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Comparison of Constant Acceleration and Dynamic Response Index Lethality Criteria

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Executive Summary

Background

There may be more accurate human lethality criterion than a fixed acceleration threshold (i.e., 23 g). The human body is a dynamic system with rate-dependent material properties and natural frequencies. Since injuries are caused by the internal stresses and strains produced by external forces, a more accurate prediction of injury should account for the body's dynamic response.

The simplest model accommodating internal body dynamics is the Dynamic Response Index (DRI), which accounts for spinal-compression forces by treating the spine as a Kelvin element (a linear spring and dashpot in parallel) connecting monolithic upper and lower body masses. Here we ask how the results of the fixed-acceleration threshold analysis relate to more complex criteria. In other words, if a system exhibits high efficiency for the 23 g criterion, how does is perform with respect to the DRI as an exemplar of dynamically sensitive criteria?

Conclusions

Choice of lethality criterion can affect the level of impulse deemed survivable. Designs at the threshold of lethality with respect to constant-acceleration threshold criterion may be lethal under dynamic lethality criteria like the DRI. Designs optimized for dynamic lethality criteria may demonstrate acceleration efficiencies higher than 1, since the point of comparison is the optimal performance under the constant-acceleration threshold criterion. The DRI approaches equivalence with the fixed-acceleration threshold criterion for impulses delivered over a timescale that is long compared with the body's natural period of oscillation.

Due to the ease of application, it may remain worthwhile to go through initial cycles of design utilizing transparent criteria like the fixed-acceleration threshold, but evaluation against dynamic criteria may be important to ensuring safety.

Contents

A. Introduction

In our previous work, we used the simplest lethality criterion, a fixed acceleration threshold (23 g).[1] This criterion has a substantial pedigree (e.g., helicopter safety work, ejection seats, etc.) and is most amenable to analysis and optimization. The 23 g standard was implemented in MIL-S-85510 as a specification for crashworthy helicopter seats.[2] The U.S. Army has acknowledged the importance of acceleration control in injury prevention, established appropriate standards, and even designed seats accomplishing much of what is desired. The 23 g criterion has experimental backing and a history of acceptably protecting occupants.[3]

There may be more accurate criteria than a simple acceleration threshold, however. The human body is a dynamic system with rate-dependent material properties and natural frequencies. Since injuries are caused by the internal stresses and strains produced by external forces, a more accurate prediction of injury might account for the body's dynamic response. To accommodate the dynamic element, various approaches have been devised, extending to complex, multicomponent finite-element analyses with detailed anatomical and material properties. On the other end of the spectrum, the simplest model accommodating internal body dynamics is the Dynamic Response Index (DRI), which assesses spinal-compression forces by treating the spine as a Kelvin element (a linear spring and dashpot in parallel) connecting monolithic upper and lower body masses. The DRI criterion is currently used by the Army Research Laboratory (ARL) to evaluate injury regimes of military vehicles during explosions.[4] The DRI approaches equivalence with the fixed-acceleration threshold criterion for impulses delivered over a timescale that is long compared with the body's natural period of oscillation. For impulses delivered rapidly, only the total impulse and not its time phasing will affect the injury outcome. Alternative simplified criteria extend the acceleration threshold by adding an approximately 7 ms minimum time window and neglecting any higher acceleration spikes of lower duration.

Defense and comparison of particular criteria are active topics of debate in the injury prevention and analysis community and beyond the present scope. In this brief paper, we

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[1] J. Teichman and J. Macheret, "Vehicle Blast Protection Efficiency Analysis and Evaluation," IDA Document D-8170 (Alexandria, VA: Institute for Defense Analyses, forthcoming).

[2] MIL-S-85510, "Military Specification: Seats, Helicopter Cabin, Crashworthy, General Specification for (19 November 1981) [S/S By JSSG-2010-7].

[3] J. Glatz, "An Analysis Resulting from the U.S. Army Study titled: H-60 Assessment of the Next Generation Troop Seat (NGTS)," Sponsored by the Assistant Secretary of the Army for Installations, Energy and Environment (ASA(IE&E)] and Deputy Assistant Secretary of the Army for Environment, Safety and Occupational Health (DASA-ESOH), May 24, 2016.

