Improving Shortfall Estimates for the National Defense Stockpile: An Optimization Approach to Material Supply Chain Modeling

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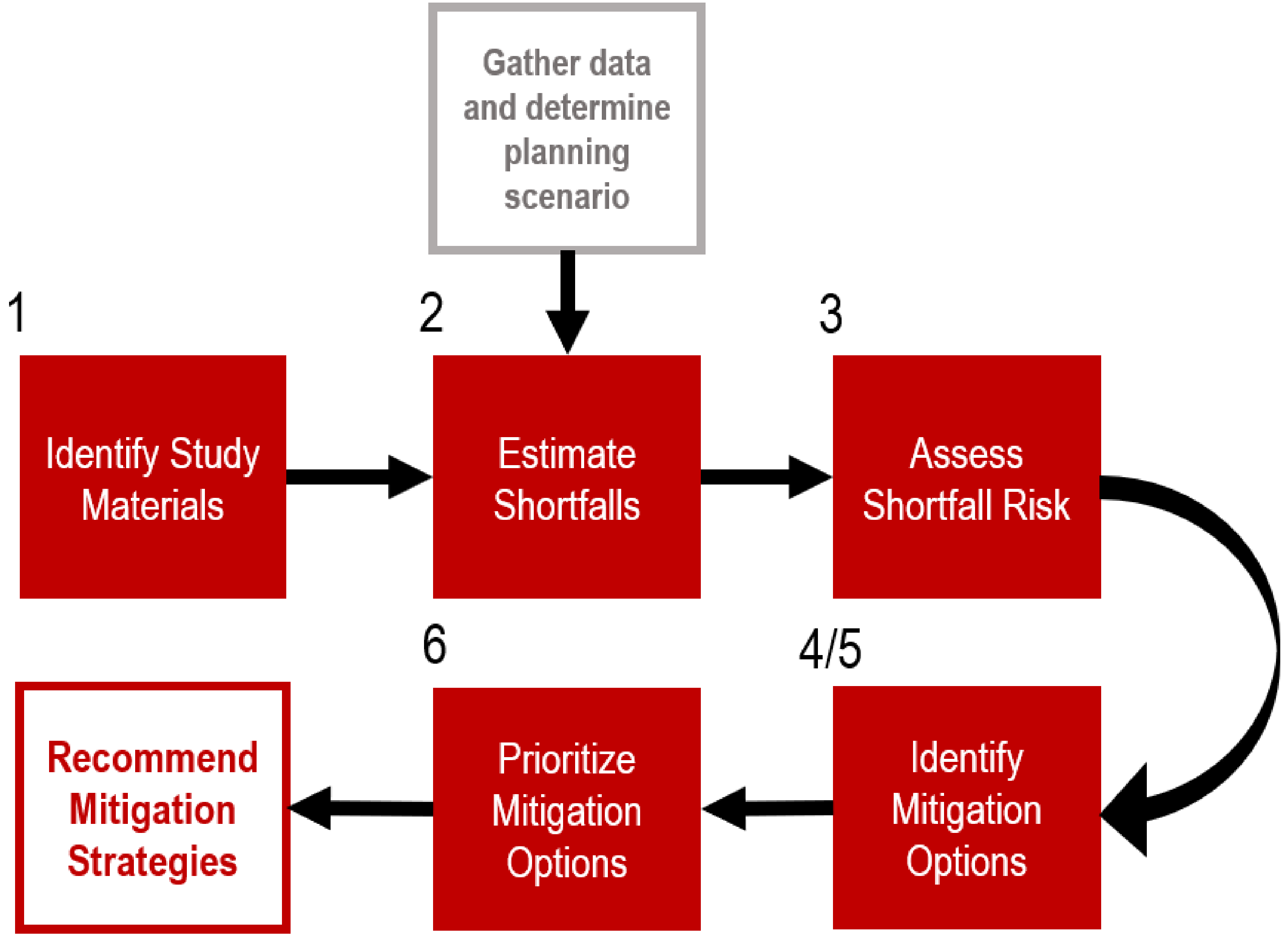
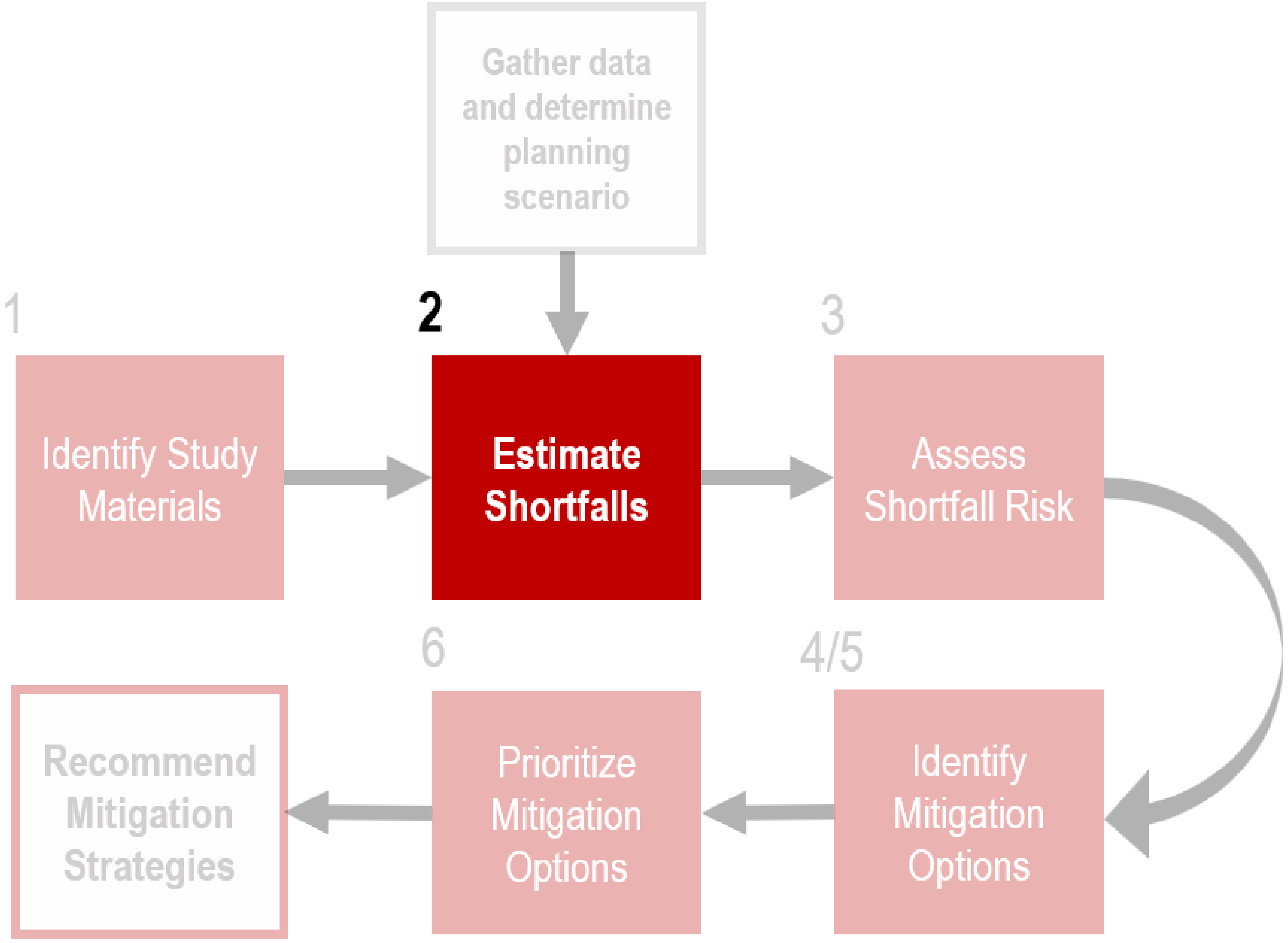
Background: National Defense Stockpile (NDS) Program

