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Memetics – Overview and Baseline Models

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The Institute for Defense Analyses is a non-profit corporation that administers three federally funded research and development centers to provide objective analyses of national security issues, particularly those requiring scientific and technical expertise, and conduct related research on other national challenges.
PREFACE

This document was prepared by the Institute for Defense Analyses (IDA) under IDA’s independent research program. The objective of this report is twofold: to document key notions and ideas that collectively comprise what is known as meme theory or memetics – an emerging area of research for potential applicability in the defense, homeland security, and intelligence arenas – and to introduce, as a platform for developing mathematical models in memetics, existing classes of mathematical models for growth, survival, and the dynamics of biological systems.

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EXECUTIVE SUMMARY

The objective of this report is twofold. The first, addressed in Chapter I of the body of this report and summarized in Section B below, is to document key notions and ideas that collectively comprise what is known as *meme theory* or *memetics* – an emerging area of research for potential applicability in the defense, homeland security, and intelligence arenas. The second, addressed in Chapter II and summarized in Section C below, is to introduce, as a platform for developing mathematical models in memetics, existing classes of mathematical models for growth, survival, and the dynamics of biological systems. Before advancing to these summary sections, Section A presents a background discussion of what we mean by the terminologies *memetics* and *memes*.

A. BACKGROUND

1. Introduction

In this report, we adopt the simple viewpoint that a *meme* is a self-reproducing and propagating information structure analogous to a gene in biology. Further, we take memetics to be the study of the replication, spread, and change of memes, e.g., how ideas proliferate or how cultures can be characterized in terms of socially transmitted perspectives and traits that evolve over time. There has been considerable debate as to whether memes are truly observable and measurable, and whether memetics constitutes a distinct scientific discipline separate from existing and well-established social science sectors such as cultural anthropology and social psychology.¹ We intentionally eschew these debates and instead concentrate on the advancement of new research frontiers that could serve to promote the utility of socio-cultural modeling. In this regard, the specific focus of this report is on taking steps toward the development of a viable analytical framework for describing and studying memes.²


Analytical models of the propagation of memes, both mathematical and statistical, would support applications of memetics in two fundamental ways. The first is to provide systematic and organized vehicles for studying and understanding specific problem settings – via combinations of theoretical abstractions and tractable simulations. The second is to develop dynamic mechanisms for tracking and managing particular implementations of memetic engineering, incorporating qualitative and quantitative insights on the degree of attained “success” (e.g., via direct polling of the populace and/or analysis of indirect performance metrics based on surrogate indicators of public attitudes).

2. Potential Applicability

Well-developed meme theories potentially could be utilized to counter terrorists and insurgents, both before and after they become entrenched in their hostile stances, by influencing their beliefs in quantifiable and measurable ways. Applied as an interventionist strategy, memetic engineering recontextualizes the content of terrorism’s message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. In order to defeat the terror meme, one essentially needs to reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered. Specific approaches that could be pursued include:

- Rewriting scripts populated with toxic means and retransmitting them through mass media.
- Editing the terror meme by eliminating its power and thus reducing its attractiveness.
- Altering the cultural and societal architecture that encourages the incubation and transmission of the terror meme.
- Destroying the linkages (or networks) that facilitate transmission of the terror meme.
- Encouraging (though not censoring) the media to portray acts of terror in a different light by not giving the terrorists the dignity of coverage, courage, machismo, and martyrdom.

More broadly, applications of memetics conceptually could be tailored to support other critical defense, security, and intelligence concerns – including peacekeeping missions, psychological and public affairs operations, training, and recruitment. Studies
of memetics also could advance scientific frontiers in the areas of brain research, cognition, and socio-cultural modeling.

B. OVERVIEW OF MEMETICS

Memes can be habits, skills, stories, or any other kind of information that is copied from host to host (i.e., person to person). A meme is meant to be understood in rough analogy to a gene. Just as genes contain the instructions for creating the bodies that carry them, memes build the culture that transmits them. Further, just as genes proliferate in the gene pool by leaping from body to body via sperm or eggs, memes propagate in the meme pool by bounding from brain to brain via an imitation process.

Meme theory posits that people are the hosts and transmitters of memes, which may be passed from person to person through text, voice, image, or sound. At times, this transmission can occur rapidly wherein an idea can parasitically infect the mind and alter behavior. On the other hand, some memes can be spread more deliberately, with selection being more closely aligned with Darwin’s theory of natural selection so that only the “best” memes survive.

