2021 Requirements Report Study Base Case. The following data were used essentially as is:

- Emergency scenario specification
- Lists of industry sectors and materials considered
- MCRs
- Goods and services demands
- Available material supplies
- Material prices.

A number of simplifying assumptions were made.

- The analysis was confined to a subset of materials (36) for which MCRs were easily computable and that had shortfalls.
- The RR21 emergency scenario is several years long, but only the supplies, demands, and shortfalls in the first year were considered. Most shortfalls occur then. Larger supplies of materials are often available later in the scenario—too late to offset first-year demands.
- All categories of demand were added together: defense plus civilian plus emergency investment plus net exports. Unlike RAMF-SM, a priority was not set on meeting defense needs, and a distinction was not made between supply that could meet all types of demand and supply that could satisfy civilian demand only.[12]
- The material demand computation procedure in RAMF-SM uses MCRs (or an equivalent algorithm) for most of the materials. For some materials with intensive military demands, the MCR method is not completely appropriate and

[12] The Base Case included one material for which overall available supply exceeded overall demand but defense-usable supply was insufficient to satisfy defense demand, creating a defense supply shortfall. That material is not being analyzed in the current paper, which is combining all kinds of available supply and all categories of demand.

RAMF-SM uses an alternative algorithm. For the current work, MCRs are used for all the materials.[13]

- RAMF-SM models several possible responses of the market to material shortfalls. These responses act to decrease demand and increase supply. "Gross shortfalls" are those before the market responses are modeled; "net shortfalls" include the market response effects. For the current work, the pre-market-response demand and supply data and gross shortfall information were used.[14] The only exceptions are the extra sell cases discussed in Chapter 5, Section F.1.

## B. Excel Solver Setup and Limits

Excel Solver was chosen as the software with which to implement the LP. The data for the Requirements Reports are maintained in Excel, and it is straightforward to transfer them to a spreadsheet on which Solver can be invoked. However, Solver can handle at most 200 decision variables and at most 100 constraints, not counting upper bounding constraints. Some truncation of the data was necessary for the problem to fit into Solver, but this should not invalidate the results.

The number of decision variables in the production formulation is equal to the number of industry sectors considered. In the budget formulation, it is equal to the number of industry sectors considered plus the number of materials considered. The number of constraints equals the number of materials considered, plus the upper bounding constraints ($x_i \leq D_i$), plus the budget constraint (for the budget formulation). The LP simplex algorithm has automated ways of handling upper bounding constraints, thus such constraints do not contribute to problem complexity in the same way that the regular constraints do.

As noted earlier, though the underlying economic models consider 352 different industry sectors, only 185 of the sectors have associated MCRs. Furthermore, 150 of those 185 sectors account for over 99% of the material demand for the 36 modeled materials. In order for the problem to be within Excel Solver's limits, only those 150 sectors were included. The production formulation thus had 150 decision variables and 36 constraints, plus upper bounding constraints. The budget formulation has 186 decision variables and 37 constraints (the 37$^{th}$ is the budget constraint), plus upper bounding constraints. (If explicit lower bounds on the industrial production amounts were included, there would also

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13 For more information about MCRs and other material demand computation algorithms, see Eleanor L. Schwartz, *The RAMF-SM Material Demand Computation Program: Documentation and User's Guide*, IDA Paper P-22689 (Alexandria, VA: Institute for Defense Analyses, March 2022).

14 For a discussion of market responses and the concept of gross vs. net shortfalls, see James S. Thomason, et al., *IDA Contributions to the Strategic and Critical Materials 2019 Report on Stockpile Requirements,* Vol. I: *Unclassified Contributions*, IDA Paper P-10727 (Alexandria, VA: Institute for Defense Analyses, October 2019).

be 150 lower bound constraints, raising the total number of constraints to beyond Excel Solver's limit.)

## C. Data Characteristics

Some of the features of the data were mentioned above. Some additional data characteristics are as follows. Total industry demand in the first scenario year is about $23 trillion (in year 2009 dollars), but only about 31% of this industry demand ($7 trillion) induces any material demand. For the modeled materials, gross shortfall is about 26% of demand, in overall dollar terms, i.e., about 74% of the material demand is satisfied. Shortfall-to-demand ratios vary widely between materials. The overall (first year) gross material shortfall amount is about $4.7B (year 2020 dollars). For the materials and industry sectors considered, there are 944 MCRs.

The budget formulation used the material prices from RR21, which are current as of Summer 2020. For guiding acquisition in the present, before a national emergency occurs, the use of current peacetime material prices is appropriate. As noted earlier, material prices in a national emergency could be quite different from the peacetime ones.

## D. Initial Results

### 1. Production Formulation Results

The production formulation LP was set up and run with the RR21 Base Case data. Some summary results are as follows.

- With the materials on hand, about 71.8% of the industrial demand can be satisfied (about the same percentage of the material demand is satisfied, 74%).
- 113 of the 150 industry sectors have their demands completely satisfied; 17 sectors partially satisfied. The LP solution specifies zero production in 20 sectors. (As noted, this situation can be addressed by imposing a lower bound on production.)
- Slack materials: of the 36 materials, 19 have supplies that are not used fully. About 20% of the total available material (measured in $M) is unused.
- The dual variables and dual-variable/market-price ratios vary widely between materials. The dual variables do not seem to be correlated with the peacetime material prices used in the RR21 study. In a national emergency, however, the market would probably be well aware of the materials that industry needed the most, and the prices would probably react accordingly.

### 2. Budget Formulation Results

As a sanity check, a budget formulation run was performed with a budget amount of zero. This run should yield the same results as the production formulation, and indeed it does.

A budget formulation run with a relatively small budget, $10M, was performed. Even that small additional acquisition amount yielded some significant results.

- Four materials are acquired (tend to have high dual-variable/market-price ratios).
- Material shortfall is just $10M less—the amount of the budget.
- Industrial production, however, is about $657 billion more (year 2009 dollars) than in the no-procurement case.
- 82.3% of industrial demand is satisfied (up from 71.8%).
- Acquisition of the four materials allowed one of the previous slack materials to become fully utilized; other slack materials are utilized to a greater extent than in the no-procurement case.