[4] R. Spink, "Injury Criteria for the Analysis of Soldier Survivability in Accelerative Events," ARL-TR-6121, 2012.

merely ask how the results of the fixed-acceleration threshold analysis relate to more complex criteria. In other words, if a system exhibits high efficiency for the 23 g criterion, how does is perform with respect to the DRI as an exemplar of dynamically sensitive criteria? Or alternatively, how would a system optimized for the DRI perform given our efficiency criteria? Note that the only relevant efficiency to compare is the acceleration efficiency. The stroke efficiency and momentum-reduction factors operate independently of the lethality criterion.[5]

Artur Iluk[6] claimed that for the DRI model (dynamic spine force injury), the optimal acceleration profile is not a constant acceleration, but rather a sudden impulse to accelerate the pelvis rapidly toward the shoulders just strongly enough to compress the spine to its maximum safe level followed by a constant acceleration to hold it there. Let us use the representative parameters given by Balandin et al. for the human body with respect to the DRI.[7] Figure 1 shows a schematic of the DRI model and the associated parameters and values.

5 Teichman and Macheret, "Vehicle Blast Protection Efficiency Analysis and Evaluation."

6 Artur Iluk, "Selected Aspects of the Control of the Human Body Motion in the Vehicle Subjected to the Blast Load," IRCOBI Conference Proceedings, No. IRC-12-48, 2012.

7 D. V. Balandin and N. N. Bolotnik, "Optimal Shock Isolation of a Two Component Viscoelastic Object," *J. Comput. Syst. Sci. Int.* 48(2006): 206.

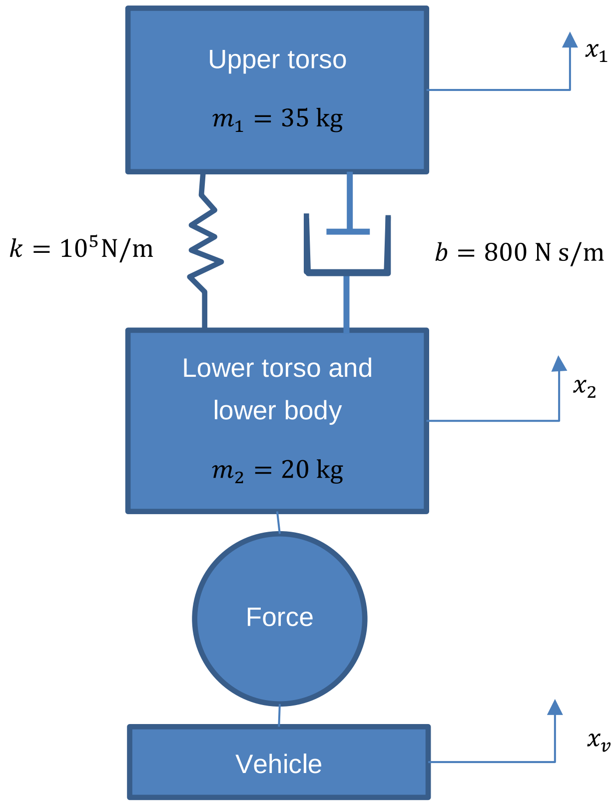

Figure 1. DRI Model Schematic

In the DRI construct, injury is determined by the spring component of the spinal force between the upper torso and the rest of the body. DRI is defined as the spring component of this force relative to the static load under gravity,

$$DRI = \frac{k\xi}{m_1 g},$$

where $\xi = x_2 - x_1$, the stroke is $d = x_v - x_2$, $\dot{x}_v(t) = \Delta v$ for $t > 0$, and F is the force produce by the damping actuator under the lower body. Typically, serious injury is accepted to be $DRI > 22.8$, which we take as approximate 23 to retain parity with the fixed-threshold acceleration criterion. In this case, at steady state (i.e., in the state of dynamic equilibrium), 23 g of acceleration produces a DRI of 23. A DRI of 23 corresponds to $\xi = 7.9$ cm.