*Meme engineering* explores how a person may pass the meme on as he or she perceives it with the aim of altering the behavior of others. Some memes are so interconnected that they become a memplex, i.e., a set of related memes that are aggregated, copied, and disseminated as a distinct group, such as religious, cultural, or political doctrines.

A memetics perspective on terrorism posits the view that terrorist acts tend to be mimicked behavior caused through the acceptance of information associated with religious fanaticism and corrupted ideologies. The underlying memes that dictate terrorist actions are labeled terrorist memes. Within this context, memetic engineering is an interventionist strategy for recontextualizing the content of a terrorist’s message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. In order to defeat the terror meme, one needs to essentially reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered.

C. BASELINE MODELS FOR STUDYING MEMETICS

One fundamental research goal (well beyond of the scope of this introductory report) is to develop new models for characterizing the growth of memes, the survival
probabilities of memes, and the dynamics between toxic memes and their defeator memes. To this end, we first consider existing sets of models that may relate to memetics, via the parallelism between memetics, genetics, growth processes, and so forth.

Many of the models introduced in Chapter II are familiar to demographers, actuaries, and mathematical biologists, while others are well known to those working in reliability and survival analysis. The classes of models include growth models (exponential, suppressed exponential, and logistic), survival models (Gompertz, Makeham, Weibull, and Pareto), predator-prey models (Lotka-Volterra), and branching process models (Markov Chain). The potential application of branching process methods, suitably modified to account for the transformation of memes throughout their propagation, appears to be a fruitful research path ahead.
I. ON MEMETICS: AN OVERVIEW

The objective of this report is to document, in a conversational and non-technical style, some of the key notions and ideas that collectively comprise what is known as meme theory or memetics. The literature here is immense (see Finkelstein 2006, 2008a, and 2008b), as the potential applicability of memetics has been discussed in numerous fields – ranging from anthropology to sociology, with recent extensions to the intelligence and counterterrorism domains.

Memetics studies the replication, spread, and change of memes, e.g., how ideas proliferate or how cultures can be characterized in terms of socially transmitted perspectives and traits that evolve over time. Well-developed meme theories potentially could be utilized to counter terrorists and insurgents, both before and after they become entrenched in their hostile stances, by influencing their beliefs in quantifiable and measurable ways. Likewise, applications of memetics conceptually could be tailored to support critical defense, security, and intelligence concerns – including peacekeeping missions, psychological and public affairs operations, training, and recruitment. Studies of memetics also could advance scientific frontiers in the areas of brain research, cognition, and socio-cultural modeling.

While much of the memetics literature is excessively verbose, our review reveals that its essence revolves around a few key ideas. Elucidation of these, postponed until Section D, is intended to create a platform upon which technical formulations and methodological developments can take place – facilitating the conversion of memetics abstractions to tractable analytical tools amenable to practitioners.

Meme theory can be better appreciated if the parallel to gene theory and genetics is understood. Likewise, the similar correspondence with germ theory (i.e., how germs incubate and spread) is illuminating. Accordingly, Sections B and C below present overviews of relevant aspects of genetics and germ theory, along with their connection to Darwinian evolution.
A. GENES AND GENETICS

The biologist Dawkins conceptualized the living world to be parsed into the two primary categories of “replicators” and “vehicles.” Humans are on the vehicle side of the ledger, and their genes are the replicators. It is the genes that propagate themselves to the next generation; thus humans can be viewed as survival machines for the genes. Collectively, genes contain instructions for building the bodies that carry them, the vehicles. In other words, genes are the true replicators that act in their own interests – the concept embraced by the title of Richard Dawkins’ (1976) famous book *The Selfish Gene*.

The interest of a gene is replication. Those genes that make more copies of themselves, those that reproduce with greater fidelity, and those that have greater longevity will leave more copies for future generations. This is the gene-centered Darwinian view that of all of the evolved species of animals and plants, those forms surviving are the ones best adapted to the environment, i.e., the “survival of the fittest” theory.

*Genetics*, the study of genes, is the branch of biology that deals with heredity and variation in similar or related animals and plants. Fundamental to genetics are *Mendel’s laws*, which prescribe the four principles of hereditary phenomena. These laws specify the means by which traits are passed from parent to offspring, and the causes of similarities and differences between related organisms. There are different kinds of genetics: e.g., clinical genetics, molecular genetics, and population genetics. There are challenging mathematical problems spawned by genetics. These have been formulated and addressed by illustrious individuals such as Malthus, Lotka, Volterra, Fisher, Hardy, Haldane, and Karlin, to name a few. Perspectives on the statistical and probabilistic aspects of genetics can be found in the notable books by Gillespi (2004) and Durret (2008), respectively.

