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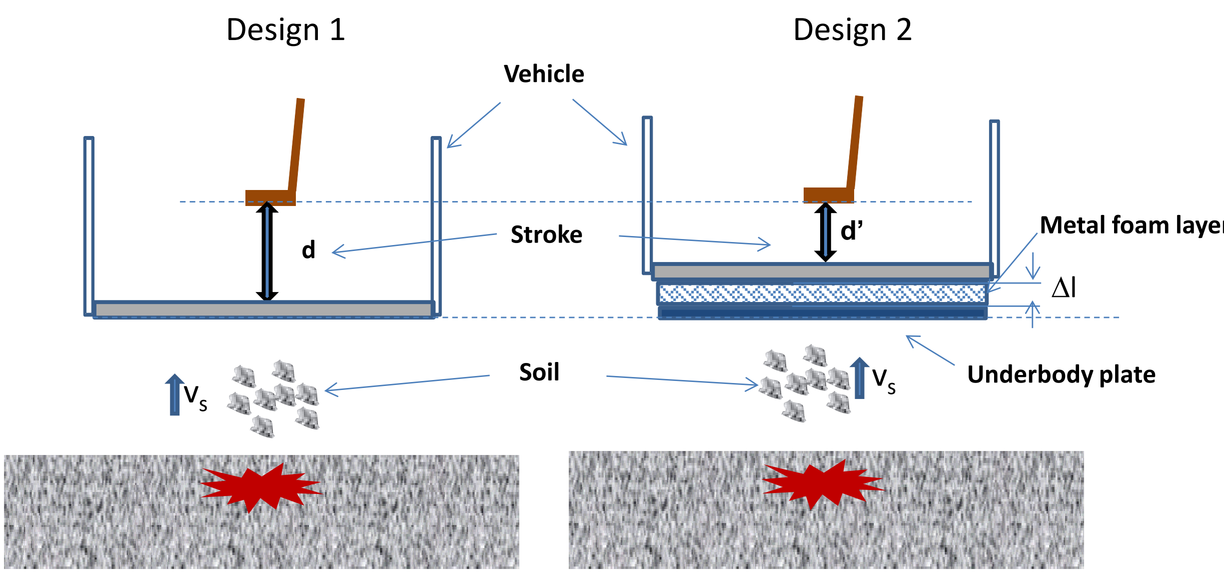
Executive Summary

We analyzed the effect of placing an energy-dissipation barrier under the vehicle to improve its blast resistance. We developed a simplified model to estimate the momentum transfer from soil to the vehicle during the impact of the fast moving soil with the underbody. The model considers a continuous collision of soil with the underbody; it accounts for the foam yield stress resisting the underbody motion and the stroke reduction that results because the foam compression distance is comparable with the original vehicle stroke and therefore not negligible.



Schematic of the Soil Impact Problem

Our results indicate that in terms of reduction of momentum transferred to the vehicle from the fast moving soil, an upper bound on improvement of the vehicle blast resistance due to placing an energy-dissipative underbody barrier is on the order of 5% to 8%.

Contents

1. Introduction

A. Background

One of the current Defense Advanced Research Projects Agency (DARPA) efforts is to develop foam-like materials that offer high kinetic energy dissipation per unit mass or volume. Currently, many types of foams exhibit constant compressive stress over an extended region of strain, thus providing a mechanism for dissipating energy [1, 2]. Figure 1 shows a typical stress vs. strain response of such materials.

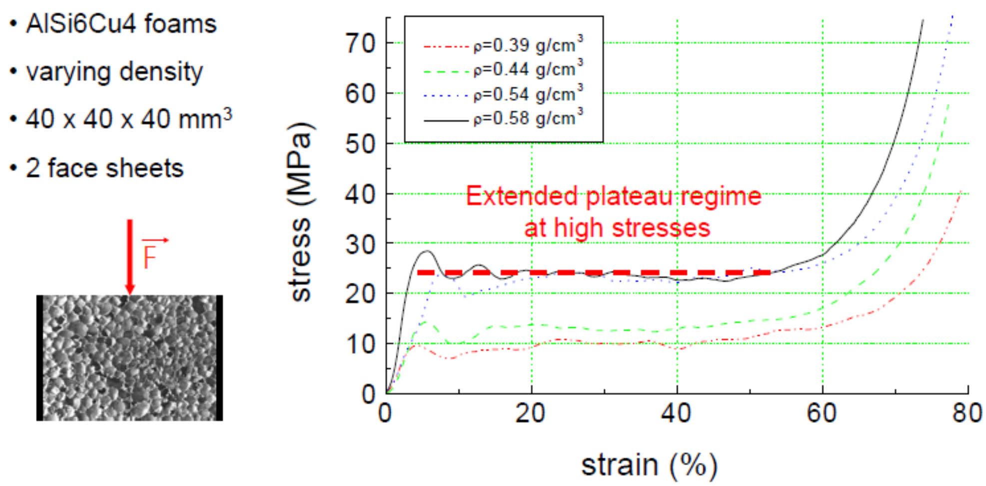


Figure 1. Typical Stress vs. Strain Response of Aluminum Foams

The goal of this DARPA project is to design materials with highest possible specific energy-dissipation characteristics. The question we pose in this investigation is whether such materials would be applicable for protecting vehicles from blast by dissipating impact energy. Several open-literature publications have reported on finite-element analysis of, and proposals for, using such materials as energy-dissipation layers under vehicles [3–6]. The objective of our analysis is to use first-principle formulation to estimate maximum possible benefits of such materials when they are used as an underbody barrier for improving vehicle blast-protection capability.

B. Approach

As is discussed in open literature [7], explosions in air and soil demonstrate that the soil is the main contributor to the total momentum imparted on the vehicle. We therefore proceed with the assumption that the underbody plate is designed to protect the vehicle from the impact by soil.

Note that we are not considering vehicle design modifications, such as increasing standoff distance between the vehicle and the ground, or various underbody shapes, such as V-hull or double-V-hull. Instead, we consider a flat sheet of material, which is installed as an underbody layer capable of dissipating the high kinetic energy of incoming soil by absorbing its impact. (Such a layer should also protect the vehicle from the breach by the impacting soil.)

We first present a solution to a simplified classic textbook two-body collision problem (with no underbody plate) with energy dissipation, which is described by a soil/vehicle restitution coefficient. Then we consider a problem of a three-body collision, in which the first collision is represented by an instantaneous momentum exchange between the soil and vehicle underbody. The subsequent collision is a completely inelastic impact between the underbody and the vehicle, so that the kinetic energy of the underbody moving relative to the vehicle/underbody center of mass system is completely dissipated by the foam.

Next, we relax the assumption of the soil/underbody collision being instantaneous and consider instead the soil to be a stream of particles moving with some average vertical velocity and impacting the underbody as the underbody acquires its own velocity due to the impact.

Finally, we account for the fact that to dissipate the kinetic energy of the relative motion of the underbody plate, the foam counteracts the motion of the underbody with a resistive force, which slows down the underbody and affects the momentum transfer to the vehicle.

To complete the analysis, we consider that the dissipation of relative kinetic energy between the underbody plate and the vehicle takes place over some distance; that is, the foam has to have appropriate thickness, which may not be negligible compared with the stroke between the vehicle and passenger seat (i.e., the distance between the seat and cabin floor). As a result, the foam effectively reduces the available vehicle stroke, which results in further decreasing the positive effect of the foam. We account for the stroke reduction in the last section of this analysis.