- DLA Strategic Materials operates and maintains a stockpile of **strategic and critical materials**; the Office of the Secretary of Defense manages the program
- Stockpiled material may be used to produce goods and services required for essential civilian and defense needs
- IDA supports DoD by helping to estimate and identify essential demands for S&CMs, safe supplies, gaps, and priorities for filling any gaps
- For this work, IDA uses a framework called "RAMF-SM" (Risk Assessment and Mitigation Framework for Strategic Materials)
- RAMF-SM informs:
  - **Biennial reports to Congress** on the National Defense Stockpile (1990s–present)
  - **Prioritized investments** for strategic materials based on an ROI strategy
  - "Deep dive" assessments and **supply chain mapping** of key strategic materials or components
  - Business case analyses identifying **risk mitigation strategies** beyond stockpiling (e.g., expanding domestic capacity, qualifying new suppliers)

RAMF-SM Steps

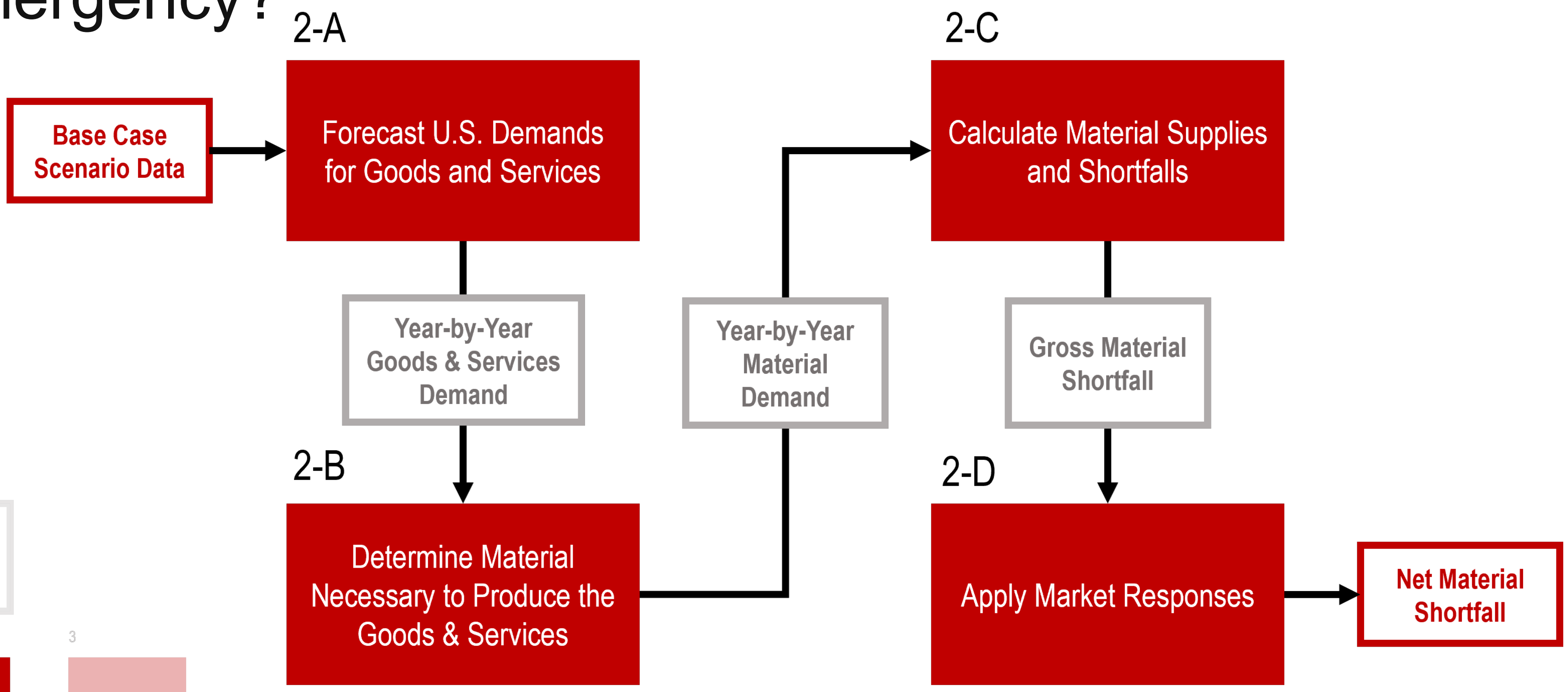


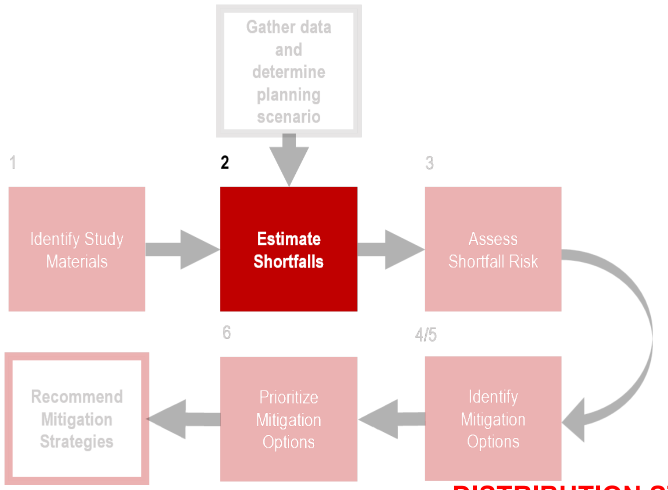
RAMF-SM Steps



Step 2 of RAMF-SM

Question: What U.S. material shortfalls would occur during a national emergency?