B. THE INCUBATION AND SPREADING OF GERMS – EPIDEMIOLOGY

Here one considers the spreading of a virus (or germ), which can be transported and transmitted under certain conditions that favor or limit its transportation and transmission. These germs can either remain at their source and be sterile, or emerge in the spreading of infection. Thus, for diffusion to take place, we must necessarily have a germ, a carrier, and receptive surroundings. The initial germ cannot travel on its own; it needs a carrier, typically, a human being moving from one place to another, or a mosquito, in the case of the malaria virus. For diffusion to take place, the environment
must be receptive; i.e., it must be suitable for the development of the germ, so that it does not remain sterile and cause merely a localized infection. Some parallels between the transmission of genes and the spread of germs are now apparent.

There are different ways to destroy and sterilize a germ, or to inhibit its diffusion. The first is to destroy the germ at its source. This could be difficult, however, because either the germ has not yet been identified, or, if it has been identified, its exact location is unknown. Further, there may be too many germs to destroy completely if the germ has multiplied extensively. The second line of defense consists of suppressing the carrier, or preventing the carrier from penetrating the territory to be preserved, i.e., imposing a quarantine of subjects. The third defensive approach is to take preventive measures via immunization and vaccination, providing immunity to potential hosts.

C. MEMES AND MEMETICS

The material of Sections A and B sets the stage for an appreciation of memes and memetics. This is because the notion of memes is a more recent one. Its development draws upon parallels with genetics and epidemiology, fields that have a mathematically mature science base.

When Dawkins (1976) introduced the idea of a meme, he defined it to be a self-reproducing and propagating unit of cultural information. He was looking for a word to describe a replicator in human culture that mimics the way a gene behaves under evolutionary biology principles. Dawkins shortened the Greek word mimeme (which means something imitated) to meme, because it sounded like a gene and therefore could convey the idea of a unit of cultural transmission. The Oxford English Dictionary (2008) defines a meme as “an element of culture that may be considered to be passed on by non-genetic means, such as imitation.” Blackmore (1998) describes the meme in terms of broadly defined instructions for behaviors and communications that can be learned by imitation and that stored in the brain. To Dawkins (1976), memes are informational replicators residing in the brain. Finkelstein (2008b) documents numerous other definitions of a meme, and introduces his own: “information which propagates, persists, and has impact.”

Memes can be habits, skills, stories, or any other kind of information that is copied from host to host (i.e., person to person). A meme is meant to be understood in rough analogy to a gene. Just as genes contain the instructions for creating the bodies that carry them, memes build the culture that transmits them. Further, just as genes
proliferate in the gene pool by leaping from body to body via sperm or eggs, memes propagate in the meme pool by bounding from brain to brain via a process that we call imitation.

Meme theory posits that people are the hosts and transmitters of memes, which may be passed from person to person through text, voice, image, or sound. At times, this transmission can occur rapidly wherein an idea can parasitically infect the mind and alter behavior. On the other hand, some memes can be spread more deliberately, with selection being more closely aligned with Darwin’s theory of natural selection so that only the “best” memes survive.

Dawkins identifies three characteristics of a successful meme replication:

- The more faithful the copy, the more will remain of the initial pattern after several rounds of copying
- The faster the copying, the more the replicator will spread
- The longer any instance of the replicating pattern survives, the more copies can be made of it.

Meme engineering explores how a person may pass the meme on as he or she perceives it with the aim of altering the behavior of others. Some memes are so interconnected that they become a memplex, i.e., a set of related memes that are aggregated, copied, and disseminated as a distinct group, such as religious, cultural, or political doctrines.

Memetics also articulates reasons for certain beliefs surviving and spreading:

- Memes survive and spread because they are helpful to the people who store them (e.g., true information about the world).
- Certain memes are numerous because they are good fits to pre-existing genetic dispositions.
- Certain memes spread because they facilitate the replication of genes that make vehicles that are good hosts for these memes (e.g., religious beliefs that urge people to have more children).
- Memes survive and spread because of self-perpetuating properties of the memes themselves.

D. MEME-GENE CO-EVOLUTION

Interactions between genes and memes can occur, for instance, when an individual’s propensity to adopt a particular meme depends on his (her) genotype (e.g.,
assertions of gender or racial superiority). Recently, branches of theoretical population genetics have focused on these sorts of phenomena, labeling them *meme-gene co-evolution*.