2. Problem Formulation

A. Nomenclature

V_v – Vehicle velocity

V_u – Underbody plate velocity

V_s – Soil velocity

m_s– Soil mass

m_u– Underbody plate mass

M_v – Vehicle mass

γ_s – Effective soil restitution coefficient

γ_p – Soil particle restitution coefficient

φ – Underbody plate-to-soil mass ratio

β – Vehicle-to-soil mass ratio

I_v– Vehicle momentum

I_s– Soil momentum

η – Momentum-transfer ratio

F_u – Force on underbody plate

ρ – Soil stream density

A – Underbody plate protection area

σ – Foam yield stress

τ – Time of soil/underbody impact

Δl – Foam thickness

d – Vehicle seat stroke

d' – Reduced vehicle seat stroke

a_c – Critical vehicle acceleration

$I_{\max}$ – Maximum allowable vehicle momentum

$I'_{\max}$ – Maximum allowable vehicle momentum with reduced stroke

a – Foam yield-strength-to-soil-stream pressure ratio

B. Formulation

To formulate the problem, consider the system shown in Figure 2. The underbody barrier consists of foam and an underbody plate, which is needed to protect the foam and the vehicle from breach by the soil. Foam is the layer between the underbody plate and vehicle compartment. It is meant to be a mechanical element capable of providing constant resistive force of appropriate magnitude over the total compression range. To absorb the soil impact, the foam compresses over a distance Δl.

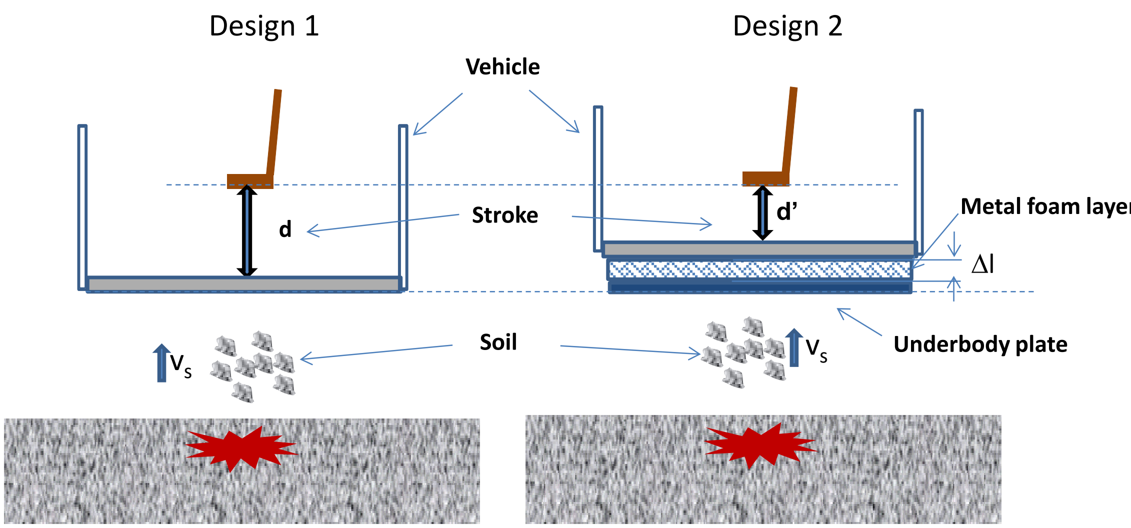


Figure 2. Schematic of the Soil Impact Problem

Also shown in Figure 2 are the cabin of appropriate mass, which is meant to represent the vehicle, and the seat, which is separated from the cabin by a stroke of magnitude d in the original vehicle and by a reduced stroke d' in the vehicle with the underbody barrier.

The problem is posed as follows. Given the magnitude of the soil impact in terms of its total momentum (i.e., specifying the mass and average vertical velocity of soil impacting the vehicle), what properties of the foam (combination of yield strength and thickness) would lead to improvement in vehicle mine-blast resistance? What is the best protection improvement possible that such foam materials could offer?

To proceed, we need to define how to evaluate and measure the vehicle blast-protection improvement. We propose to answer this question by computing the ratio of the momentum transferred from the soil to the vehicle with and without the protective underbody barrier. The smaller the ratio, the more benefit the energy-dissipation barrier provides.

C. Two-Body Instantaneous Collision

In the case of no underbody plate, after the soil instantaneously impacts the vehicle at velocity V_s, the vehicle starts moving in the direction of the soil velocity. The expression for vehicle velocity V_v includes the restitution coefficient γ_s, which describes how much energy is dissipated during the collision. When it is equal to one, the impact is perfectly elastic with no energy dissipation. When it is zero, all the kinetic energy of the relative motion of the soil (motion of the soil relative to the center of mass of the soil/vehicle system) is dissipated. In reality, γ_s varies between zero and one. The vehicle velocity can be easily determined from the conservation of linear momentum as follows:

$$V_v = (1+\gamma_s)\cdot\frac{m_s}{m_s+M_v}V_s = \frac{(1+\gamma_s)}{1+\beta}V_s\,, \tag{1}$$

where the mass ratio β is defined as

$$\beta \equiv \frac{M_v}{m_s}.$$

The momentum transferred to the vehicle as a result of the impact is therefore given by the expression

$$I_v = (1+\gamma_s)\frac{\beta}{1+\beta}I_s\,. \tag{2}$$

D. Three-Body Collision

Consider a problem of three-body collision: soil, underbody plate, and the vehicle, as shown in Figure 3a and b. Assume that the soil/underbody collision is instantaneous. If the soil bounces back after the impact (i.e., reverses its direction of motion), the underbody gains some momentum in excess of the original soil momentum, and then it proceeds to collide with the vehicle. In the underbody/vehicle impact, the restitution coefficient is zero (the foam is designed to ensure a completely inelastic collision), and the underbody/vehicle system therefore acquires the underbody momentum. As can be seen from the expressions below, for $\gamma_S \neq 0$, the vehicle momentum is less than that gained from the collision with the soil without the underbody; hence the underbody does indeed result in decreasing the momentum transferred to the vehicle. (When $\gamma_S = 0$, the momentum-transfer coefficient is one, meaning that the underbody provides no benefit in reducing the momentum transfer from soil to the vehicle.)

3a. Before Impact **Figure 3b. After Impact**

Figure 3. Three-Body Collision: Soil, Underbody Plate, and Vehicle

The velocity of the underbody after impact is

$$V_u = \frac{(1+\gamma_s)}{1+\varphi} V_s, \tag{3}$$

where the mass ratio φ is defined as

$$\varphi = \frac{m_u}{\mathrm{m_s}} . \tag{4}$$

When there is no secondary impact between the soil and the underbody,

$$I_v = I_u = m_u V_u = (1+\gamma_S)\frac{\varphi}{1+\varphi} I_s. \tag{5}$$

The momentum-transfer ratio in this case is

$$\eta = \frac{\varphi}{1+\varphi}\frac{1+\beta}{\beta}. \tag{6}$$

To account for a possible secondary underbody/vehicle impact, we find the vehicle and soil velocities after the first impact, V_v and V'_s, respectively:

$$V_v = \frac{m_u}{M_v} V_u = \frac{m_u}{M_v}\frac{(1+\gamma_s)}{1+\varphi} V_s , \tag{7}$$

$$V'_s = \frac{1-\varphi\gamma_s}{1+\varphi} V_s . \tag{8}$$

The secondary impact occurs when $V'_s > V_v$, that is, when

$$\varphi < \frac{\beta}{1+\gamma_s+\beta\gamma_s} . \tag{9}$$

When the soil impacts the plate a second time, the vehicle momentum is computed as

$$I_v = (1 + \gamma_s)\left(\frac{\beta}{1+\beta} - \gamma_s \frac{\varphi}{1+\varphi}\right) I_s. \quad (10)$$