Several budget formulation runs were then performed, gradually increasing the budget amount. The percent of industrial demand satisfied grows as the budget increases, but diminishing returns quickly set in. The number of slack materials decreases when available quantities of other materials increase, allowing the slack materials to be used. A budget of $4.7B, the initial material shortfall amount, eliminates all shortfalls.

The table below shows some summary results of these runs.

**Table 1. LP Results Under Increasing Acquisition Budgets**

| Budget ($2020M) | Percent of industry demand satisfied (150 sectors) | Number of sectors completely satisfied (of 150) | Number of materials procured | Number of slack materials | % of material unused |
|---|---|---|---|---|---|
| 0 | 71.8 | 113 | 0 | 19 | 20 |
| 10 | 82.3 | 113 | 4 | 18 | 15 |
| 25 | 87.0 | 117 | 9 | 12 | 12 |
| 100 | 89.5 | 126 | 14 | 9 | 11 |
| 500 | 93.2 | 130 | 20 | 6 | 8 |
| 1,000 | 95.7 | 135 | 25 | 3 | 1 |
| 2,000 | 98.1 | 143 | 33 | 1 | 0.1 |
| 4,700 | 100 | 150 | 36 | 0 | 0 |

## E. Minimum Satisfaction Fractions

The work in Chapter 4 introduced the idea of a "minimum satisfaction fraction" $\alpha$, and derived the concept of deficit materials, i.e., materials for which the available supply was insufficient to meet fraction $\alpha$ of demand in each industry sector. Let the terminology be the same as in Chapter 4. Using the RR21 data, for various levels of $\alpha$ it was determined which materials would be in deficit. The corresponding total additional material requirement $R$ was computed. These computations did not necessitate re-solving any linear programming problems. The table below shows the results. A value of $\alpha$ just above 11% resulted in one material having a deficit; for values of $\alpha$ lower than that, the relation $\alpha Q_m \leq S_m$ holds for all materials $m$. Note that the total material shortfall is about \$4.7B; that would be the amount of additional requirement that would be necessary to ensure that all the industrial demands were satisfied (i.e., the case $\alpha = 1$).

**Table 2. Material Deficits as a Function of the Minimum Satisfaction Fraction α**

| Value of *α* | Number of Materials (of 36) with Deficit | Total Deficit Amount *R* ($M) |
|---|---|---|
| ≤11% | 0 | 0 |
| 20% | 4 | 3.73 |
| 30% | 5 | 17.32 |
| 40% | 7 | 47.14 |
| 50% | 15 | 238.96 |
| 60% | 19 | 712.47 |
| 70% | 22 | 1296.70 |
| 80% | 28 | 2,020.30 |
| 90% | 31 | 3,118.05 |
| 100% | 36 | 4,699.90 |

Excel Solver was set up to perform the substitution formulation LP discussed in Chapter 4, Section C. For each Solver run, a minimum satisfaction fraction $\alpha$ was specified. The amounts of each material required to satisfy this fraction of industrial demand were computed and subtracted from the initial available supply values; some of the resultant available supply values were negative. The budget was set to the total deficit amount shown in Table 2. The decision variables in the LP were:

- The *incremental* amount $u_i$ of industrial output in sector $i$ to produce over and above fraction $\alpha$ of the total demand, for each sector, and
- The amounts of each material to procure, subject to the budget constraint.

For each sector, the variables $u_i$ are constrained to be less than $(1 - \alpha)D_i$, i.e., the industrial demand remaining after fraction α is satisfied. The objective function is to maximize the sum of the $u_i$. Since the budget was set to the total deficit amount in $M (Table 2), procurement of the deficit amounts of the materials with deficits is necessary for there to be any feasible solution to the LP. The Solver results were consistent with this condition; exactly the deficit amounts were procured.

Letting $\{u_i\}$ denote the optimal solution to the LP, the total production in sector $i$ is equal to $\alpha D_i + u_i$. The fraction of total demand satisfied is $(\sum_i(\alpha D_i + u_i))/(\sum_i D_i)$ and the fraction of *incremental* demand satisfied equals $(\sum_i u_i)/(\sum_i(1-\alpha)D_i)$. All the industry sectors have at least the fraction $\alpha$ of demand satisfied, but the incremental amount of production, $u_i$, might be zero for some sectors. Conversely, for some sectors $i$, $u_i$ might equal $(1 - \alpha)D_i$, so all the demand in that sector is satisfied.

Table 3 shows the results of the LP solutions for this formulation for several different values of $\alpha$, including those shown in Table 2. The values in the second and third columns of Table 3 are as in Table 2. The subsequent columns show the fraction of total demand satisfied, fraction of incremental demand satisfied, the number of sectors (of 150) with demand fully satisfied, and the number of sectors with zero incremental production. In the bounding case, shown in the bottom row of the table, the full material shortfall amount is procured and there is no unsatisfied industry demand.

**Table 3. LP Solution Results for Various Minimum Satisfaction Fractions**

| Minimum Satisfaction Fraction α (%) | Procurement Budget ($M) | Number of materials procured | Fraction of Total Demand Satisfied (%) | Fraction of Incremental Demand Satisfied (%) | Number of sectors with demand fully met | Number of sectors with zero incremental production |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 71.83 | 71.83 | 113 | 20 |
| 10 | 0 | 0 | 69.30 | 65.89 | 111 | 25 |
| 11 | 0 | 0 | 67.45 | 63.42 | 105 | 31 |
| 20 | 3.73 | 4 | 66.29 | 57.86 | 95 | 45 |
| 30 | 17.32 | 5 | 68.39 | 54.85 | 80 | 60 |
| 40 | 47.14 | 7 | 53.09 | 21.82 | 21 | 123 |
| 50 | 238.96 | 15 | 59.06 | 18.13 | 16 | 133 |
| 60 | 712.47 | 19 | 67.19 | 17.98 | 15 | 134 |
| 70 | 1,296.70 | 22 | 75.17 | 17.23 | 12 | 137 |
| 80 | 2,020.30 | 28 | 80.73 | 3.66 | 5 | 144 |
| 90 | 3,118.05 | 31 | 90.16 | 1.63 | 1 | 148 |
| 100 | 4,699.90 | 36 | 100.00 | n/a | 150 | 150 |