The works of Cavalli-Sforza and Feldman (1981), Boyd and Richerson (1985), and Feldman and Laland (1996) utilize population genetics models to investigate the evolution and dynamics of cultural traits. They explore phenomena so diverse as linguistics, social values and customs, epidemics, and the diffusion of innovations. The said literature postdates Dawkins (1976), but does not explicitly use the term meme. Instead, it refers to *cultural trait or cultural phenotype*.

**E. THE TERRORIST MEME AND MEMETIC ENGINEERING**

Currently, an era of growing terrorist threats confronts intelligence and security agencies. A memetics perspective on terrorism posits the view that terrorist acts tend to be mimicked behavior caused through the acceptance of information associated with religious fanaticism and corrupted ideologies. The underlying memes that dictate terrorist actions are labeled *terrorist memes*.

Within this context, *Memetic Engineering* is an interventionist strategy for recontextualizing the content of a terrorist’s message – thereby disengaging the susceptible mind from the terror meme, and thus reducing the potential for acts of terrorism, especially those attributed to mimicry. Many factors contribute to the incubation and replication of the terrorist memes. One of these is the advances in information technology and the strong focus of the media in reporting terrorist acts. Another factor for a meme to successfully replicate from mind to mind is *resonance* with the existing mental constructs; i.e., the meme must fit with a human need or desire.

Memetic engineering can be seen as entailing two phases: mapping the threat, and defeating the terror meme. With respect to mapping the threat, three elements have been recognized:

- A person must be *susceptible* to the message of the meme. That is, the person must be open to the intent of the message and the form in which the message is communicated.
- The person must be *immune* to alternative messages that might be inflicted upon the person by different cultures, education, and socialization.
- The meme must *resonate* with some intrinsic emotion or value possessed by the host.
In order to defeat the terror meme, one essentially needs to reverse-engineer it – first explaining what makes the terror meme attractive, and then identifying the individual components of the terror meme so that they can be manipulated and altered. Some of the components of the terror meme include a desire to be part of something larger, venting frustration and anger, finding absolution, fulfilling a fantasy, and fulfilling a need for power.

Defeating the terror meme could entail several approaches, including the following:

- Rewriting scripts populated with toxic memes and retransmitting them through mass media.
- Editing the terror meme by eliminating its power and thus reducing its attractiveness.
- Altering the cultural and societal architecture that encourages the incubation and transmission of the terror meme.
- Destroying the linkages (or networks) that facilitate transmission of the terror meme.
- Encouraging (though not censoring) the media to portray acts of terror in a different light by not giving the terrorists the dignity of coverage, courage, machismo, and martyrdom.

F. SUMMARY AND CONCLUSION

One overall objective of our research is to develop mathematical models of memetics for its use in the intelligence and national security domains. Mathematical models serve several purposes. The first is a systematic and organized vehicle for understanding and unambiguously communicating a problem. The second is to develop mechanisms for managing the problem and putting it to use in practical settings.

The science of memetics is relatively new. As a consequence, a mathematical platform that articulates the essence of the issues underlying memetics remains to be developed. Since memetics has its genesis in genetics, however, the well developed mathematics of genetics could be a starting point for developing the mathematics of memetics. Like genetics, memetics appears to also have parallels with germ theory and the spread of germs in the context of infectious diseases. Here again, we have the mathematics of germ spread (epidemiology), which could be relevant. Then there is the gene-meme co-evolution theory whose mathematics goes under the label of cultural evolution theory. The underlying premise here is that there is an interaction between
genes and memes; such interactions produce cultural and sociological traits. In particular, one’s disposition to embrace certain memes could be a function of one’s genotype. Finally, there is the notion of memetic engineering. This takes a systems engineering-type approach to the memetics of terror, and endeavors to destroy the terror meme by first understanding its constituent elements and then defeating them.

Thus it appears that there is a collection of available mathematical and engineering technologies that can be brought to bear on memetics to begin to transform it from a descriptive field to a more quantitative science. As of this writing, it is not clear to us whether the science of memetics is rich enough to spawn mathematics that is fundamentally new.

G. REFERENCES


Chapter I provided an introductory overview of memetics and its relationships with cultural evolution theories. It motivates us in this chapter to look at mathematical models for growth, survival, and the dynamics of biological systems as a platform for developing mathematical models in memetics. Of interest here is the growth of memes, the survival probabilities of memes, and the dynamics between toxic memes and their defeator memes.