The expressions for the momentum-transfer ratio can summarized as follows:

$$\eta = \begin{cases} \frac{\varphi}{1+\varphi}\frac{1+\beta}{\beta}, \varphi > \frac{\beta}{1+\gamma_s+\beta\gamma_s} \\ 1 - \gamma_s \frac{\varphi}{1+\varphi}\frac{1+\beta}{\beta}, \varphi < \frac{\beta}{1+\gamma_s+\beta\gamma_s} \\ \frac{1}{1+\gamma_s}, \varphi = \frac{\beta}{1+\gamma_s+\beta\gamma_s} \end{cases}. \quad (11)$$

To plot the behavior of the momentum-reduction ratio, we need to consider realistic values of the model parameters for current vehicles. According to open-literature publications [2], the velocity of soil particles a short time after explosion is on the order of 700–800 m/s. For current vehicles of about 20,000 kg, vehicle initial velocity after experiencing underbody mine explosions is on the order of 6–8 m/s [3]. Therefore, we estimate the mass of soil impacting the vehicle to be on the order of 200 kg. The underbody plate has to protect the vehicle from breach. Assuming the needed coverage area under the vehicle is at least 2 × 3 m, and the underbody plate is made of steel sheet with thickness of 4 to 5 cm, the mass of such a plate should be at least 2000 kg. Hence the underbody-plate-to-soil mass ratio parameter is on the order of 10 and above. The expressions (11) are plotted in Figure 4 for M_v = 20,000 kg, m_s = 200 kg, and selected values of γ_s.

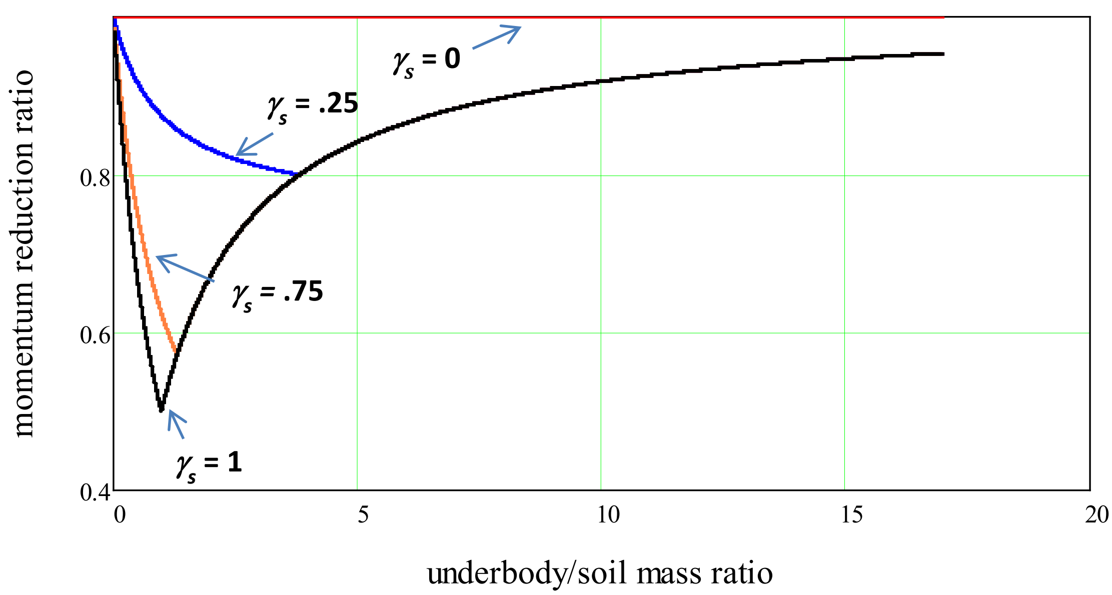


Figure 4. Effect of Underbody/Soil Mass Ratio (φ) on Vehicle Momentum-Transfer Ratio

As can be seen from Figure 4, when there is a secondary soil impact with the underbody, the momentum-transfer ratio depends on the restitution coefficient between the soil and the underbody, the mass of the soil and the vehicle, and the underbody-plate-to-soil-mass ratio φ. When there is no secondary soil impact, the momentum-transfer ratio only depends on φ. When the underbody-to-soil-mass ratio is very large, the underbody is no different to the soil than the vehicle itself, and no gain is achieved as a result of such a design. When the restitution coefficient between the soil and the underbody is zero, again, no momentum reduction is achieved, since the vehicle gains the original soil momentum in both cases. The most momentum reduction is obtained when the mass ratio $\varphi = \frac{\beta}{1+\gamma_S+\beta\gamma_S} \approx \frac{1}{\gamma_S}$. The larger the soil/underbody restitution coefficient (the more elastic is the collision between the soil and underbody), the more momentum reduction is achieved. When the impact is perfectly elastic (i.e., γ_S= 1), the momentum-reduction ratio is 50%, which is the maximum that can be achieved.

Note that for realistic scenarios of soil-to-mass ratios of 10 and above, the momentum-reduction ratio is on the order of 0.9, irrespective of all other parameters.

E. Continuous Soil/Underbody Collision Model

1. No Resistive Force

In this section we account for the fact that the collision is not instantaneous, but is extended in time, as it is accomplished by a stream of soil particles continuously impacting the underbody plate over some finite time interval, during which the underbody plate gains some momentum. To estimate the momentum exchange during such a collision, we propose the following model, schematically shown in Figure 5.

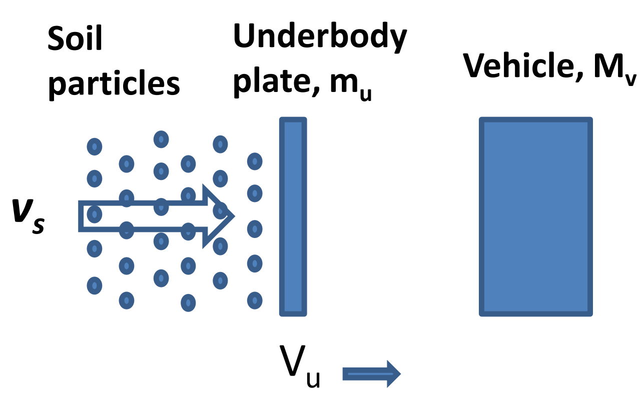


Figure 5. Continuous Soil/Underbody Plate Impact Model

The soil particles are moving with average velocity V_s in a direction perpendicular to the underbody plate. The restitution coefficient describing energy dissipation during collision of individual soil particles with the underbody plate is γ_p. Note that there is no reason for this restitution coefficient to be the same as the one we utilized earlier in the classical two-body collision model (γ_s), where the lump of soil impacted the underbody instantaneously. (We show how the two are related in the next section.) We summarize our model assumptions as follows:

- The soil moves with some average velocity that is uniform in space and constant in time.
- The soil density is constant.
- The soil that initially impacts the underbody does not interfere with subsequent soil particle impact.
- The soil impact area is approximately equal to the underbody plate area.