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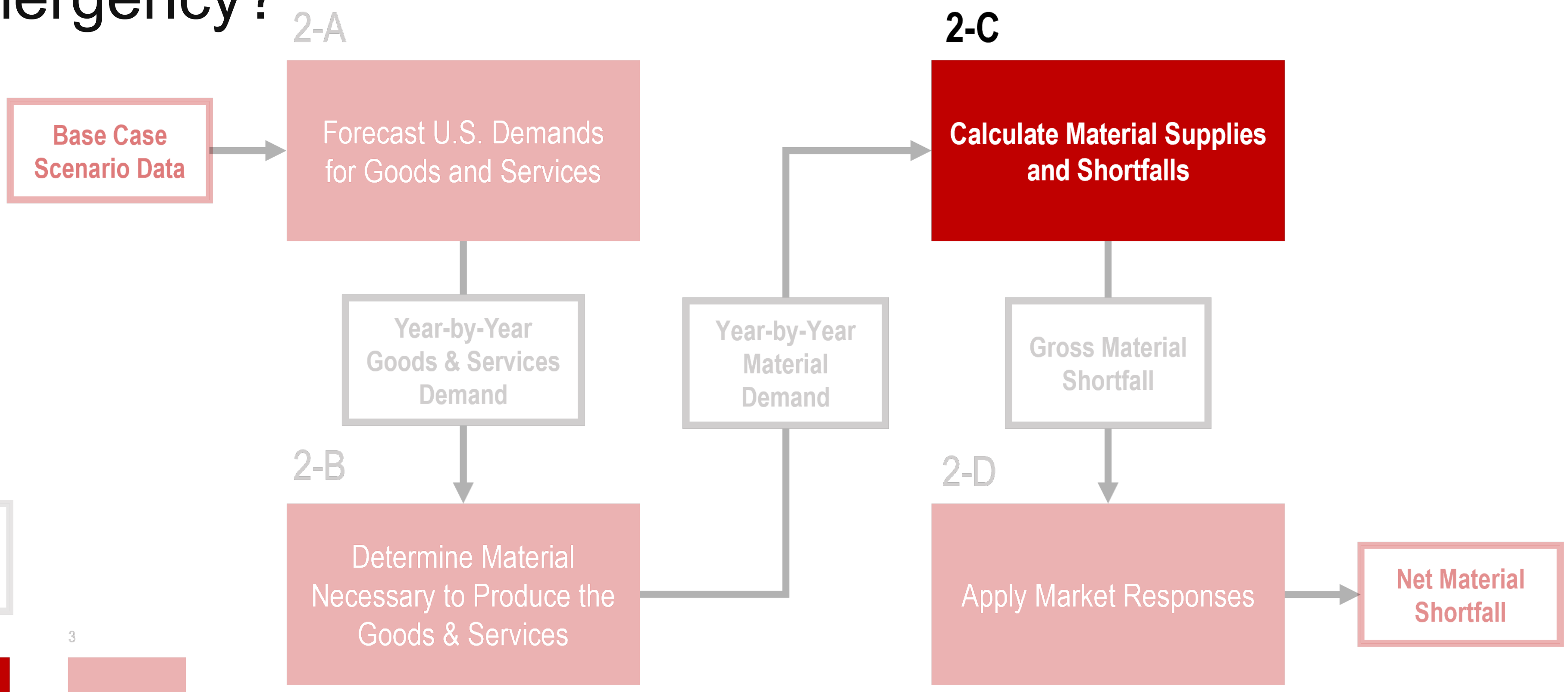
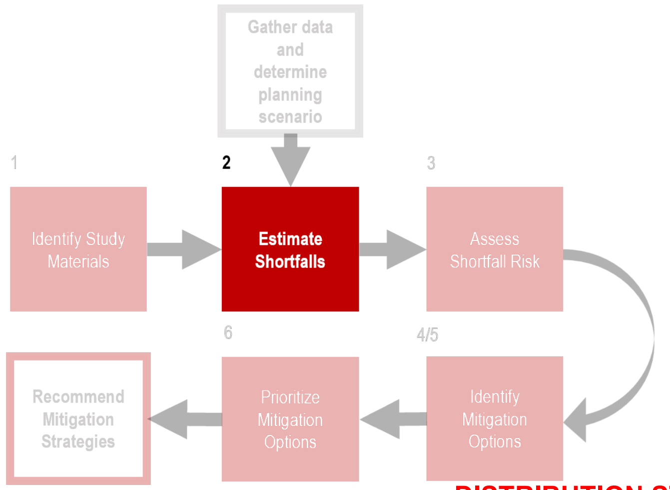




Motivation for Optimization-Based Modeling

- Global supply chains of mined materials are complex, and material flow is susceptible to various decrement factors
- Material is demanded in manufacture-ready form (e.g., metal wire, sheet, rod), but supply is often examined in a different form (e.g., ore)
- While demand for finished metal is related to demand for ore, explicitly tracing the stages of production may lead to new insights
  - Material loss or supply bottlenecks may occur at any stage of production

Goal: Develop a general mathematical framework that models the downstream flow of material production, **formally** capturing refining capability and constraints.

Two-Stage Production Model

11A 11B

- Consider a simple two-stage production process (e.g., mining and refining) of a single material over a four-year planning scenario
- Given the production capacities of each producer, is there a feasible route of material flow to meet U.S. demand?
  - If not, what is the shortfall? And where is the bottleneck in production?

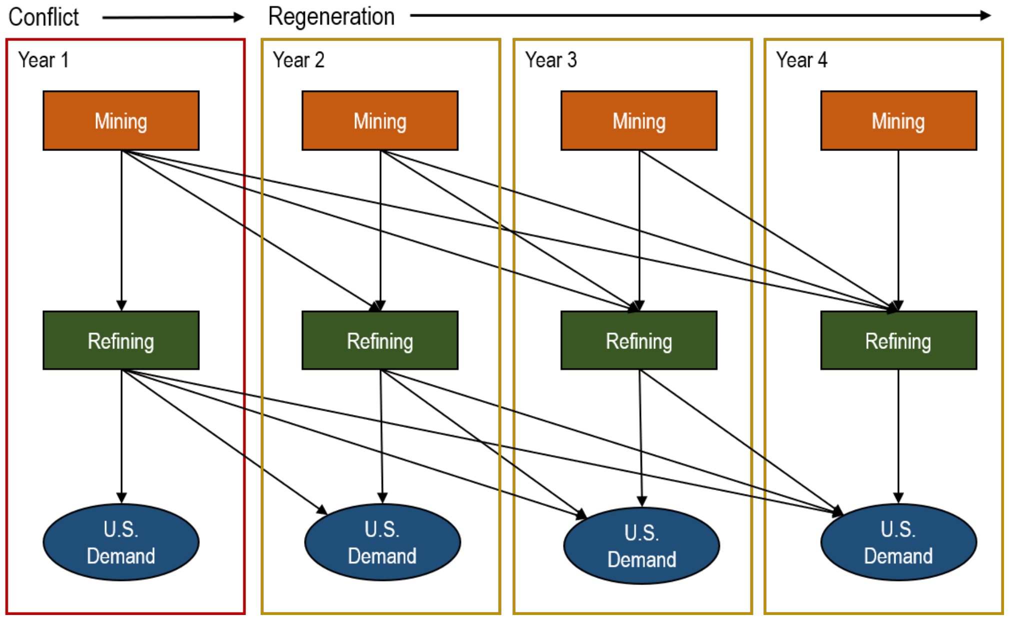


Figure. Time-space network representation of raw material supply chain.