The models to be described here are familiar to demographers, actuaries, and mathematical biologists; some of these are also recognizable to those working in reliability and survival analysis. These models are intended to bring some synergy to the science of memetics, and it is with this point of view that we give an overview of such models.

A. POPULATION GROWTH MODELS

One of the simplest models for population growth, called the law of growth, is prescribed by a differential equation of the form \( \frac{dN(t)}{dt} = rN(t) \), where \( N(t) \) denotes the size of a population at time \( t \), and \( r \) is a constant called the Malthusian parameter. Essentially, the model prescribes that the rate of change of a population is proportional to its present size.

The solution of the above differential equation follows by integration. Specifically,

\[
\int_{N(0)}^{N(t)} \frac{dN(t)}{N(t)} = \int_{0}^{t} r \, dt ,
\]

where \( N(0) \) is the initial size of the population. Thus, \( \ln(N(t)/N(0)) = rt \), yielding \( N(t) = N(0) \cdot e^{rt} \).

A slightly more complicated version of the exponential growth model attempts to suppress the long run growth. Here, the governing differential equation is of the form

\[
\frac{dN(t)}{dt} = N(t) \cdot [(rt - 1)/t] ,
\]
with \( r > 1 \). Its solution, also obtained by direct integration, is simply \( N(t) = N(0) \cdot e^{rt} / t \), where the \( t \) in the denominator clearly suppresses the exponential growth of the preceding result.

A third model for population growth, is the *logistic growth model*, popular in biology. Here, the governing differential equation is of the form

\[
dN(t)/dt = rN(t)(K - N(t))/K,
\]

where \( K \) is some constant. Its solution is

\[
N(t) = \frac{K}{1 + \left( \frac{K}{N(0)} - 1 \right) \cdot e^{rt}}.
\]

**B. LIFE EXPECTANCY CURVES**

These curves are used in actuarial science to specify mortality laws. They are also used by survival analysts in biomedicine to manage health care, as well as in engineering reliability to facilitate maintenance planning. Their relevance to memetics could be in terms of the amount of time a toxic meme could prevail in a population.

Let \( S(x) \) denote the probability that a newborn reaches age \( x \); i.e., it survives to \( x \). Then, the *Gompertz Law of Survival* (1832) states that

\[
S(x) = \exp[-m(C^x - 1)], \ x \geq 0,
\]

for some constants \( C > 1 \) and \( m \geq 0 \). Observe that \( S(x) = 1 \) when \( x = 0 \), and that \( S(x) \downarrow 0 \) as \( x \uparrow \infty \).

A variation of the above is *Makeham’s Law of Survival* (1860), wherein

\[
S(x) = \exp[-Ax - B(C^x - 1)], \ x \geq 0,
\]

for some constants \( C > 1, B > 0, \) and \( A \geq -B \). Here again \( S(x) = 1 \) when \( x = 0 \), and \( S(x) \downarrow 0 \) as \( x \uparrow \infty \). When \( B = 0 \) and \( A > 0 \), \( S(x) \) yields an exponential survival function commonly used in reliability and life-testing. Some other commonly used survival curves are the Weibull and the Pareto.

**C. THE PREDATOR-PREY MODELS OF LOTKA AND VOLterra**

To describe the dynamics of biological systems, Lotka (1925) and Volterra (1926) proposed a system of non-linear differential equations wherein two species, one a predator and the other a prey, interact within an ecological system. These equations can
be germane to memetics if memes can be categorized as being either toxic or as being antidotes to toxic memes, as is conceptualized in memetic engineering. From the point of view of military intelligence, a toxic meme can be viewed as the prey and its antidote meme considered to be the predator. This makes sense because in a biological context, the prey of a species tends to grow exponentially (in population size) and is supposed to have an unlimited supply of food. The predator of the species is smaller in size and depends on the prey for its supply of food.

1. The Lotka-Volterra Equations

Let \( x(t) \) denote the number of the prey of a species at time \( t \), and \( y(t) \) the number of its predators at that same point in time \( t \). Then the dynamics of the interaction between the species is described by the following system of differential equations:

\[
\frac{dx(t)}{dt} = x(t)[\alpha - \beta y(t)] \quad \text{and} \quad \frac{dy(t)}{dt} = -y(t)[\gamma - \delta x(t)],
\]

where \( \alpha, \beta, \gamma, \) and \( \delta \) are parameters characterizing the interaction. The first equation represents the feature that the change in the prey’s number is given by its own growth minus the rate at which it is preyed upon. Similarly, the second equation characterizes the change in the predator’s population to be its growth minus its natural death, namely \( \gamma y(t) \).