The fraction of total demand satisfied does not exhibit monotone behavior as $\alpha$ increases. Note, however, that two different factors are varying, the minimum satisfaction fraction and the procurement budget amount. One might expect that the incremental production would be small since after procurement, only enough of the deficit materials are available to produce exactly the fraction $\alpha$ of the industrial demand. Any incremental production must occur in industries that use only the non-deficit materials. In this vein, it is worth noting that the optimal LP solutions have many slack materials, i.e., much of the available material supply is not used. (The deficit materials are like the chocolate in the brownie example in Chapter 3, Section C.)

To try to separate the results of the two factors, two additional series of runs were performed. In the first series, the procurement budget was set at $300M and the minimum satisfaction fraction was varied within a range where feasible solutions to the LP existed. Table 4 shows the results. With the budget held constant, the fraction of total demand satisfied is indeed monotone. It decreases as the severity of the minimum satisfaction constraint increases.

**Table 4. Effects of Increasing *α* for Constant Procurement Budget**

| Minimum Satisfaction Fraction α (%) | Procurement Budget ($M) | Number of materials procured | Fraction of Total Demand Satisfied (%) | Fraction of Incremental Demand Satisfied (%) | Number of sectors with demand fully met | Number of sectors with zero incremental production |
|---|---|---|---|---|---|---|
| 0 | 300 | 16 | 91.54 | 91.54 | 127 | 9 |
| 10 | 300 | 16 | 91.29 | 90.32 | 125 | 11 |
| 11 | 300 | 16 | 91.25 | 90.17 | 125 | 11 |
| 20 | 300 | 17 | 90.90 | 88.62 | 123 | 14 |
| 30 | 300 | 17 | 90.46 | 86.37 | 120 | 17 |
| 50 | 300 | 19 | 84.32 | 63.64 | 87 | 59 |

In the second series of runs, the minimum satisfaction fraction was held constant at 50% and the procurement budget amount varied. The first run was as listed in Table 3, with the budget set at the minimum feasible amount. Relatively slight increases in the budget amount led to considerable increases in the fraction of total demand satisfied. Table 5 shows the results.

**Table 5. Effects of Increasing Procurement Budget for Constant α**

| Minimum Satisfaction Fraction α (%) | Procurement Budget ($M) | # of materials procured | Fraction of Total Demand Satisfied (%) | Fraction of Incremental Demand Satisfied (%) | # of sectors with demand fully met | # of sectors with zero incremental production |
|---|---|---|---|---|---|---|
| 50 | 238.96 | 15 | 59.06 | 18.13 | 16 | 133 |
| 50 | 250.00 | 15 | 73.79 | 47.58 | 51 | 93 |
| 50 | 300.00 | 19 | 84.32 | 63.64 | 87 | 59 |

## F. Additional Cases of Interest

A number of additional cases have been performed with MPLP, as summarized in the following sections.

### 1. Extra Sell

Upon noticing a potential material shortfall, the market might act to reduce that shortfall. RAMF-SM (Substep 2d) can model three different such market responses:

- Thrift—using less of a material in the production of goods and services;
- Substitution—using substitute material(s) for a material that might be in shortfall;
- Extra sell—obtaining extra material from certain countries.

The first two market responses act to reduce demand for materials; the third tends to increase the available supply. A gross shortfall model run has none of the responses; the corresponding net shortfall run has all of them.[15] Because of the way RAMF-SM computes demand for materials,[16] it is not straightforward to integrate the thrift and substitution modeling into the MPLP framework. As noted in Chapter 5, Section A, most of the computational results reported in this paper, including the Base Case analysis, use the gross shortfall data.

The extra sell market response involves U.S. manufacturers obtaining preferential access to currently-unused capacity. A manufacturer might identify unused foreign material productive capacity and make a special agreement for the producer to produce at or near capacity, with the manufacturer buying the extra production amount. (For details

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[15] For more information about the market responses, see James S. Thomason et al., *IDA Contributions to the Strategic and Critical Materials 2019 Report on Stockpile Requirements,* Vol. I: *Unclassified Contributions*, in particular Chapter 8.

[16] For more information about RAMF-SM's computation of material demand, see Eleanor L. Schwartz, *The RAMF-SM Material Demand Computation Program*.

about the actual algorithm, see IDA Paper P-10727[17] and IDA Paper P-22696.[18] The seller is the material producer and the buyer is the manufacturing company. In the 2021 Base Case, extra sell is restricted to a small subset of foreign countries, those with which the U.S. has explicit understandings or arrangements.

The extra sell market response is straightforward to implement in MPLP—increase the amount of available supply to include material obtained via extra sell. For the 36 materials under consideration, available supply with extra sell was about 3.8% more than the Base Case supply. Including the extra sell made a small difference in the material shortfalls and also made a small difference in the percentage of industry demand satisfied (72.1%, up from 71.8% in the Base Case). There were the same 19 slack materials as in the Base Case, and 23% of the available material supply was unused, up from 20% in the Base Case. Some of the extra sell amounts were for slack materials, and thus had no effect on industrial production. (A manufacturer might not attempt to obtain an extra sell of a slack material.)

### 2. Inclusion of Existing NDS Inventory

When considering the acquisition of additional material, it would seem prudent to do this on top of what is already in the National Defense Stockpile. This was *not* done for the Base Case analysis. It was therefore desired to explore what would happen if the NDS inventory amounts for the 36 materials in question were added to the available supply. A number of these 36 materials do have inventories, with a total value of about $717M. When the inventory amounts were included, the available material supply increases by about 6% from the Base Case, and some of the individual material shortfalls went away—the NDS inventory was sufficient to cover them.