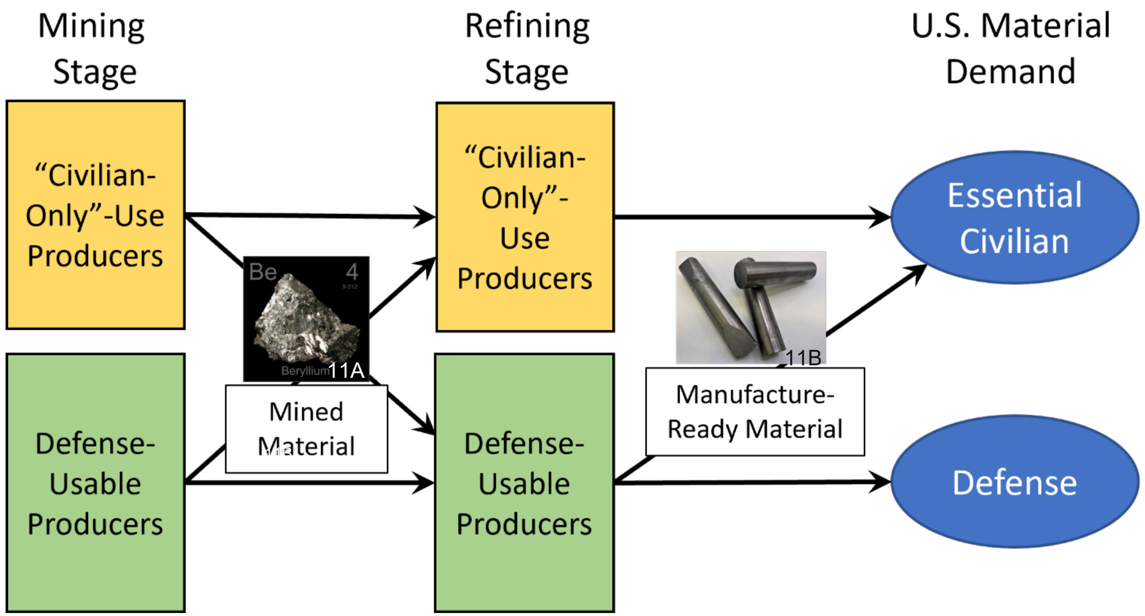
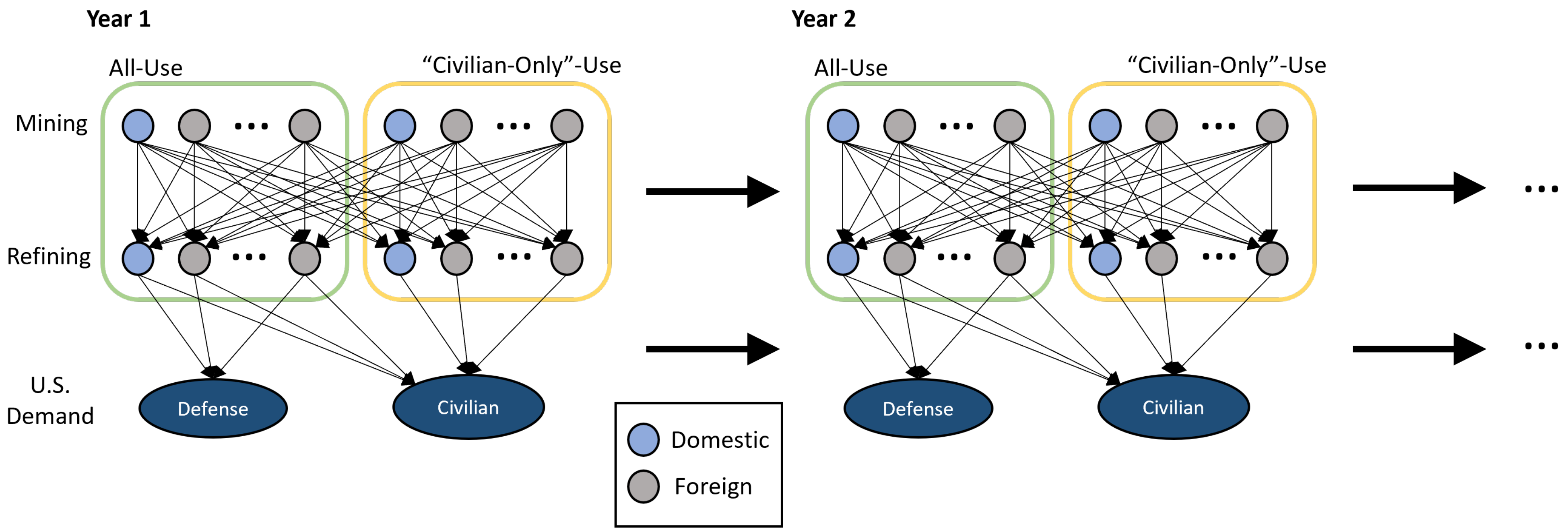


Figure. Usability classifications and flow restrictions.

Multi-Commodity Network Flow Approach

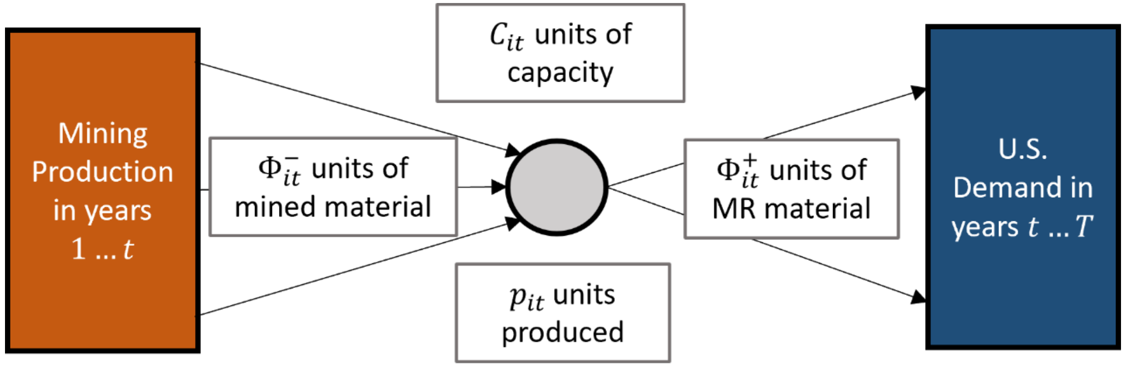
1. Create a **directed graph** to represent the global supply chain
2. Formulate **flow constraints** and incorporate **decrements to supply**
3. Construct an **optimal flow** on the network (i.e., numerical values on each edge) via linear programming