Linear momentum imparted by a soil mass element Δm_s on the underbody plate in time *dt* is

$$\Delta I_u = \Delta m_s \cdot \frac{(1+\gamma_p)(V_s - V_u)}{\Delta m_s + m_u} \cdot m_u. \tag{12}$$

The force on the underbody, therefore, can be found as

$$F_u = \lim_{\Delta t \to 0} \left(\frac{\Delta m_s \cdot \frac{(1+\gamma_p)(V_s - V_u)}{\Delta m_s + m_u} \cdot m_u}{\Delta t} \right), \tag{13}$$

where ρ is the density of the incoming soil stream and

$$\Delta m_s = \rho A (V_s - V_u) \Delta t. \tag{14}$$

The expression for the underbody force can be rewritten as the differential equation of motion of the underbody plate:

$$\dot{V}_u = \frac{(1+\gamma_p)}{m_u} \rho A (V_s - V_u)^2. \tag{15}$$

The solution is then obtained by simple integration with the initial condition $V_u = 0$ at $t = 0$:

$$V_u(t) = V_s \frac{(1+\gamma_p)t}{(1+\gamma_p)t + \phi\tau}, \tag{16}$$

where we denote by τ the duration of the loading of the underbody by the soil stream and use the identity

$$\tau = \frac{m_s}{\rho A V_s}. \qquad (17)$$

2. Effective Soil Restitution Coefficient

As we mentioned in the beginning of this section, γ_p is the restitution coefficient describing the impact of individual soil particles with the underbody, while γ_s is some effective soil restitution coefficient describing instantaneous impact of the lump mass of soil with the underbody (or vehicle). We show that the two are related as follows.

For instantaneous impact of soil with the underbody:

$$V_u = \frac{(1+\gamma_s) m_s V_s}{m_u + m_s} = \frac{(1+\gamma_s)}{1+\varphi} V_s. \qquad (18)$$

For continuous impact of soil with the underbody:

$$V_u(t) = V_s \frac{(1+\gamma_p)t}{(1+\gamma_p)t + \phi\tau}. \qquad (19)$$

Equating these two expressions at $t = \tau$ allow us to relate γ_s and γ_p as follows:

$$\gamma_s = \frac{\phi \gamma_p}{1+\gamma_p+\varphi}. \qquad (20)$$

We plot this relationship in Figure 6. For the underbody-to-soil-mass ratio of 10 and below, there is a relatively large difference between γ_p and γ_s, especially for more elastic impacts, when γ_p is close to 1. As the mass ratio φ increases, the difference between the two restitution coefficients decreases. For the mass ratio of 100, there is practically no difference between γ_p and γ_s for impacts when the restitution coefficient is between .25 and .75 (i.e., in realistic collisions). When soil impacts the vehicle (the case of no underbody energy-dissipation barrier), the mass ratio is on the order of 100 and above, and the two restitution coefficients are therefore practically identical. We will make use of this fact later in the paper to estimate the ratio of the momentum imparted by the soil on the vehicle with and without the energy-dissipation barrier.

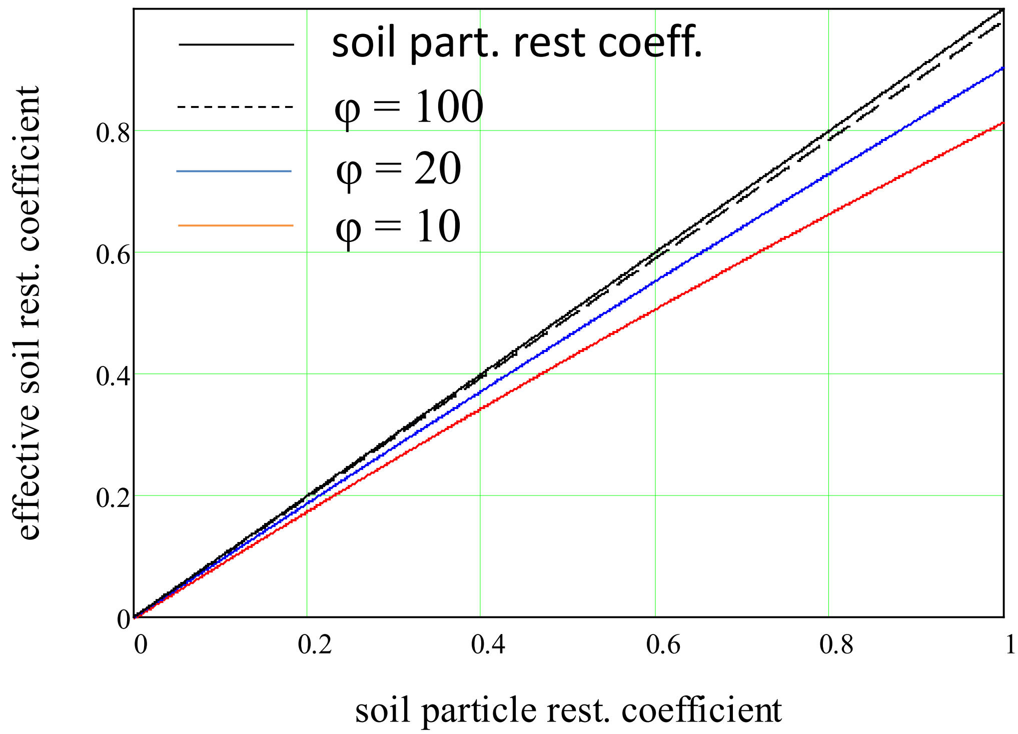


Figure 6. Relationship between Soil Particle and Effective Restitution Coefficient

3. Inclusion of the Resistive Foam Force

Finally, we account for the fact that to dissipate the kinetic energy of the underbody moving toward the vehicle, a resistive force has to be applied to the underbody to decelerate its motion and eventually equalize its velocity with that of the vehicle. We assume this force is provided by a foam layer positioned between the underbody and the vehicle; the foam behaves as a damper with constant resistive force (provided by the underbody yield stress), which is constant over the distance traveled by the underbody. The model in this case is exactly the same as the soil stream loading model described previously, but there is a constant force term added to the right side of the underbody equation of motion to account for the foam yield stress.

We return to the problem in which the underbody plate is acted upon by the impacting soil and the foam with constant yield stress, as shown in Figure 7.

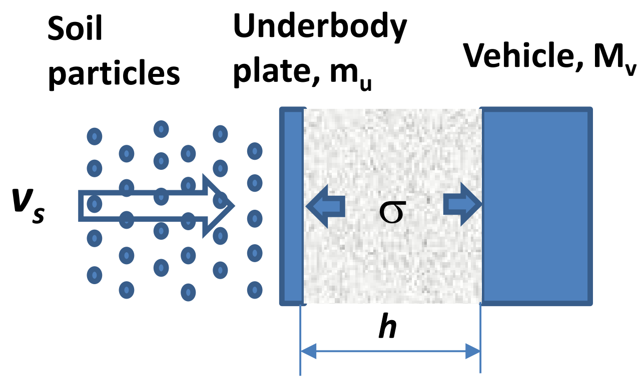


Figure 7. Continuous Soil/Underbody Plate Impact Model with Resistive Force

The force on the vehicle underbody in this case can be expressed as follows:

$$F_u = \frac{d}{dt}\left[\left\{\Delta m_s \cdot \frac{(1+\gamma_p)(V_s - V_u)}{\Delta m_s + m_u}\right\} \cdot m_u\right] - \sigma A. \tag{21}$$

The equation of motion in this case becomes

$$\dot{V}_u = \frac{(1+\gamma_p)}{m_u}\rho A (V_s - V_u)^2 - \sigma A/m_u \,. \tag{22}$$

The solution to this equation can be readily obtained as follows:

$$V_u = V_s\left(1 + \sqrt{a}\,\frac{\frac{1-\sqrt{a}}{1+\sqrt{a}}e^{\frac{-2\sqrt{a}(1+\gamma_p)}{\varphi\tau}t} + 1}{\frac{1-\sqrt{a}}{1+\sqrt{a}}e^{\frac{-2\sqrt{a}(1+\gamma_p)}{\varphi\tau}t} - 1}\right), \tag{23}$$

where a is the ratio of the foam yield stress to soil stream pressure:

$$a = \frac{\sigma}{(1+\gamma_p)\rho {V_s}^2} = \frac{\sigma A \tau}{(1+\gamma_p) m_s V_s}. \tag{24}$$

Equation (23) can be simplified by expanding the exponential terms in Taylor series and retaining two first terms, resulting in the expression below

$$V_u(t) = V_s(1-a)\frac{(1+\gamma_p)t}{(1+\gamma_p)t + \phi\tau}\,. \tag{25}$$

The exact and approximate solutions, (23) and (25), are compared in Figure 8. They are practically identical when φ is larger than 10, which the region of our interest. The simplified expression readily reveals the effect of the foam yield stress-to-soil stream pressure ratio a; when $a = 0$ the expression reduces the solution for the case of no foam, equation (16). As the yield stress increases, the underbody plate velocity decreases. In the subsequent analysis, we will utilize the approximate solution.