The percentage of industry demand satisfied, however, increases only slightly, from 71.8% to 73.2%. This is comparable to the increase from extra sell. Many of the NDS inventories are for slack materials, so adding them in does not increase the industrial output that can be produced. There were 20 slack materials, and 21% of the available material supply was unused.

### 3. Extra Sell and Existing NDS Inventory

An excursion was done adding both the extra sell amount and the NDS inventory to the Base Case available supply. The percentage of industry demand satisfied was 73.3%—

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17 James S. Thomason, et al., *IDA Contributions to the Strategic and Critical Materials 2019 Report on Stockpile Requirements,* Vol. I: *Unclassified Contributions*.

18 Eleanor L. Schwartz and James S. Thomason, *The RAMF-SM Stockpile Sizing Module: Updated Documentation and User's Guide*, IDA Paper P-22696 (Alexandria, VA: Institute for Defense Analyses, April 2022).

not much larger than the effect of either case separately. The incremental effects of these two extra supply measures appear to overlap. There were 20 slack materials, and 23% of the available material supply was unused.

### 4. New Acquisitions Using Value of Existing Inventory

As mentioned, the NDS inventory for the 36 materials totaled about $717M. What if that inventory were sold and the proceeds were used to buy materials that would be more directly useful in industrial production? To address this question, a run was done with the budget LP formulation using the Base Case available material supplies and a budget of $717M. The percentage of industrial demand satisfied was 94.5%. As expected, this percentage lies between those for the $500M budget and $1,000M budget cases shown in Table 1. The number of materials acquired was 22. Some of these were materials with existing NDS inventory, but many were not. The results showed only four slack materials, and only 2% of the available material supply was unused. Of the 150 industry sectors considered, 129 have their demand fully satisfied and 11 have their demand partially satisfied; the LP solution specifies zero production for the ten remaining sectors. As noted in Chapter 4, zero production is troublesome from a real-world perspective, and it would be meaningful to impose a lower bound on production in each industry sector. Some explorations in this direction appeared in Chapter 5, Section E.

### 5. More Severe Case

As part of a study of NDS inventory evaluation, a number of cases more stressful than the Base Case were examined.

A problem with calculating the MPLP results for those cases was that many of them had more than 50 shortfall materials. In conjunction with 150 industry sectors, that would raise the number of decision variables in the LP to above Solver's limit of 200. One of the "mildly more severe" cases was chosen; it had 40 shortfall materials with a first-year material shortfall of $10.1B, in contrast to $4.7B for the Base Case (year 2020 dollars).

The MPLP differences from the Base Case are not as dramatic as one might expect. The percentage of industrial demand met is 68.0% (down from 71.8% in the Base Case). There are four more shortfall materials, so some results aren't directly comparable. There are fewer slack materials (17 as opposed to 19 in the Base Case), but a significantly higher percentage of available material supply is not usable (34% as opposed to 20%). The reasons for these disparities are unclear and should be explored further.

# 6. Lower Bounds and Sensitivity Analysis

Chapter 5 has presented some actual results from the 2021 Requirements Report study at an aggregated level. A nonproprietary test database was constructed using unclassified, economic data and random numbers to more fully explore the interplay between the constraints, the shadow prices (dual variables), and the objective function values of the linear program. Below are the results of some analyses with the test database, which considers 40 industry sectors and ten fictive materials.

## A. Lower Bounds

Lower bounds can be placed on output requirements. The same lower bounds are used here for all 40 industries. Table 6 gives results for lower bound percentages 0, 10, 20, 30 and 40. When a 50% lower bound is imposed, there is no feasible solution—it is impossible to achieve at least the specified amount given the available materials. When the lower bound is zero, there are 12 industries with no production. After that, production is at least as great as the lower bound. The number of fully-satisfied sectors decreases as the lower bounds increase.

**Table 6. Results for Selected Lower Bounds on Industrial Production**

| Lower Bound (Percent of Requirement) | Industrial Output ($ Billions) | Peacetime Production Achieved (Percent of Requirement) | Number of Sectors Fully Satisfied |
|---|---|---|---|
| 0 | 1,331 | 71.7 | 22 |
| 10 | 1,312 | 70.7 | 20 |
| 20 | 1,287 | 69.3 | 17 |
| 30 | 1,258 | 67.8 | 16 |
| 40 | 1,149 | 61.9 | 8 |

In order to achieve more that 40% of demand for all industries, it is necessary to have more material available.

## B. Sensitivity to Budgets

A notional budget of $1.0 Billion is supplied to buy materials. With this budget, it is possible to achieve lower bounds of 70% of requirements.

Table 7 shows that for a material budget of $1.0 Billion and a lower bound of 0.7 industrial output is $1,537 Billion. The shadow price of 776 indicates that a budget increase of $100 Million to $1.1 Billion will generate an increase of $77.6 Billion in output. This holds true in Table 7. Also shown are results for two more budget cases, $1.160 Billion and $2.0 Billion, which are discussed below.

**Table 7. Results of Increasing Material Acquisition Budgets for Lower Bounds of 70 Percent of Peacetime Production**

| Budget for Materials ($ Billion) | Industrial Output ($ Billion) | Peacetime Production Achieved (Percent of Requirement) |
|---|---|---|
| 1.0 | 1,537 | 82.8 |
| 1.1 | 1,615 | 87.0 |
| 1.160 | 1,661 | 89.5 |
| 1.2 | 1,690 | 91.1 |

When the budget is $1.0 Billion, Industry 3 is constraining. It uses Materials 5, 9, and 10, and Materials 5 and 9 have available slack capacity. Therefore, Material 10 is the constraining material and is procured.

When the budget is increased to $1.1 Billion, production in Industry 3 increases, Material 10 is still constraining and is procured, and Materials 5 and 9 still have some slack. The shadow price is still 776.

When the budget is increased to $1.160 Billion, production in Industry 3 increases and Material 10 is still constraining and is procured. Slack Materials 5 and 9 are used, and the slack in Material 9 is exhausted. But now there is also increased production in Industries 19 and 20, for which Materials 1 and 10 are procured and slack Materials 3 and 7 are used. The shadow price is reduced from 776 to 755.