Production and Feedstock Constraints

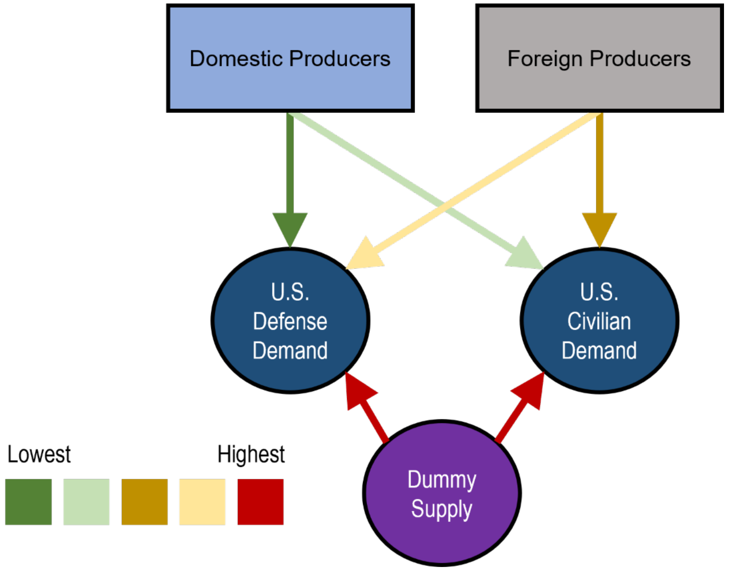
- Let C_{it} denote the production capacity of refinery i in year t
  - Decremented by producer's ability factor
- Let Φ_{it}^{-} denote the amount of mined material feedstock arriving at refinery i in year t
  - Decremented by shipping loss factor
- Let Φ_{it}^{+} denote the amount of manufacture-ready (MR) material leaving refinery i in year t
  - Decremented by material wastage factor
- Refining production p_{it} is limited by **both** refinery capacity and arriving feedstock
  - For refinery i in year t, we must have

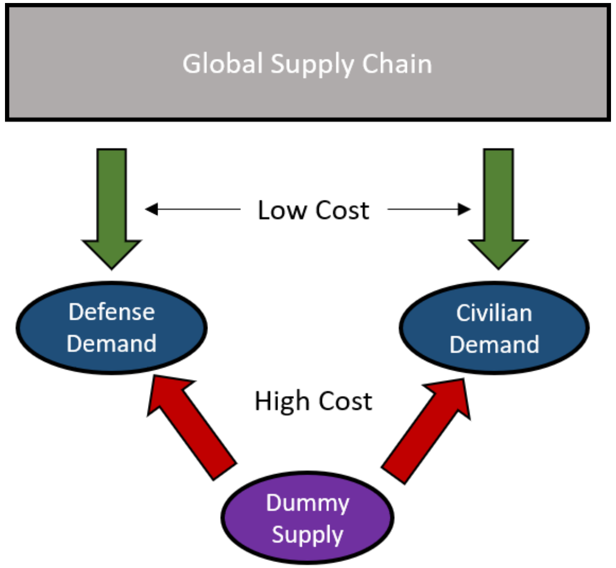
$$p_{it} \leq \Phi_{it}^{-} \quad \text{and} \quad p_{it} \leq C_{it}$$



Objective Function and Arc Costs

- **Objective:** Minimize the total cost of material flow across the network
- Arc costs are applied to each directed edge to **prioritize** or **restrict** certain material flows
- The **dummy supply node** is used to model material shortfall
  - It contains an unlimited amount of supply, but the cost of flow is set to be high relative to other sources of material flow
- The optimal solution will have positive flow from the dummy supply node *only* if there is no other feasible flow pattern to satisfy demand
  - Demand satisfaction is enforced through LP equality constraints





General LP Formulation

At a high level, the LP formulation of the downstream material production model is formulated as:

$$\text{Minimize:} \quad \textit{Total Cost of Material Flow} \quad \}\mathbf{c}^\top\mathbf{x}$$

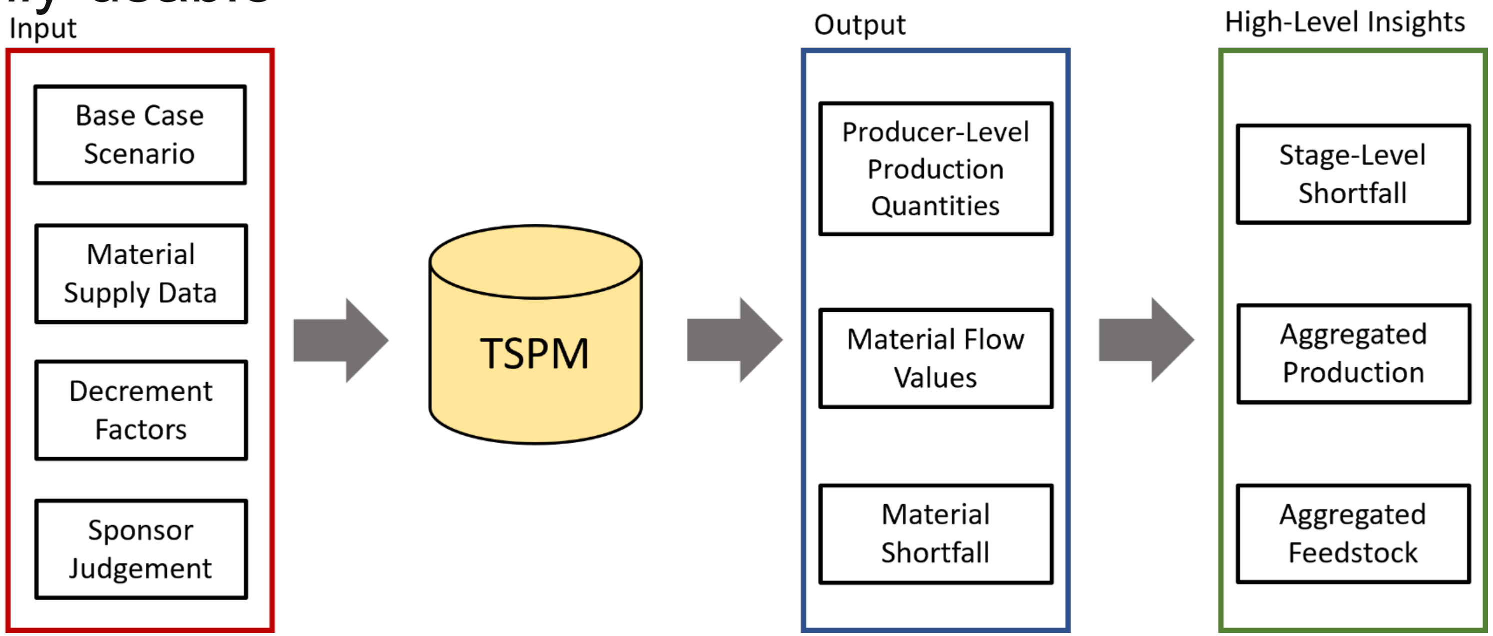
$$\text{Subject to:} \quad \left.\begin{array}{l} \textit{Flow Feasibility} \\ \textit{Usability Considerations} \\ \textit{Production Capacity} \\ \textit{Demand Satisfied} \end{array}\right\} A\mathbf{x} \leq \mathbf{b}$$

Output:

- Production level of each producer during all planning years
- Units of material flow between producers
- Year-by-year supply to U.S. demand categories
- Year-by-year shortfalls of manufacture-ready material

High-Level Insights

- Total defense-usable and civilian-only-usable mined material feedstock by year
- Stage-level shortfalls; identification of bottlenecks in supply chain
  - Can inform domestic production efforts and determinations of which material form is best to stockpile
- Aggregated production quantities: domestic, foreign, defense-usable, and civilian-only-usable



Conclusion and Model Impact

- The Two-Stage Production Model enhances supply and shortfall calculations in RAMF-SM by **explicitly** incorporating refining capacities and capabilities.
- Delivers more accurate shortfall estimates, enhancing the quality of risk mitigation options provided to DoD.