The solution of these equations is periodic. It yields a simple harmonic motion with the population of predators following that of prey; see Figure II-1.

![Figure II-1. Predator-Prey Population Dynamics](image)
The portrayed dynamics show that predators thrive when prey are plentiful, but then the predators eventually outstrip their food supply and begin to decline. As the predator population shrinks down, the prey population thrives again and the process repeats itself cyclically. There is no prediction whenever \( x(t) \) or \( y(t) \) is zero.

2. Population Equilibrium

Population equilibrium means the absence of any growth or decay in the population size. This is tantamount to setting \( \frac{dx(t)}{dt} = 0 = \frac{dy(t)}{dt} \), which yields the non-trivial solution \( x = \gamma / \delta \) and \( y = \alpha / \beta \) for all \( t > 0 \). It can be shown that the predator and prey populations cycle and oscillate around the curve

\[
y(t)^{\alpha} e^{-\beta x(t)} x(t)^{\gamma} e^{-\delta y(t)} = \text{constant}.
\]

The extinction of both species is difficult. It can happen only if the prey are completely eradicated by artificial means; this will cause the predators to die of starvation. If the predators are eradicated, then the prey population grows unchecked (in this model).

D. THE PROPAGATION OF A TOXIC MEME – A BRANCHING PROCESS MODEL

From a historical perspective, the study of branching processes arose from a somewhat sexist context of evaluating the probability that a man’s direct male line of descendents eventually dies out. The problem is attributed to Sir Francis Galton and the Rev. H. W. Watson (1874). Its more recent impetus has come from a study of the behavior of “particles,” such as bacteria, genes, and neutrons, that generate new particles of the same type. Since memes transform themselves as they propagate, studying their behavior via standard branching process theory seems inappropriate. The standard theory needs to be enhanced to account for the generation of transformed “particles.” To do so, however, one needs to gain an appreciation of the essentials of the technology of the standard theory because it could serve as a foundation for the development of an enhanced theory.

The material that follows is motivated with the above consideration in mind. Section 1 introduces preliminary notation, terminology, and key results from branching processes theory. Section 2 gives an overview of probability generating functions (pgf), a key technical tool for studying standard branching processes.
1. Preliminaries and the Branching Process Model

Consider a toxic meme (e.g., anti-American sentiment), henceforth the **parent meme**, that can spawn $\xi_1$ offspring memes, all judged to be identical in their toxicity to the parent meme. We assume that $\xi_1$ is random with $P(\xi_1 = n) = p_n$, $n = 0,1,2,...$. The parent meme is said to belong to the 0\textsuperscript{th} generation, and its offspring are said to belong to the 1\textsuperscript{st} generation. Each first generation offspring meme can generate additional offspring, with memes being identical to the parent meme. The additional offspring memes are said to belong to the 2\textsuperscript{nd} generation. This process of memes generating offspring memes continues like the branches of a tree, and thus the label “branching processes.” Figure II-2 portrays an illustrative example.

![Figure II-2. Schemata of Toxic Meme Generation](image)

In principle, the above process of toxic meme generation can continue indefinitely, so that, after several cycles of meme generation, one could have an essentially infinite population of toxic memes. However, it is possible that not all memes spawn offspring so that the meme tree could be *de facto* pruned. For example, in Figure II-2, the 3\textsuperscript{rd} generation consisting of only two memes does not spawn additional memes; thus the process of toxic meme generation is extinguished.

To model mathematically the above process, let $X_j$ denote the total number of (toxic) memes in the $j$-th generation, $j = 0,1,2,...$. In our case, $X_0 = 1$, and in Figure II-2 we have $X_1 = 2$, $X_2 = 4$, $X_3 = 2$, and $X_4 = 0$, since the process gets extinguished at the 4\textsuperscript{th} generation. The collection of random variables \( \{X_j, j = 0,1,2,...\} \) constitutes a stochastic
process, of which the Markov Chain is an important sub-class. Thus, we ask, under what conditions will the process \( \{X_j, j = 0,1,2,\ldots\} \) be a Markov Chain?