When the budget is increased to $1.2 Billion, production in Industry 3 continues to increase and Materials 9 and 10 are procured. Industry 19 production increases, and Materials 1 and 10 are procured, while slack Material 5 is used. There is no additional production in Industry 20. The shadow price is reduced from 755 to 729.

This illustrates how the myriad of relationships among industries and materials is revealed by the linear program, and how the sensitivity analysis addresses the changes in the mixes of industrial outputs and material inputs.

# 7. Satisfying Final Demands

## A. Introduction

The foregoing discussion has the objective of maximizing production of industrial output, which corresponds to total requirements demand. But ultimately, it is the essential *final* demands for goods and services that matter. The inter-industry demands are a means to an end. It might not be desirable to have a policy that enables industrial capacity that does not contribute to satisfying final demand. (There is an analogy to not using a procurement budget to purchase slack materials.) One could make the decision variables the amounts of final demand to satisfy. Via a linear transformation (the Leontief inverse matrix), the final demands generate total requirements demands, which in turn generate material demands. This structure can be formulated as a linear program as indicated below.

## B. Production LP Formulation

Let all notation be as defined previously. In what follows, matrices will be shown as bold capital letters, and vectors as bold underlined letters (lowercase or uppercase).

Let $C_i$ denote the final demand in industry sector *i*, net of imports, so $\underline{\mathbf{C}}$ is a vector of final demands (net of imports).[19] Let $\mathbf{W} = \|w_{ij}\|$ denote the Leontief inverse matrix.[20] $\mathbf{W}$ is an $I$ by $I$ matrix. Let $\mathbf{P} = \|\rho_{im}\|$ denote the matrix of MCRs. Note that an MCR represents the material required to make a million dollars' worth of *industrial output*, which corresponds to total requirements demand. The symbol $\mathbf{P}$ is to be interpreted as an uppercase Greek rho, not an English letter. The matrix $\mathbf{P}$ has $I$ rows and $M$ columns. Its transpose, $\mathbf{P}^\mathbf{T}$, is $M$ by $I$; the rows of $\mathbf{P}^\mathbf{T}$ correspond to materials. Let $y_i$ be the amount of *final* demand in sector *i* that is to be satisfied. The $y_i$ are the decision variables in the linear program. Each $y_i$ must be nonnegative and should be constrained to be less than the final demand $C_i$ (to avoid imbalances). (The vector $\underline{\mathbf{D}}$ of total requirements demands, which was used in previous formulations, is equal to $\mathbf{W}\underline{\mathbf{C}}$.)

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[19] Some of the final demand is satisfied by imported goods. It is the remainder, which need to be made in the United States, that induce a demand by U.S. manufacturers for materials. Also, goods manufactured for export constitute a source of demand on U.S. industry. The formulation therefore works with final demands minus imports where final demands include exports. Equivalently, the formulation works with domestic final demands plus net exports.

[20] Traditionally, economic input-output theory has used the symbol $(\mathbf{I}-\mathbf{A})^{-1}$ for the Leontief inverse matrix where $\mathbf{I}$ denotes the identity matrix and $\mathbf{A}$ is a matrix of inter-industry flows. That is, the Leontief inverse is the matrix inverse of the matrix $\mathbf{I}-\mathbf{A}$. Since this paper has already used the symbol $I$ to denote the number of industry sectors, a different symbol for the Leontief inverse matrix is used here.

The vector $\underline{\mathbf{y}}$ of final demands induces a vector $\mathbf{W}\underline{\mathbf{y}}$ of total requirements demands, which in turn induces a vector $\mathbf{P}^{\mathbf{T}}\mathbf{W}\underline{\mathbf{y}}$ of material demands. The $m^{th}$ component of this vector must be less than the available material supply, $S_m$. The objective function is to maximize the total final demand satisfied. In matrix terms, we have the LP formulation:

maximize $\underline{\mathbf{e}}^{\mathbf{T}}\underline{\mathbf{y}}$

subject to

$\mathbf{P}^{\mathbf{T}}\mathbf{W}\underline{\mathbf{y}} \leq \underline{\mathbf{S}}$

$\underline{\mathbf{0}} \leq \underline{\mathbf{y}} \leq \underline{\mathbf{C}}$

where $\underline{\mathbf{e}}$ denotes a length-$I$ vector, all the elements of which are one.

The matrix constraint $\mathbf{P}^{\mathbf{T}}\mathbf{W}\underline{\mathbf{y}} \leq \underline{\mathbf{S}}$ encompasses $M$ different constraints, one for each material. The formula for the $m^{\text{th}}$ constraint is not straightforward. The $(m,i)^{\text{th}}$ element of the product matrix $\mathbf{P}^{\mathbf{T}}\mathbf{W}$ is given by the inner product of the two length-$I$ vectors $(\rho_{1m}, \rho_{2m},\ldots, \rho_{Im})$ and $(w_{1i}, w_{2i}, \ldots, w_{Ii})$. That is, $(\mathbf{P}^{\mathbf{T}}\mathbf{W})_{mi} = \sum_{j=1}^{I} \rho_{jm} w_{ji}$. The constraint for material $m$ then becomes $\sum_{i=1}^{I}\left(\sum_{j=1}^{I} \rho_{jm} w_{ji}\right) y_i \leq S_m$. Using regular algebraic terminology, the LP formulation is then:

maximize $\sum_{i=1}^{I} y_i$

subject to

$\sum_{i=1}^{I}\left(\sum_{j=1}^{I} \rho_{jm} w_{ji}\right) y_i \leq S_m \quad m=1,\ldots, M$