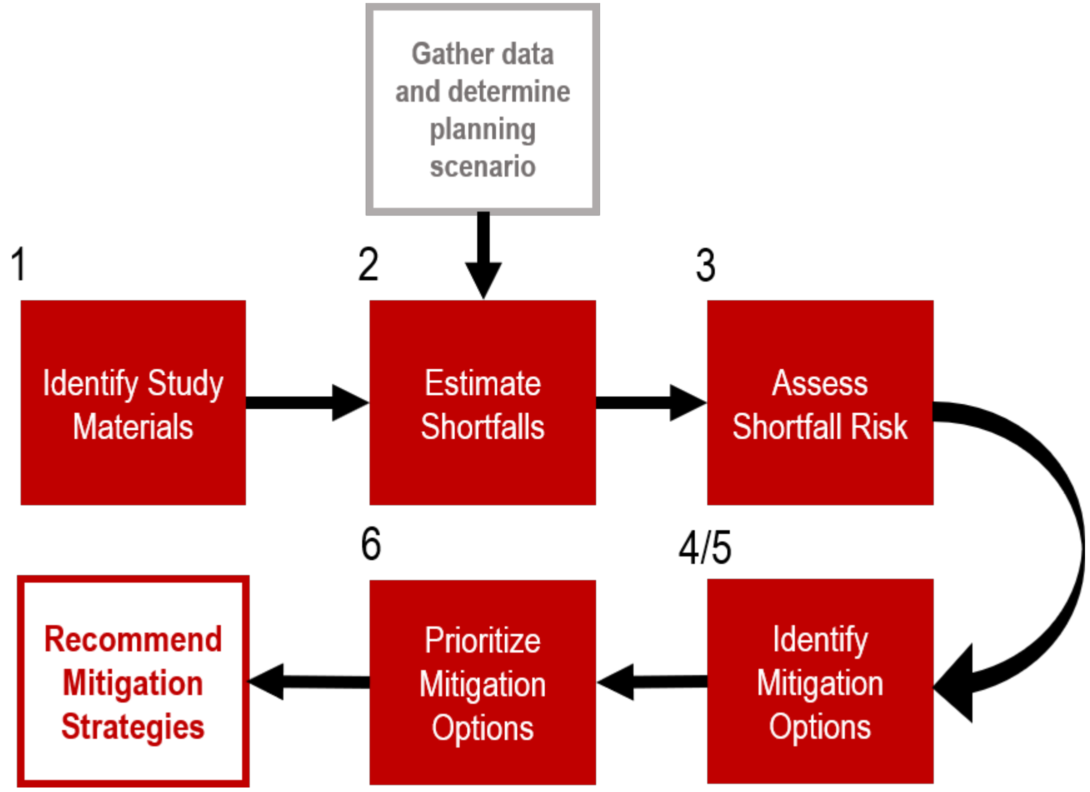


Figure. Steps of RAMF-SM

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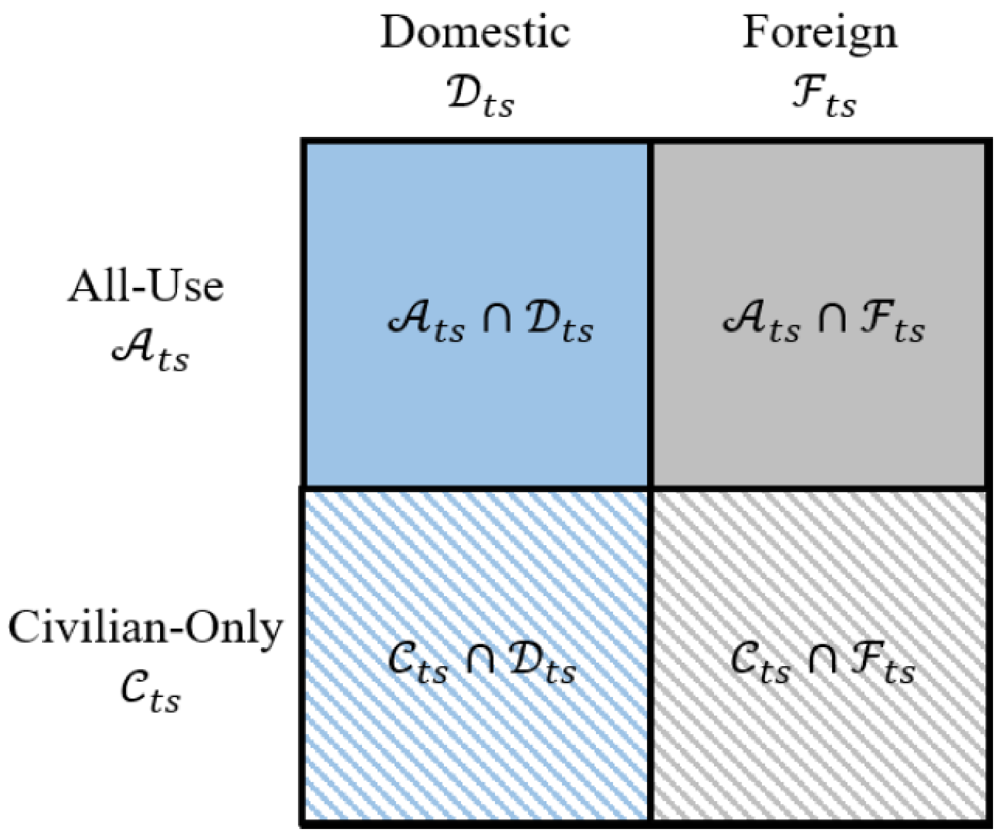
Notation Summary

Symbol	Description
	Producer Sets
$\mathcal{P}_{ts}$	Set of all stage-s producers during year t, indexed $1, \ldots, m_s$
$\mathcal{A}_{ts}$	Set of stage-s all-usable producers during year t, indexed $1, \ldots, k_s$ for $k_s \leq m_s$
$\mathcal{C}_{ts}$	Set of stage-s civilian-only-usable producers during year t, indexed $k_s + 1, \ldots, m_s$
$\mathcal{D}_{ts}$	Set of domestic stage-s producers during year t
$\mathcal{F}_{ts}$	Set of foreign stage-s producers during year t
	Input Parameters
$Cits$	Production capacity of stage-s producer i in year t, given as units of material
$D_t^{\text{(def)}}$	U.S. defense demand in year t, given as units of material
$D_t^{\text{(civ)}}$	U.S. civilian demand in year t, given as units of material
K_{uv}	Arc cost on arc $(u, v) \in \mathcal{E}$. For more details on arc notation for specific arcs, see Table 1