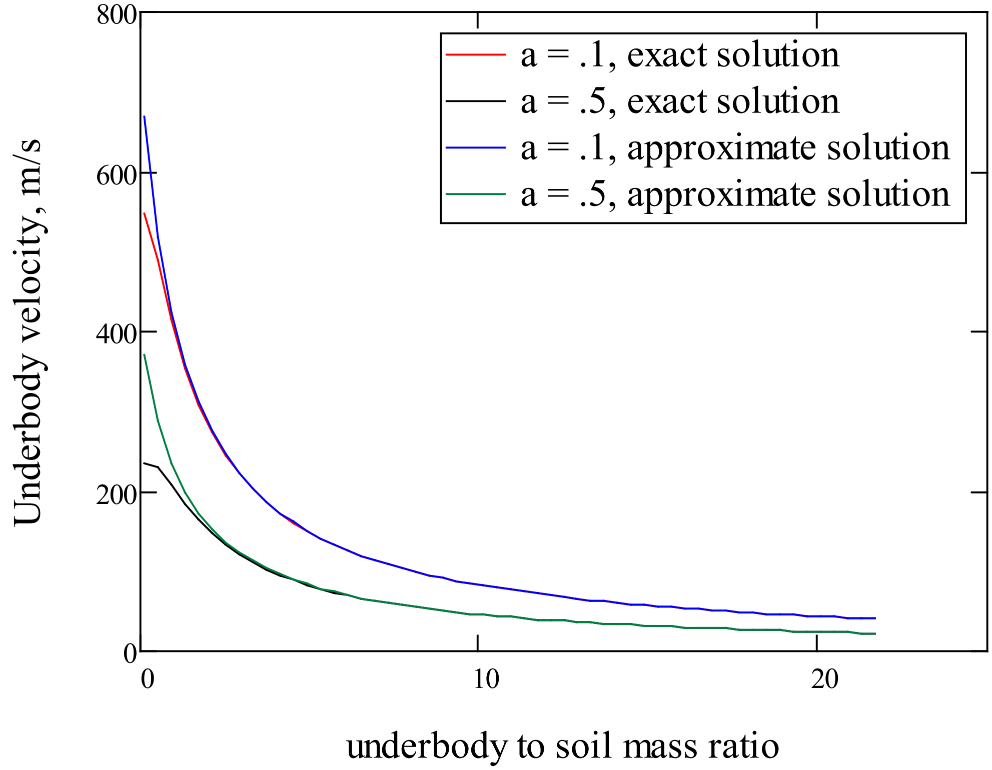


Figure 8. Comparison of the Exact and Approximate Underbody Velocity Solutions

Final momentum imparted to the vehicle at $t = \tau$ with the underbody plate and resistive foam is

$$I_v = I_u + \sigma A \tau = m_u V_s (1 - a) \frac{1+\gamma_p}{1+\gamma_p+\phi} + \sigma A \tau. \tag{26}$$

Recall the expression for momentum imparted to the vehicle without the bottom plate:

$$I_v \approx (1 + \gamma_s) m_s V_s = I_s (1 + \gamma_s). \tag{27}$$

As we have shown earlier, the effective soil and soil particle restitution coefficients are nearly the same for the mass ratio of the vehicle to the soil that we are considering here. Therefore, we simply denote the restitution coefficient by γ and express the momentum-transfer ratio as shown by the expression below. Since we do not know exactly the value of γ, we use it as a parameter that varies in some realistic range (0.25 to 0.75):

$$\eta = \frac{\varphi(1-a)}{1+\gamma+\varphi} + a. \tag{28}$$

Note that this expression explicitly tells us how the back pressure from the foam on the underbody affects the momentum transfer. This relationship is shown in Figure 9.

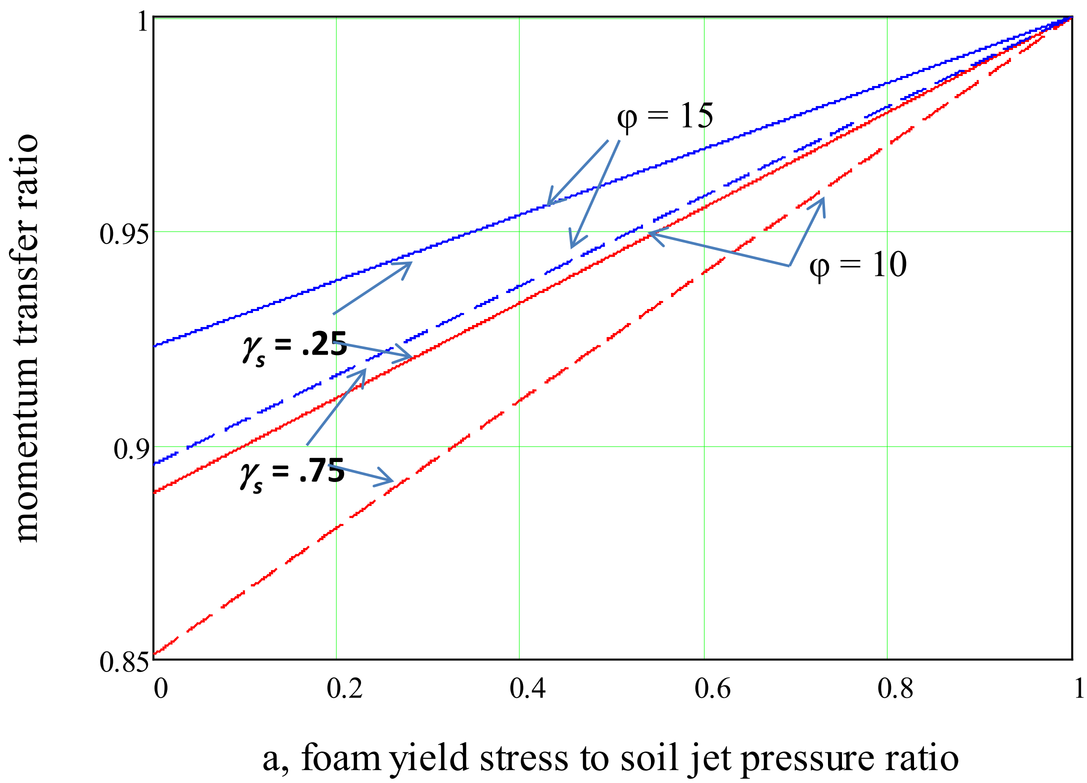


Figure 9. Effect of Resistive Force on Momentum Transfer

We already considered the case a = 0 (no foam) in the previous section. As a increases, the resisting force on the underbody increases, which is similar to the effect of increasing its mass, and the momentum transfer to the vehicle increases. The case of a = 1 is the limiting one when the underbody is no longer moving during the impact, effectively removing the underbody from the system and reverting to the original case when no underbody was present. All the momentum in this case is transferred to the vehicle (i.e., the transfer coefficient is one).