It is well known that the sum of independent, identically distributed discrete random variables constitutes a Markov Chain. Thus, if we suppose that meme \( i \) gives rise to \( \xi_i \) memes for the next generation, where \( \xi_i \) is a discrete random variable, and if the various \( \xi_i \)'s are independent and identically distributed, then \( \{X_j, j = 0,1,2,\ldots\} \) will be a Markov Chain. Specifically, \( \xi_1 \) is the number of memes spawned by the parent meme, \( \xi_2 \) is the number of memes spawned by the first offspring (which necessarily is in the 1st generation of memes), \( \xi_3 \) is the number of memes spawned by the second offspring (which may be 1st or 2nd generation of memes), and so on.

The Markov Chain \( \{X_j, j = 0,1,2,\ldots\} \) has the set of non-negative integers as its state space, with 0 as an absorbing state. Its transition function is given by the expression

\[
p(x,y) = P(\xi_1 + \ldots + \xi_x = y),
\]

for \( x \geq 1 \). Here \( p(x,y) \) denotes the probability that the chain in state \( x \) transitions to state \( y \) in one step of the iteration, i.e., a generation with \( x \) offspring memes is followed with the next generation having exactly \( y \) offspring memes. Interest focuses on \( \rho \), the probability that the parent meme and all its offspring become extinct, i.e., the Markov Chain starting in state 1 gets absorbed in state 0. Once \( \rho \) can be determined, the branching process model construction is such that were one to start with \( x \) toxic genes, all identical to each other, then the probability that all the \( x \) toxic genes and their descendents get extinguished is \( \rho^x \).

We have assumed that the \( \xi_i \)'s are independent and identically distributed, and that \( P(\xi_i = n) = p(n) \), for \( n = 0,1,2,\ldots \), and all \( i \). If \( p(1) = 1 \), that is, each meme will generate exactly one offspring meme, then the resulting Markov (Branching) Chain is degenerate, i.e., every state is an absorbing state. Thus, we require that \( p(1) < 1 \). When such is the case, 0 is an absorbing state, and every other state is a transient state. A consequence is that the chain is either absorbed at 0 or it approaches \( +\infty \), i.e., it explodes. Indeed, it can be shown

\[
P_x \left( \lim_{j \to \infty} X_j = +\infty \right) = 1 - \rho^x, \quad x = 1,2,\ldots
\]

where \( P_x(A) \) is the probability of some event \( A \) of interest when the Markov Chain is in the state \( x \). Thus the probability that a single toxic meme eventually spawns an infinite number of toxic memes is \( (1 - \rho) \). Consequently, it is important to know whether \( \rho = 1 \) or \( \rho < 1 \). The answer to this question is provided via pgf's.
Specifically, write $\phi(t)$, the pgf of the random variable $\xi_i$, as

$$
\phi(t) = p(0) + \sum_{y=1}^{\infty} p(y)t^y, \quad 0 \leq t \leq 1,
$$

and note that $\mu = \sum_{n=0}^{\infty} np(n)$ is the mean of $\xi_i$. Then, the behavior of $\rho$ is governed by the roots of the equation $\phi(\rho) = \rho$.

When $\mu \leq 1$, $\phi(\rho) = \rho$ has no roots in $[0,1)$, and thus $\rho = 1$. This implies that the extinction of a toxic meme is certain if $\mu \leq 1$ and $p(1) < 1$. When $\mu > 1$, $\phi(\rho) = \rho$ has a unique root $\rho_0$ in the interval $[0,1)$. Thus to control the spread of toxic memes, we must have $\mu \leq 1$, i.e., on average, a toxic meme should spawn at most one offspring toxic meme. Memetic engineering therefore should focus on the control of $\mu$. The case $\mu \leq 1$ is known as the subcritical case, and the case $\mu > 1$ is referred to as the supercritical case.

2. Probability Generating Functions

The pgf of the random variable $\xi_i$, $\phi(z) = \sum_{n=0}^{\infty} p(n)z^n$, is absolutely convergent for $|z| \leq 1$. Moreover, $\phi(1) = 1$, $\phi(0) = p(0)$, $\phi'(0) = p(1)$, and, in general, $\phi^n(0) = n!p(n)$, where $\phi^n$ denotes the $n$-th derivative of $\phi$ with respect to $z$. Thus, knowing $\phi(z)$ in some neighborhood of 0, we know $p(n)$ for all $n$.

Also, $\phi'(z) = \sum_{n=0}^{\infty} nz^{n-1} p(n)$ implies $\phi'(1) = \sum_{n=0}^{\infty} np(n) \equiv \mu$, the mean of $\xi_i$. Similarly, we can see that $\sigma^2$, the variance of a $\xi_i$ is $\sigma^2 = \phi^2(1) + \phi'(1) - (\phi'(1))^2$. Accordingly, the pgf encompasses all the key features of $\xi_i$.