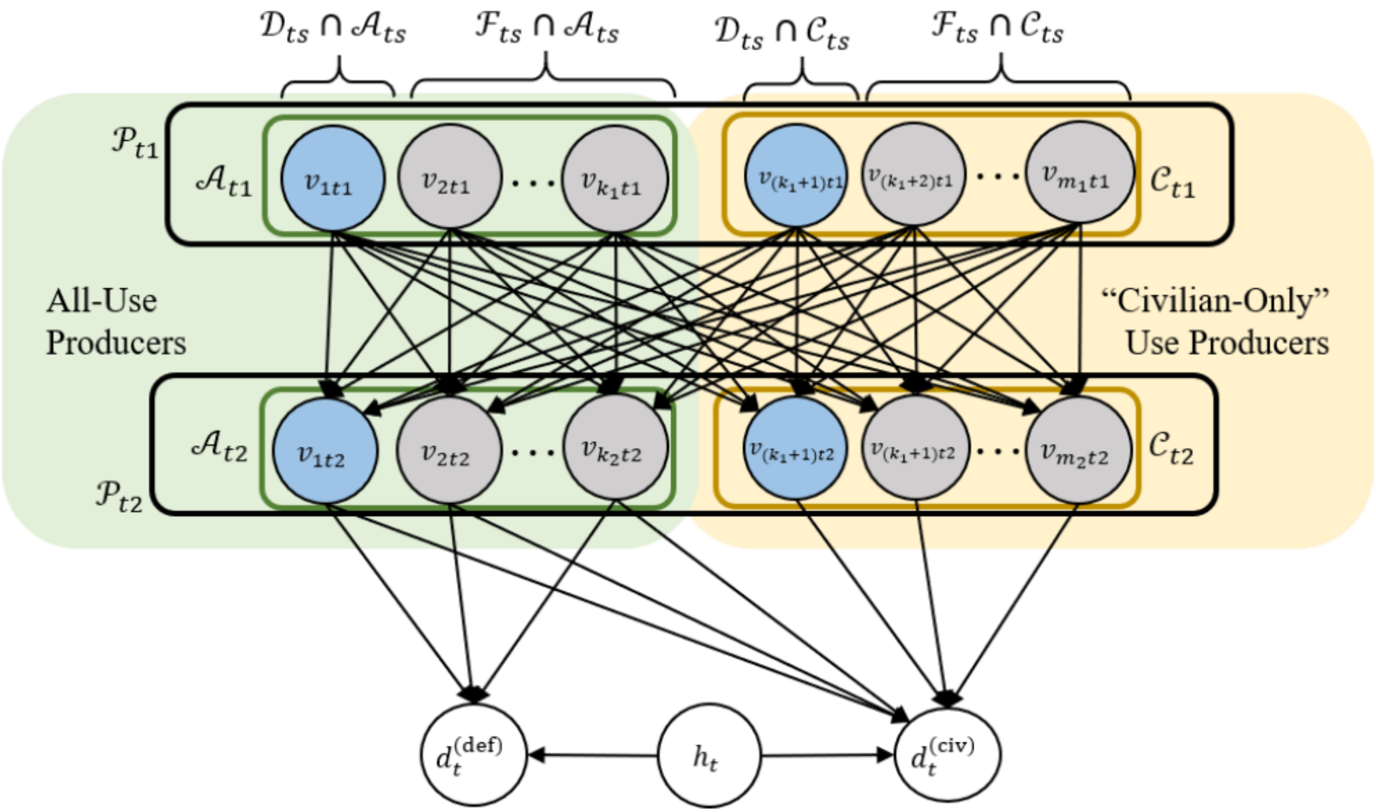
Classification of Stage-s Producers in Year t



Note: Producers fall into one of four categories based on usability and location (counterclockwise, from upper left): Domestic All-Usable, Domestic Civilian-Only-Usable, Foreign Civilian-Only Usable, and Foreign All-Usable.

Figure 2. Characterization of Stage-s Producers at Time t

Network Representation of Supply Chain with Decision Variables



Note: Here, we assume that there are $|\mathcal{A}_{ts}| = k_{ts}$ all-use producers, indexed $1, \ldots, k_{ts}$ and $|\mathcal{C}_{ts}| = m_{ts} - k_{ts}$ civilian-only-use producers, indexed $k_{ts} + 1, \ldots, m_{ts}$ in stage-s. Blue and gray nodes represent domestic and foreign producers, respectively.

Figure 3. Visualization of the Network Representation of the Supply Chain at a Fixed Time *t*

Decrement Factors

	Decrement Factors
$\hat{\beta}_{ijt}$	Shipping loss factor between stage-1 producer i and stage-2 producer j for material processed by producer i in year t
$\bar{\beta}_{it}$	Shipping loss factor between stage-2 producer i and the U.S. for material processed by producer i in year t
α_{its}	Ability factor of producer i in year t, computed as the product of multiple capacity reduction factors
δ_{its}	Willingness factor of stage-s producer i in year t, interpreted as the fraction of material not subject to delay
ρ_t	Market share factor for production in year t
γ_{it}	Wastage factor for stage-2 material processed by producer i in year t

Linear Program Decision Variables

Decision Variables	
$f_{itj\tau}$	Units of material flow from stage-1 producer $i \in \mathcal{P}_{t1}$ in year t to stage-2 producer $j \in \mathcal{P}_{\tau 2}$ in year τ
p_{its}	Units of material production from stage-s producer $i \in \mathcal{P}_{ts}$ in year t
$x_{it\tau}^{(\text{def})}$	Units of material processed by stage-2 producer i in year t flowing into U.S. defense demand node in year τ
$x_{it\tau}^{(\text{civ})}$	Units of material processed by stage-2 producer i in year t flowing into U.S. civilian demand node in year τ
$\sigma_t^{(\text{def})}$	Units of material flow from dummy supply node into U.S. defense demand node in year t
$\sigma_t^{(\text{civ})}$	Units of material flow from dummy supply node into U.S. civilian demand node in year t

Linear Programming Formulation (No Decrements)

$$\text{Minimize} \quad z(\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p}) \qquad (6)^{12}$$

$$\text{subject to:} \quad \sum_{t=t_0}^{T} \sum_{j \in \mathcal{P}_{t2}} f_{it_0jt} \leq p_{it_01} \qquad \forall i \in \mathcal{P}_{t_01}, t_0 \in [T] \qquad (7)$$

$$\sum_{t=t_0}^{T} \left[x_{it_0t}^{(\text{def})} + x_{it_0t}^{(\text{civ})} \right] \leq p_{it_02} \qquad \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] \qquad (8)$$

$$p_{it_02} \leq \sum_{t=1}^{t_0} \sum_{j \in \mathcal{P}_{t1}} f_{jtit_0} \qquad \forall i \in \mathcal{P}_{t_02}, t_0 \in [T] \qquad (9)$$

$$p_{it_0s} \leq C_{it_0s} \qquad \forall i \in \mathcal{P}_{ts}, s \in [2], t_0 \in [T] \qquad (10)$$

$$\sum_{t=t_0}^{T} x_{it_0t}^{(\text{def})} \leq \sum_{t=1}^{t_0} \sum_{j \in \mathcal{A}_{t1}} f_{jtit_0} \qquad \forall i \in \mathcal{A}_{t_02}, t_0 \in [T] \qquad (11)$$