Shown in Figure 10 are the plots of the momentum-transfer ratio as a function of the underbody/soil mass ratio and restitution coefficient. As we have discussed previously, the realistic values of φ are in the range of 10 to 15. For these values, the momentum-transfer ratio is somewhere in the range of 0.85 to 0.98. In addition, it is clear from the figure that accounting for the foam resistive force leads to reducing the beneficial effect of the foam (i.e., the larger the yield stress, the less momentum reduction achieved). Note, however, that so far we have not accounted for the loss of stroke necessary to accomplish the energy dissipation in the foam. As the yield strength increases, less distance is necessary to dissipate the same energy, leading to a smaller stroke penalty. We account for this effect next.

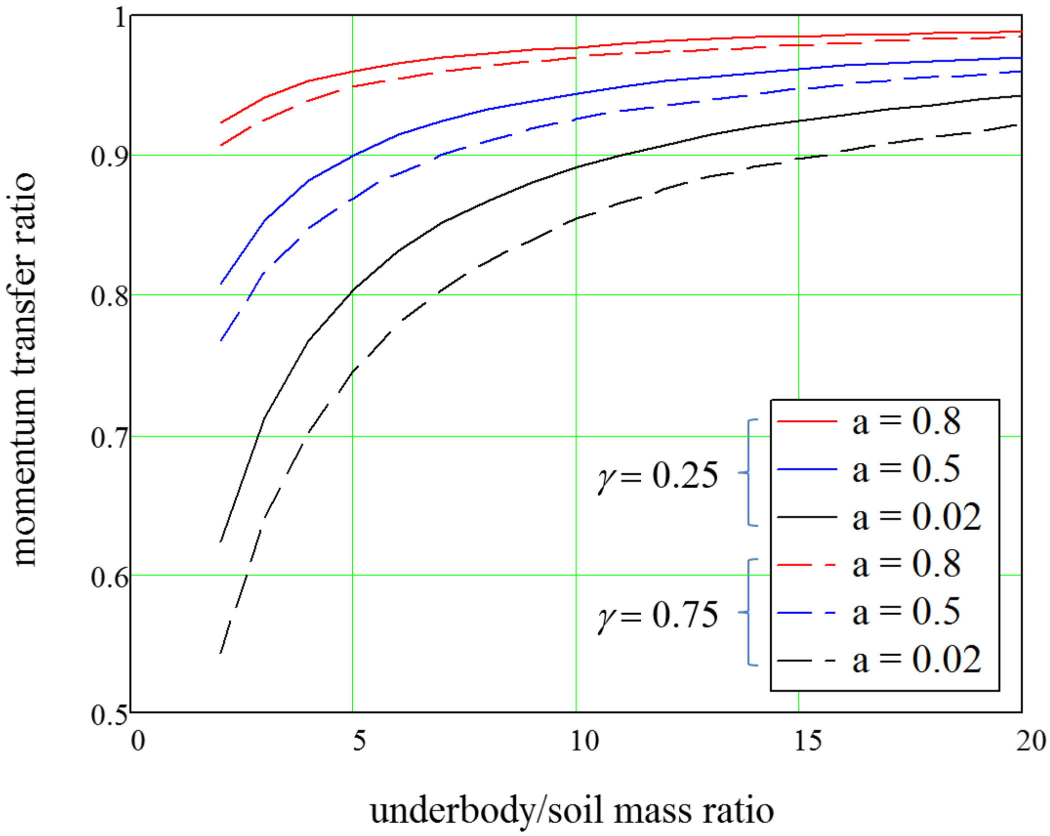


Figure 10. Effect of Underbody-to-Soil-Mass Ratio on Momentum Transfer

F. Estimation of Stroke-Reduction Penalty

The amount of stroke needed to dissipate the relative kinetic energy of the underbody is computed as follows. Consider the length over which the energy is dissipated (foam thickness Δl), which is no longer available for damping between the vehicle and the seat, thus leading to the reduction of the maximum allowable momentum that can be safely applied to the vehicle. In this case there is no initial gap between the underbody and foam, and the resistive force on the underbody is applied immediately after the soil impact. (The case when there is an initial gap between the underbody and foam is less advantageous, as we show in the appendix A.) It is easy to show that

$$\Delta l = \frac{\frac{1}{2} m_u {V_u}^2}{\sigma \cdot A} \left(\frac{\beta}{\phi + \beta} \right). \tag{29}$$

In reality, the foam compresses to a final length that should be added to Δl to compute the total stroke penalty. Since we are interested in the upper bound estimate, however, we neglect that part of the stroke reduction for the sake of simplicity. As the foam yield stress increases (i.e., a increases), the required foam thickness decreases; they are, in fact, almost inversely proportional (except for V_u being weakly affected by σ). We show the relationship between them in Figure 11 for the underbody-to-soil mass ratio of 10.

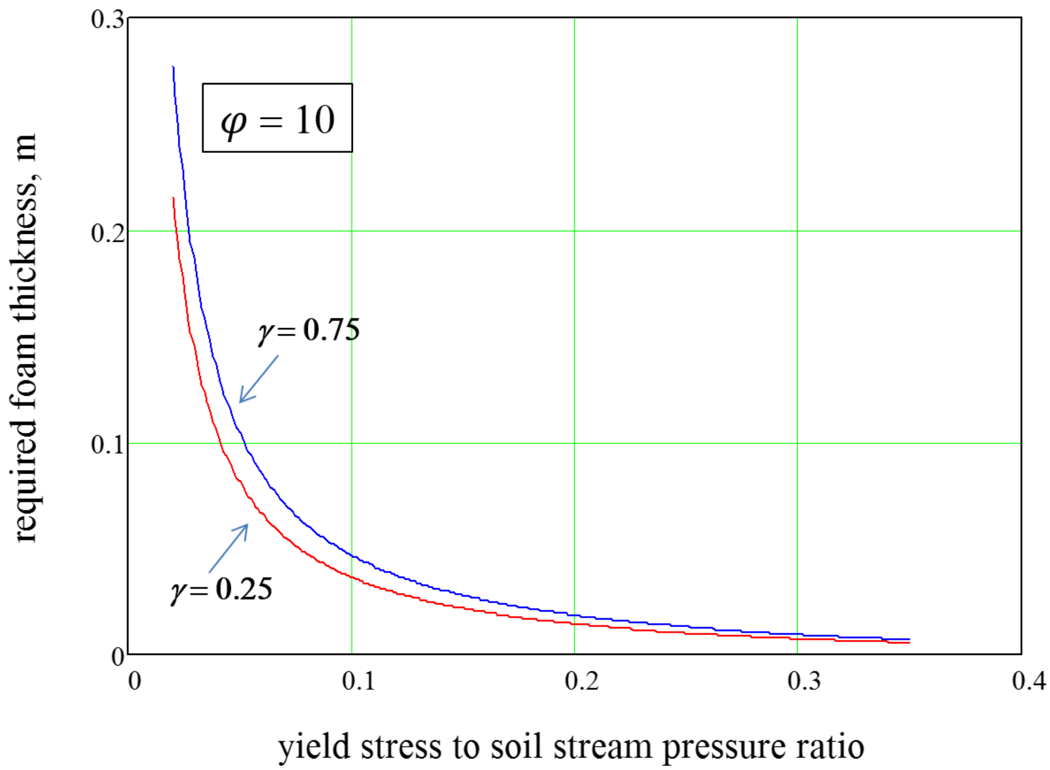


Figure 11. Relationship between Required Foam Thickness and Yield Stress to Soil Stream Pressure Ratio

For values of a below 0.1, the required foam thickness to dissipate the underbody kinetic energy begins to increase above 0.05 m. Considering a stroke of 0.25 m to be a typical value for today's vehicles, this represents a stroke penalty of 20% and above. From the plot shown in Figure 10, we see that increasing a leads to an increase of the momentum transfer to the vehicle. These results suggest that there is an optimum yield stress (and a corresponding value of Δl) for minimizing the momentum-transfer ratio. We determine the optimum value of the foam yield stress and parameter a from the following considerations.