There are some other properties of pgf’s that are germane to us in a study of branching processes. For example, suppose that a discrete random variable $\xi_1$ has pgf $\phi_1(z)$, and another discrete random variable $\xi_2$ has pgf $\phi_2(z)$. Then, if $\xi_1$ and $\xi_2$ are independent, the pgf of their sum $\xi_1 + \xi_2$, denoted by say $\phi_{1,2}(z)$, is $\phi_{1,2}(z) = \phi_1(z) \cdot \phi_2(z)$. An induction argument similarly establishes that the pgf of the sum of $k$ independent and identically distributed discrete random variables $\xi_1 + \xi_2 + \cdots + \xi_k$ is $(\phi(z))^k$, where $\phi(z)$ is the pgf of each $\xi_i$.

More generally, it can be proved that, if the number of memes in a population has pgf $Q(z)$, and if the number of offspring spawned by each meme has pgf $\phi(z)$, then the
pgf for the total number of offspring memes is $Q(\phi(z))$. Thus, if the 0th generation has one toxic meme, then the number of toxic memes spawned by the first toxic meme has pgf $\phi(z)$, and the total number of toxic memes in the second generation will have pgf $\phi(\phi(z))$. Continuing in this fashion, the total number of toxic memes in the $m$-th generation will have pgf

$$\phi_m(z) = \phi(\phi(\cdots(\phi(z))\cdots)),$$

where the function $\phi$ appears exactly $m$ times in the right-hand-side of the equation, $\phi_m(z) = \phi(\phi_{m-1}(z))$, and $\phi_0(z) = z$.

Let $\pi_m$ be the probability the offspring of toxic genes spawned by a single 0th generation toxic gene gets extinguished at or before the $m$-th generation. Then, clearly

$$\pi_0 = 0, \pi_1 = p(0) = \phi(0), \text{ and } \pi_m = \phi_m(0) = \phi(\phi(\cdots(\phi(0))\cdots)).$$

Thus $\pi_{m+1} = \phi(\pi_m)$. However, because $\phi'(z) = z = \sum_{n=0}^{\infty} nz^{n-1} p(n) > 0$, it follows that $\phi$ is an increasing function of $z$. Consequently, since $\pi_1 > \pi_0$, it follows by induction that we must have $\pi_{m+1} > \pi_m$. Moreover, $\pi_m$ is a probability that is bounded above by 1. Consequently, $\pi_m \to \rho$ for some $\rho$, and continuity arguments readily establish that $\phi(\rho) = \rho$, a result we used in Section 1.

3. Some Examples

Suppose that each toxic meme spawns offspring memes with their number governed by a Poisson distribution $p(n) = \lambda^n e^{-\lambda} / n!$. Then $\phi(z) = \exp(\lambda(z - 1))$, and we seek the roots of $z = \exp(\lambda(z - 1))$. The mean number of offspring is $\lambda$. With $\lambda = 1$, we obtain $\pi_1 = .3679, \pi_2 = .6259, \pi_31 = .9411, \pi_{127} = .9854$, and $\pi_\infty = 1 = \rho$. When $\lambda = 1.01$, the corresponding extinction probabilities are .3642, .6197, .9729, .9729, and .9803.

Lotka fitted the U.S. population via a pgf representation of the form $p(0) = \alpha$ and $p(n) = \beta n^{-1}$. For this representation, $\phi(z) = \alpha + \beta z (1 - \gamma z)^{-1}$ and $\phi'(z) = \beta (1 - \gamma z)^{-2}$. Incorporating the earlier general result $\phi(1) = 1$, the equation $\phi(z) = z$ here can be simplified to obtain the relationship $(\gamma z - \alpha)(z - 1) = 0$. Thus, the unique root is the minimum of 1 and $\alpha/\gamma$. With Lotka’s fitted values $\alpha = .4825, \beta = .2126$, and $\gamma = .5893$, the estimated extinction probability is $\alpha/\gamma = .8188$.

E. REFERENCES


**Title and Subtitle:**
Memetics – Overview and Baseline Models

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**Abstract:**
This report documents key notions and ideas that collectively comprise what is known as meme theory or memetics – an emerging area of research for potential applicability in the defense, homeland security, and intelligence arenas – and introduces existing classes of mathematical models for growth, survival, and the dynamics of biological systems as a platform for developing mathematical models in memetics.

**Subject Terms:**
Meme, memetics, meme engineering, memplex, terrorist meme