$0 \leq y_i \leq C_i \quad i=1,\ldots, I.$

## C. Budget and Minimum Satisfaction Fraction Formulations

It is straightforward to implement the procurement budget and minimum satisfaction fraction concepts in the final demand formulation. Consistent with the notation defined previously, let $\theta_m$ be the market price of material $m$, let $B$ be the total budget for procurement, and let $\alpha$ represent the minimum fraction of *final* demand in each industry that must be satisfied. The decision variable $z_m$ represents the amount of additional material to acquire. The linear programming problem then becomes:

maximize $\sum_{i=1}^{I} y_i$

subject to

$\sum_{i=1}^{I}\left(\sum_{j=1}^{I} \rho_{jm} w_{ji}\right) y_i \leq S_m + z_m \quad m=1,\ldots, M$

$\sum_{m=1}^{M} \theta_m z_m \leq B$

$\alpha C_i \leq y_i \leq C_i \quad i=1,\ldots, I$

$z_m \geq 0 \quad m=1,\ldots, M .$

The above LP uses explicit constraints to model the minimum satisfaction fraction, but the substitution technique described in Chapter 4, Section C could be adapted for use in the final demand formulation. The vector $\alpha\underline{\mathbf{C}}$ of final demands induces the material demand vector $\alpha\ \mathbf{P}^{\mathrm{T}}\mathbf{W}\underline{\mathbf{C}}$. The substitution technique involves subtracting this vector from the initial available material supply $\underline{\mathbf{S}}$ and using the result in the right hand side of the material constraints. The decision variables would then represent the incremental amount of final demand in sector *i* to satisfy; they would be constrained to be less than or equal to $(1-\alpha)\underline{\mathbf{C}}$.

## D. Size and Complexity Issues

The formulation above is indeed an LP, but it might be a large, dense one. All the industry sectors must be considered, since final demand in any particular sector can induce total requirements demand in many different sectors, including some that use materials. The production formulation has *I* variables and *M* constraints, plus upper bounding constraints. The budget formulation has *I* variables and *M*+1 constraints, plus upper bounding constraints. Implementing the minimum demand satisfaction condition via explicit lower bound constraints (as opposed to the substitution technique) adds *I* constraints. In the 2021 Requirements Report data, *I* is 352; this value puts the problem outside of Solver's limits. The approach taken in Chapter 5, Section B, in which only the subset of major demanding sectors was considered, will most likely not capture the full detail of the problem.

The matrix **P** of MCRs is sparse, but the Leontief inverse matrix is generally dense. The Leontief inverse matrices used in the Requirements Report analyses have about 85% nonzero elements. The test computations performed so far indicate that the product matrix $\mathbf{P}^{\mathrm{T}}\mathbf{W}$ has virtually no zero elements. Such matrix density might tend to make the LP computations slow.

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# 8. Conclusions

This paper has developed and expanded upon concepts first explored by Barnett and Bracken as part of the 2015 Requirements Report.[21] The work is based on a principle that underlies the Stock Piling Act and IDA's strategic materials analyses but is rarely explicitly articulated: Materials are valuable because they are used to produce essential goods and services.

Shortfalls of strategic materials in a national emergency constitute a potential source of risk to national security. One quantitative measure of such risk is the fraction of material demand that is unsatisfied. This paper develops an additional quantitative measure—the fraction of essential goods and services demand that is unsatisfied, given that the available materials are used as efficiently as possible.

IDA's previous work on material shortfall analysis has tended to examine different materials independently of one another. The modeling developed in this paper treats materials in combination because an industry generally needs a set of several different materials in specific proportions to produce its output. An important finding that arises from this combined treatment is that even if a material is in shortfall, acquiring more of it may not necessarily increase the amount of industrial output that can be produced. The linear programming formulations developed in this paper can identify such slack materials. A useful extension of the basic LP model is as follows: Given a budget for acquisition of additional materials, the "budget formulation LP" can identify a set of material acquisitions, within that budget, that maximizes the additional amount of industrial output that can be produced. A further elaboration of the LP model, developed in this paper, involves maximizing final demand, which is a crucial component of industrial production. Further applications of the LP model could include selling the portion of the stockpiled materials that would not be used in industrial production in a national emergency (the slack materials), and using the funds thus obtained to optimally buy needed materials.

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[21] Appendix 20 of James S. Thomason et al., *Analyses for the 2015 National Defense Stockpile Requirements Report*.

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# Appendix A.
# A Precis of RAMF-SM

The Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM) is a suite of procedures, models, and databases that can be used to assess shortfalls of strategic materials and the risks of such shortfalls. It can also be used to develop and assess strategies to help reduce those risks.

RAMF-SM and its precursors have played a key role in the analyses that have supported the biennial Reports to Congress concerning requirements for the NDS of strategic and critical non-fuel materials.[22] RAMF-SM, which was developed by IDA and is discussed more fully in IDA Paper P-5190[23], has six major steps:

1. Identify (and select for study) materials of concern to the U.S. national security community;
2. Compute material shortfalls to assess whether there could be significant problems in a planning scenario (such as a national emergency scenario) in meeting critical demands for materials with supplies of materials likely to be available to the United States;
3. Assess the importance of overcoming (or the risks to the United States of not overcoming) those shortfalls by deliberate government mitigation actions;
4. Identify various promising government mitigation options to address any important shortfalls;
5. Assess and compare the specific costs and mitigation effects of these government mitigations options, both individually and together; and
6. Identify priorities among the materials for investments of taxpayer dollars, whether through stockpiling or other government investments, to mitigate important potential shortfalls.

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[22] The National Defense Stockpile was established in the World War II era and has been managed by the Department of Defense (DOD) since 1988. By law, DOD is required to submit periodic reports to Congress stating which materials, and in what amounts, the stockpile should contain. The most recent such report as of this writing is Office of the Under Secretary of Defense for Acquisition and Sustainment, *Strategic and Critical Materials 2021 Report on Stockpile Requirements*.

[23] James S. Thomason, et al., *Analyses for the 2015 National Defense Stockpile Requirements Report*.