The original maximum allowable momentum (with no underbody) that can be safely applied to the vehicle is

$$I_{\max} = M_v \sqrt{2 a_c d}, \tag{30}$$

where d is the original vehicle stroke. After the stroke is reduced by Δl, the new maximum safe momentum is lower:

$$I'_{\max} = M_v\sqrt{2a_c(d - \Delta l)}, \tag{31}$$

where $\Delta l < d$.

Substituting velocity of the underbody after impact in the expression (29) and utilizing the definition of parameter *a*, we derive the expression for the required length to dissipate the energy of the underbody:

$$\Delta l = \frac{1}{2}\varphi \frac{v_s\tau(1+\gamma)\beta}{(1+\gamma+\varphi)^2(\beta+\phi)}\frac{(1-a)^2}{a}. \tag{32}$$

Although reducing the stroke reduces the allowable (not actual) momentum, for consistency, we treat the stroke reduction as an equivalent change in applied momentum. The momentum-reduction ratio is computed by:

$$\eta = \left[\frac{\varphi(1-a)}{1+\gamma+\varphi} + a\right]\sqrt{\frac{d}{d-\Delta l}}. \tag{33}$$

To determine the optimum design parameters of the foam, namely yield stress and required thickness, we plot the momentum-transfer ratio η as a function of a, shown in Figure 12. We use d = .25 m to represent a typical value of the vehicle stroke. As we show in Appendix A, a reasonable estimate for the time of impact is 100 to 150 microseconds; we take τ = 125 microseconds in our analysis. We note here that η can be differentiated with respect to σ (keeping γ and φ constant), since both a and Δl are explicit functions of σ. Setting the derivative to zero and solving for σ would then give us the optimum value of σ. We prefer to show a graphical solution, however, since it gives a better understanding of the result. We observe that η is a weak function of a when it is larger than about 0.2. The optimum value of a is somewhere between 0.25 and 0.3, depending on the soil-restitution coefficient and underbody-to-soil-mass ratio φ. We also note that under the most favorable conditions (i.e., soils with a high restitution coefficient of 0.75 and relatively light underbody with φ = 10), the minimum momentum-transfer ratio is slightly above 90%.

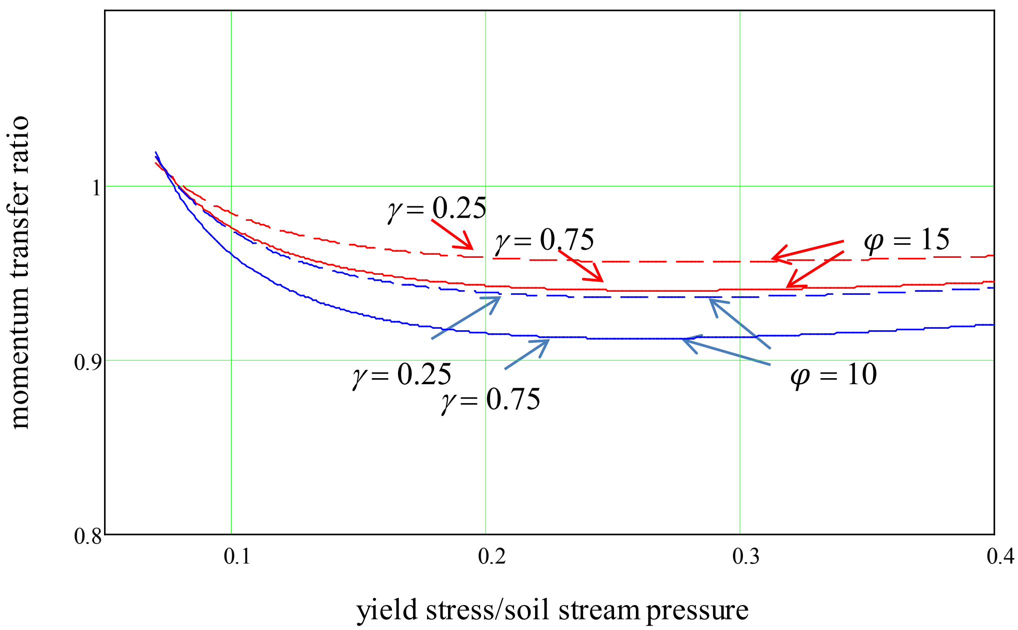


Figure 12. Region of Optimum Values of Foam-Yield-Stress-to-Soil-Jet-Pressure Ratio

We choose a = 0.3 and show how the momentum-transfer ratio depends on the underbody-to-soil-mass ratio for such foams in Figure 13. (We anticipate that the protection from the vehicle breach may require a thicker underbody panel and values of φ larger than 10.)

Similarly to our previous results, as the underbody-to-soil-mass ratio φ increases, the beneficial effect of the energy-dissipation barrier decreases. For values of φ between 10 and 15, the momentum-transfer ratio from the soil to the vehicle is about 92% to 95%, which can be restated in terms of improvement in vehicle blast resistance as 5% to 8%. In addition, shown in Figure 13 is the plot of momentum transfer as a function of φ for the case of two-body instantaneous impact. Note that the continuous-impact analysis accounts for the stroke-reduction penalty and values of soil-restitution coefficient. In addition, the model allows us to estimate the optimum design parameters of the foam, that is, yield-stress-to-soil-stream-pressure ratio and associated required foam thickness. Otherwise, for all practical purposes, the results of the two analyses are similar.

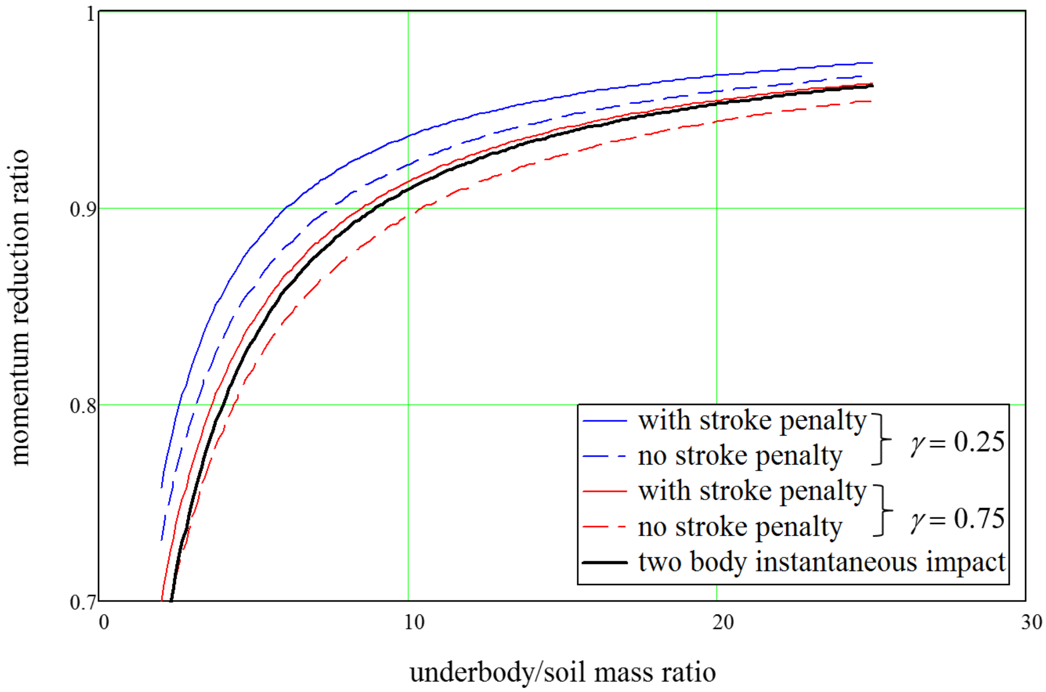


Figure 13. Effect of Underbody-to-Soil-Mass Ratio on Momentum Transfer with Energy-Dissipation Stroke Penalty