$$\sigma_{t_0}^{(\text{def})} + \sum_{t=1}^{t_0} \sum_{j \in \mathcal{A}_{t2}} x_{jtt_0}^{(\text{def})} = \mathrm{D}_{t_0}^{(\text{def})} \qquad \forall t_0 \in [T] \qquad (12)$$

$$\sigma_{t_0}^{(\text{civ})} + \sum_{t=1}^{t_0} \sum_{j \in \mathcal{P}_{t2}} x_{jtt_0}^{(\text{civ})} = \mathrm{D}_{t_0}^{(\text{civ})} \qquad \forall t_0 \in [T] \qquad (13)$$

$$\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p} \geq 0 \qquad (14)$$

Linear Programming Formulation (With Decrements)

$$\text{Minimize} \quad z(\mathbf{f},\mathbf{x},\boldsymbol{\sigma},\mathbf{p}) \tag{34}$$

subject to:

$$\sum_{t=t_0}^{T}\sum_{j\in\mathcal{P}_{t2}} f_{it_0jt} \le p_{it_01} \qquad \forall i\in\mathcal{P}_{t_01}, t_0\in[T] \tag{35}$$

$$\sum_{j\in\mathcal{D}_{t_02}} f_{it_0jt_0} \le \delta_{it_01}\cdot p_{it_01} \qquad \forall i\in\mathcal{F}_{t_01}, t_0\in[t] \tag{36}$$

$$\sum_{t=t_0}^{T}\left[x_{it_0t}^{(\text{def})} + x_{it_0t}^{(\text{civ})}\right] \le \gamma_{it_0}\cdot p_{it_02} \qquad \forall i\in\mathcal{P}_{t_02}, t_0\in[T] \tag{37}$$

$$x_{it_0t_0}^{(\text{def})} + x_{it_0t_0}^{(\text{civ})} \le \gamma_{it_0}\cdot\delta_{it_02}\cdot p_{it_02} \qquad \forall i\in\mathcal{F}_{t_02}, t_0\in[T] \tag{38}$$

$$p_{it_02} \le \sum_{t=1}^{t_0}\sum_{j\in\mathcal{P}_{t1}} \hat{\beta}_{jit}\cdot f_{jtit_0} \qquad \forall i\in\mathcal{P}_{t_02}, t_0\in[T] \tag{39}$$

$$p_{it_0s} \le \alpha_{it_0s}\cdot C_{it_0s} \qquad \forall i\in\mathcal{P}_{ts}, s\in[2], t_0\in[T] \tag{40}$$

$$\sum_{t=t_0}^{T} x_{it_0t}^{(\text{def})} \le \gamma_{it_0}\sum_{t=1}^{t_0}\sum_{j\in\mathcal{A}_{t1}} \hat{\beta}_{jit}\cdot f_{jtit_0} \qquad \forall i\in\mathcal{A}_{t_02}, t_0\in[T] \tag{41}$$

$$\sigma_{t_0}^{(\text{def})} + \sum_{t=1}^{t_0}\left[\left(\sum_{k\in\mathcal{D}_{t2}\cap\mathcal{A}_{t2}} \bar{\beta}_{kt}\cdot x_{ktt_0}^{(\text{def})}\right) + \rho_t\left(\sum_{j\in\mathcal{F}_{t2}\cap\mathcal{A}_{t2}} \bar{\beta}_{jt}\cdot x_{jtt_0}^{(\text{def})}\right)\right] = \mathrm{D}_{t_0}^{(\text{def})} \qquad \forall t_0\in[T] \tag{42}$$

$$\sigma_{t_0}^{(\text{civ})} + \sum_{t=1}^{t_0}\left[\left(\sum_{k\in\mathcal{D}_{t2}} \bar{\beta}_{kt}\cdot x_{ktt_0}^{(\text{civ})}\right) + \rho_t\left(\sum_{j\in\mathcal{F}_{t2}} \bar{\beta}_{jt}\cdot x_{jtt_0}^{(\text{civ})}\right)\right] = \mathrm{D}_{t_0}^{(\text{civ})} \qquad \forall t_0\in[T] \tag{43}$$

$$\mathbf{f},\mathbf{x},\boldsymbol{\sigma},\mathbf{p} \ge 0 \tag{44}$$

Formal Objective Function Expression

Table 1. Relationships Among LP Flow Variables, Network Arc Construction, and Arc Costs

LP Variable	Network Representation	Arc Cost	Description
$f_{itj\tau}$	$(v_{it1}, v_{j\tau 2}) \in \mathcal{E}^1 \sqcup \mathcal{E}^4$	$K^f_{itj\tau}$	Material flow from stage-1 producer i in year t to stage-2 producer j in year $\tau \geq t$
$x^{(\text{def})}_{it\tau}$	$\left(v_{it2}, d^{(\text{def})}_\tau\right) \in \mathcal{E}^2 \sqcup \mathcal{E}^5$	$K^{x(\text{def})}_{it\tau}$	Material flow from stage-2 producer i in year t to U.S. defense demand node in year $\tau \geq t$
$x^{(\text{civ})}_{it\tau}$	$\left(v_{it2}, d^{(\text{civ})}_\tau\right) \in \mathcal{E}^2 \sqcup \mathcal{E}^5$	$K^{x(\text{civ})}_{it\tau}$	Material flow from stage-2 producer i in year t to U.S. civilian demand node in year $\tau \geq t$
$\sigma^{(\text{def})}_t$	$\left(h_t, d^{(\text{def})}_t\right) \in \mathcal{E}^3$	$K^{\sigma(\text{def})}_t$	Material flow from dummy supply node in year t to U.S. defense demand node in year t
$\sigma^{(\text{civ})}_t$	$\left(h_t, d^{(\text{civ})}_t\right) \in \mathcal{E}^3$	$K^{\sigma(\text{civ})}_t$	Material flow from dummy supply node in year t to U.S. civilian demand node in year t

$$
\begin{aligned}
z(\mathbf{f}, \mathbf{x}, \boldsymbol{\sigma}, \mathbf{p}) = & \sum_{t=1}^{T} \Big[\left(K^{\sigma(\text{def})}_t \cdot \sigma^{(\text{def})}_t + K^{\sigma(\text{civ})}_t \cdot \sigma^{(\text{civ})}_t \right) + \sum_{\tau=t}^{T} \sum_{i \in \mathcal{P}_{t1}} \sum_{j \in \mathcal{P}_{\tau 2}} \left(K^f_{itj\tau} \cdot f_{itj\tau} \right) \\
& + \sum_{\tau=t}^{T} \sum_{i \in \mathcal{P}_{t2}} \left(K^{x(\text{def})}_t \cdot x^{(\text{def})}_{it\tau} + K^{x(\text{civ})}_t \cdot x^{(\text{civ})}_{it\tau} \right) \Big]
\end{aligned}
$$

Image References

11A: Oat_Phawat/iStock via Getty Images. Lump of silver or platinum or rare earth minerals on black background.

11B: Dheo Tegar Pratama/iStock via Getty Images. Closeup of a bunch of metal pipes stacked on top of each other.