Step 2 of RAMF-SM involves assessing material shortfalls in a national emergency scenario. This process has four parts, or substeps:

- Substep 2a. Identify the demand for goods and services (defense and essential civilian) in the scenario.
- Substep 2b. Determine the amounts of S&CMs needed by U.S. firms to manufacture these goods and services (i.e., the demand for S&CMs).
- Substep 2c. Determine the supply of S&CMs available to the U.S. in the scenario and compare that supply with the demand to determine material shortfalls.
- Substep 2d. Model the effect of market responses (as opposed to government mitigation actions) on material shortfalls. This substep is implemented by changing the material demands and supplies in Substeps 2b and 2c and rerunning the shortfall computation procedure.

The vast majority of the work done with RAMF-SM so far has concerned Step 2. Several mathematical models and dozens of databases, encompassing thousands of data items, support the Step 2 computations. However, some work has been done with the other steps of RAMF-SM.[24] The work reported in the current paper applies to Step 3 in that unsatisfied industrial demand is a measure of risk arising from shortfalls of strategic materials. It also applies to Step 4 in that it suggests a mitigation option that involves selective acquisition of strategic materials under a budget constraint.

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[24] A linear programming model was developed to address Steps 5 and 6. James S. Thomason, D. Sean Barnett, James P. Bell, Jerome Bracken, and Eleanor L. Schwartz, *Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM)*, IDA Document D-4811 (Alexandria, VA: Institute for Defense Analyses, April 2013).

# Appendix B.
# Equivalence of Two Linear Programming Formulations that Impose a Minimum Percentage of Demand that Must Be Satisfied

Chapter 4, Section C suggests two different LP formulations that impose a minimum fraction of industrial demand that must be satisfied. This appendix presents the problems as formal LPs and demonstrates that they give equivalent solutions.

## Notation and Assumptions

Define the following notation, which is the same as in the main paper. All quantities are assumed to be nonnegative.

$I$ = the total number of industry sectors considered. Sectors without MCRs do not enter into the analysis.

$i$ = index for industry sector ($i$ = 1,…, $I$).

$M$ = total number of materials considered.

$m$ = index for material ($m$ = 1,…, $M$).

$D_i$ = the full amount of demand for industrial output from industry sector $i$ (in the case of interest), measured in millions of constant-year dollars.

$\rho_{im}$ = the material consumption ratio for industry sector $i$ for material $m$, measured in mass units of material $m$ needed to produce a million dollars of output from industry sector $i$.

$S_m$ = supply of material $m$ available (measured in mass units of material $m$).

$\alpha$ = a parameter between 0 and 1 representing the minimum fraction (or equivalent percentage) of industrial demand that is to be satisfied in each sector.

Define, for each material $m$,

$$Q_m = \sum_{i=1}^{I} \rho_{im} D_i .$$

$Q_m$ is the amount of material $m$ that is needed to satisfy the full amount of demand for industrial output. By linearity of the MCR assumption, to produce the fraction $\alpha$ of the full demand amount in each sector requires $\alpha Q_m$ units of material $m$. An underlying assumption is that all the materials are in shortfall, thus for each material $m$, $Q_m$ exceeds the available supply $S_m$ for that material. Assume, however, that $\alpha$ is low enough such that $\alpha Q_m \leq S_m$ for *all* materials $m$. This assumption is necessary for the LPs shown below to make sense.

## LP Formulations

This section presents two different LP formulations that impose a lower bound on the fraction of industrial demand that must be satisfied. (These are both variations of the "production" formulation and do not model the acquisition of additional material.)

### Formulation 1

In the first formulation, the decision variable $x_i$ represents the amount of industrial output in industry sector $i$ to be produced. The LP contains an explicit constraint that $x_i$ exceed the minimum percentage of demand. The objective is to maximize the total industrial production. The formulation is:

maximize $\sum_{i=1}^{I} x_i$

subject to

$\sum_{i=1}^{I} \rho_{im} x_i \leq S_m \quad m=1,\ldots, M$

$x_i \geq \alpha D_i \quad i=1,\ldots, I$

$x_i \leq D_i \quad i=1,\ldots, I.$

The constraint $x_i \geq \alpha D_i$ implies that $x_i \geq 0$. The assumption that that $\alpha Q_m \leq S_m$ for all $m$ is necessary for there to be a feasible solution to the LP.

### Formulation 2

The second LP formulation assumes that industry sector $i$ will produce the baseline amount $\alpha D_i$, for each $i$, and the decision variable $u_i$ represents the incremental amount, over the baseline, that sector $i$ will produce. The total amount of material $m$ used will be at least $\alpha Q_m$; the assumption that $\alpha Q_m \leq S_m$ guarantees that there is enough material supply on hand to meet this minimum amount. The amount of material left to satisfy any incremental production is thus $S_m - \alpha Q_m$, a nonnegative quantity. The objective is to maximize the total incremental industrial production. The LP formulation is:

maximize $\sum_{i=1}^{I} u_i$

subject to

$\sum_{i=1}^{I} \rho_{im} u_i \leq S_m - \alpha Q_m \quad m=1,\ldots, M$

$u_i \leq (1-\alpha) D_i \quad i=1,\ldots, I.$

$u_i \geq 0 \quad i=1,\ldots, I$

Essentially, Formulation 2 is making the change of variable $u_i = x_i - \alpha D_i$.

**Propositions of Equivalence**

Are the two formulations essentially the same problem? The meaning of "essentially the same problem" is formalized by the following two propositions.

**Proposition 1.** If $\{x_i^*\}$ is an optimal solution to Formulation 1 and $u_i$ is defined by $u_i = x_i^* - \alpha D_i$, for each $i$, then the $\{u_i\}$ constitute an optimal solution to Formulation 2.

**Proposition 2.** If $\{u_i^*\}$ is an optimal solution to Formulation 2 and $x_i$ is defined by $x_i = u_i^* + \alpha D_i$, for each $i$, then the $\{x_i\}$ constitute an optimal solution to Formulation 1.

A formal proof of Proposition 1 appears below; the proof of Proposition 2 is similar. One can also construct actual examples and verify that they give equivalent solutions.