3. Conclusions

In this investigation we analyzed the effect of placing an energy-dissipation barrier under the vehicle to improve its blast resistance. We developed a simplified model to estimate the momentum transfer from soil to the vehicle during the impact of the fast moving soil with the underbody. The model considers a continuous collision of soil with the underbody; it accounts for the foam yield stress resisting the underbody motion and the stroke reduction that results because the foam compression distance is comparable with the original vehicle stroke and therefore not negligible.

Our results indicate that in terms of reduction of momentum transferred to the vehicle from the fast moving soil, an upper bound on improvement of the vehicle blast resistance due to placing an energy-dissipative underbody barrier is on the order of 5% to 8%.

Appendix A
Gap vs. No-Gap Design

In the main body of our paper we considered the case when there is no gap between the foam and the underbody, so that the foam starts resisting the underbody motion immediately, increasing the effective restitution coefficient (i.e., making the impact more elastic). That means that the momentum-transfer ratio is also increased.

Another case that has to be considered is when there is a gap between the underbody and foam, such that there is no resistive force acting on the underbody while the soil stream is loading it. In this case, the restitution coefficient is lower than in the previous case, and the momentum transfer is reduced. On the other hand, this case requires larger stroke to dissipate the energy, because of the initial gap as well as higher underbody velocity after its impact with the soil. Which design is more advantageous?

To answer this question, we simply compute the momentum reduction in both cases and compare the results.

When there is an initial gap, the underbody acquires its velocity V_u after impact with the soil stream (while there is no foam yield stress acting on it). The stroke penalty is then a sum of two terms:

$$\Delta l = \int_0^\tau V_u(t)dt + \frac{\frac{1}{2}m_u V_u(\tau)^2}{\sigma \cdot A}\frac{\beta}{\phi+\beta}\,. \tag{A1}$$

The underbody velocity V_u is computed by the expression (16) in the main paper, and the integral in the expression (A1) can be easily evaluated as follows:

$$\int_0^\tau V_u(t)dt = V_s\tau\left(1 + \frac{\varphi}{(1+\gamma)}\ln\left[\frac{\varphi}{1+\gamma+\varphi}\right]\right). \tag{A2}$$

The momentum reduction is then computed by the expression below:

$$\eta = \left[\frac{\varphi}{1+\gamma+\varphi}\right]\sqrt{\frac{d}{d-\Delta l}}\,, \tag{A3}$$

where Δl is given by the expression (A1).

To calculate the stroke-reduction penalty, we need to estimate the time of loading. We assume that after the explosion all the soil particles travel with approximately the same velocity of 800 m/s; we estimate that the distance of soil travel during the impact is at least the overburden dimension of the soil above the mine, that is, about 0.1 m. The impact duration is then on the order of 100–150 microseconds. (This duration corresponds to the soil stream density of 300 to 400 kg/m^3.)

We plot the expression (A3) for $\tau = 125$ microseconds with Δl given by (A1) in Figure A-1. In addition, we plot the momentum reduction for the case when the there is no gap, given by the expression (33), and Δl given by expression (32) in the main paper. To compare the results, we use the value of foam yield stress in the expression (A1) to be 100 Mpa, which corresponds to approximately an average value of foam yield stress when a = 0.3 and the restitution coefficient varying between 0.25 and 0.75.

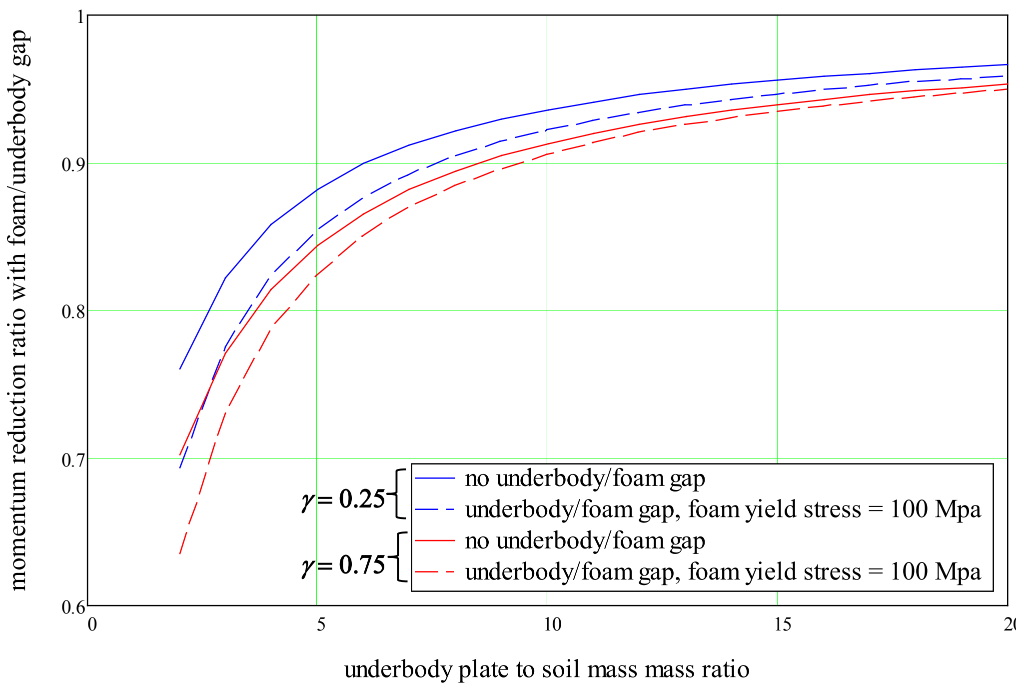


Figure A-1. Comparison of the Momentum-Reduction Ratio for the Underbody/Foam "Gap vs. No Gap" Designs. No Gap design: underbody-to-soil-mass ratio, *a* = 0.3; gap design: σ = 100 Mpa.

As can be seen from Figure A-1, the difference between momentum reductions in the region $\varphi > 10$ is not significant. The most improvement with the gap design is only on the order of at most few percent. Given that the required gap distance depends on the underbody travel during its loading by the soil, this distance is affected by many parameters, such as soil overburden and density, standoff distance, etc. Considering the complexity of such a design with the resulting only (very) minor benefits leads us to conclude that the more robust no-gap design is preferable.

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14. ABSTRACT

We analyzed the effect of placing an energy-dissipation barrier under a vehicle to improve its blast resistance. We developed a simplified first-principle model to estimate the momentum transfer from soil to the vehicle during the impact of the fast moving soil with the underbody. The model considers a continuous collision of soil stream with the underbody; it accounts for the foam yield stress resisting the underbody motion and the stroke-reduction penalty that results from the finite foam compression distance. Our results indicate that in terms of reduction of momentum transferred to the vehicle from the fast moving soil, an upper bound on improvement of the vehicle blast resistance due to placing an energy-dissipative underbody barrier is on the order of 5% to 8%.

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