**Proof of Proposition 1.** It is clear that the $\{u_i\}$ constitute a feasible solution to Formulation 2. Because $\{x_i^*\}$ is optimal for Formulation 1, then for any other feasible solution $\{x_i\}$ to Formulation 1,

$$\sum_{i=1}^{I} x_i^* \geq \sum_{i=1}^{I} x_{i.}$$

Let $v_i$ be some feasible solution to Formulation 2, and define $x_i$ by $x_i = v_i + \alpha D_i$, for each $i$. It clear that the $\{x_i\}$constitute a feasible solution to Formulation 1. The quantity $\sum_{i=1}^{I} \alpha\, D_i$ is a constant. Subtracting it from both sides of the above inequality yields

$$\sum_{i=1}^{I} x_i^* - \sum_{i=1}^{I} \alpha\, D_i \geq \sum_{i=1}^{I} x_i - \sum_{i=1}^{I} \alpha\, D_i$$

i.e.,

$$\sum_{i=1}^{I} (x_i^* - \alpha D_i) \geq \sum_{i=1}^{I} (x_i - \alpha D_i).$$

The terms in the left-hand side sum are the $\{u_i\}$ as defined, and the terms in the right-hand side sum are the $\{v_i\}$, which represents any feasible solution to Formulation 2. The $\{u_i\}$ are thus optimal for Formulation 2.

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# Appendix C.
# Illustrations

## Tables

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# Appendix D.
# References


Ao, Wallice, Justin M. Lloyd, Amrit K. Romana, and Eleanor L. Schwartz. *Methods in Macroeconomic Forecasting Uncertainty Analysis: An Assessment of the 2015 National Defense Stockpile Requirements Report*. IDA Paper P-5310. Alexandria, VA: Institute for Defense Analyses, March 2016.

Gass, Saul I. *Linear Programming: Methods and Applications*. Third Edition. New York City, NY: McGraw-Hill, 1969.

Meade, Douglas S. *The INFORUM LIFT Model*. Technical Documentation. College Park, MD: Inter-industry Forecasting Project, University of Maryland, November 3, 2017. https://www.researchgate.net/publication/266049640_The_LIFT_Model.

Meade, Douglas S., et al. *ILIAD*. College Park, MD: Inter-industry Forecasting Project, University of Maryland, December 2011.

Miller, Ronald E. and Peter D. Blair. *Input-Output Analysis: Foundations and Extensions*. Second edition. New York City, NY: Cambridge University Press, 2009.

Office of the Under Secretary of Defense for Acquisition and Sustainment. *Strategic and Critical Materials 2021 Report on Stockpile Requirements*. U.S. Department of Defense. n.p., February 2021.

Schwartz, Eleanor L. *The RAMF-SM Material Demand Computation Program: Documentation and User's Guide*. IDA Paper P-22689. Alexandria, VA: Institute for Defense Analyses, March 2022.

Schwartz, Eleanor L. and James S. Thomason. *The RAMF-SM Stockpile Sizing Module: Updated Documentation and User's Guide*. IDA Paper P-22696. Alexandria, VA: Institute for Defense Analyses, April 2022.

Thomason, James S., D. Sean Barnett, James P. Bell, Jerome Bracken, and Eleanor L. Schwartz. *Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM)*. IDA Document D-4811. Alexandria, VA: Institute for Defense Analyses, April 2013.

Thomason, James S., Wallice Ao, Robert J. Atwell, D. Sean Barnett, Jerome Bracken, Kevin E. Burns, Caroline R. Earle et al. *Analyses for the 2015 National Defense Stockpile Requirements Report to Congress on Strategic and Critical Materials*. Vol. I, *Material Assessments and Associated Analyses*. IDA Paper P-5190. Alexandria, VA: Institute for Defense Analyses, August 2015.

Thomason, James S., Eli Alster, Maxwell W. Camp, Jacqueline DuBois, Caroline R. Earle, Anthony C. Hermes, Eliza Johannes et al. *IDA Contributions to the Strategic and Critical Materials 2019 Report on Stockpile Requirements.* Vol. I, *Unclassified Contributions*. IDA Paper P-10727. Alexandria, VA: Institute for Defense Analyses, October 2019.

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# Appendix E.
# Abbreviations

| | |
|---|---|
| DOC | Department of Commerce |
| DOD | Department of Defense |
| IDA | Institute for Defense Analyses |
| ILIAD | Inter-industry Large-scale Integrated and Dynamic Model |
| INFORUM | Inter-industry Forecasting Project at the University of Maryland |
| LIFT | Long-term Inter-industry Forecasting Tool |
| LP | Linear programming, linear programming problem |
| $M | Millions of dollars |
| MCR | Material consumption ratio |
| MPLP | Material Prioritization using Linear Programming |
| NDS | National Defense Stockpile |
| RAMF-SM | Risk Assessment and Mitigation Framework for Strategic Materials |
| RR21 | 2021 Requirements Report |
| S&CM | Strategic and critical material |
| U.S.C. | United States Code |

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## 14. ABSTRACT

This Institute for Defense Analyses paper develops several linear programming (LP) formulations that can be used to assess the effects of shortfalls of strategic and critical materials in a national emergency. The analysis proceeds from the basic premise that materials are valuable because they are used to produce essential goods and services. Each industrial sector of the U.S. economy is assumed to need a particular mix of materials, in particular proportions, in order to produce its output. In a national emergency, there might not be sufficient materials available (because of increased demand and/or reductions in supply) to produce all of the needed goods and services. The basic LP formulation determines how many of these goods and services can be produced if the available materials are allocated to the industrial sectors in an optimal manner. One paradoxical finding is that for some materials, not all of the available supply is necessarily used in the optimal solution, even if the material is in overall shortfall. This paper develops several additional LP formulations, including allocating a material acquisition budget. Illustrative results are presented, using data from the Department of Defense Strategic and Critical Materials 2021 Report on Stockpile Requirements.

15. SUBJECT TERMS

strategic materials, critical materials, linear programming, linear programming applications, U.S. industry, material requirements, National Defense Stockpile, Defense Logistics Agency, mathematical modeling, methodology

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