B. Fixed Acceleration Threshold Evaluated Against DRI

Is the 23 g criterion ever lethal under the DRI criterion? The most stressing case is the constant application of 23 g of bodily acceleration over a long period of time. In this case, with a net body mass of 55 kg, 23 g of bodily acceleration is produced by $F_0 =$

12.4 kN of force. The internal dynamics of the body under the DRI model are described by

$$M\ddot{\xi} = -k\xi \; - b\dot{\xi} + \frac{m_1}{m_1 + m_2} F_0$$

where

$$M = \frac{m_1 m_2}{m_1 + m_2}$$

subject to initial conditions

$$\xi(0) = 0,$$

$$\dot{\xi}(0) = 0.$$

For the body parameters of Figure 1, the body is underdamped, and the solution to the governing differential equation is given by

$$\xi(t) = \frac{m_1}{m_1 + m_2} \frac{F_0}{k} \left[e^{\omega_0 t} \left(\frac{\omega_0}{\omega_1} \sin \omega_1 t - \cos \omega_1 t \right) + 1 \right],$$

where

$$\omega_0 = -\frac{b}{2M}$$

and

$$\omega_1 = -\omega_0 \sqrt{\frac{4kM}{b^2} - 1}\,.$$

Maximum ξ is attained when $\dot{\xi} = 0$, for which

$$t = \frac{\pi}{\omega_1}$$

and

$$\xi_{\max} = \frac{m_1}{m_1 + m_2} \frac{F_0}{k} \left[e^{\frac{\omega_0}{\omega_1}\pi} + 1 \right].$$

Computing the DRI,

$$DRI = \frac{k\xi_{\max}}{m_1 g} = \frac{a_c}{g} \left[e^{\frac{\omega_0}{\omega_1}\pi} + 1 \right],$$

We see that according to the DRI criterion, where $\frac{a_c}{g}$ is the lethal threshold, a constant-acceleration seat damper can be lethal. At the given values of the parameters, DRI will eventually exceed the lethal threshold by 30% $\left(e^{\frac{\omega_0}{\omega_1}\pi} + 1 = 1.3 \right)$ if the constant acceleration is continued for long enough. Spine compression over time for constant acceleration of a_c is shown in Figure 2. The red line indicates spinal compression corresponding to a DRI of 23.

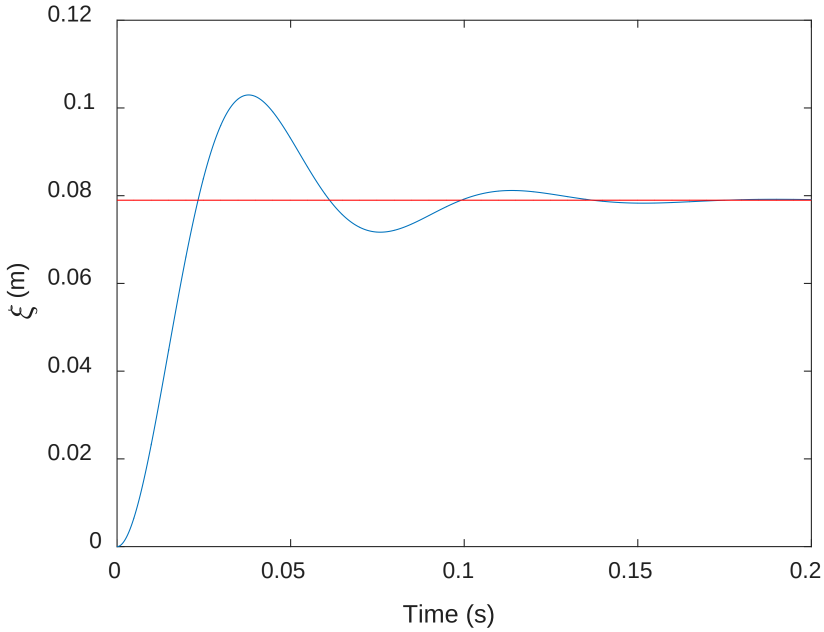

Figure 2. Spinal Compression under Constant Acceleration

Using the above expression for the DRI allows us to determine the maximum constant acceleration that would lead to the critical DRI = 23; it is simply 23 g/1.3, which is about 17.7 g.

How long can a constant acceleration of 23 g be applied to the passenger without causing DRI to rise above 23? While the force is being applied to the pelvis,

$$\xi_1(t) = \frac{m_1}{m_1 + m_2}\frac{F_0}{k}\left[e^{\omega_0 t}\left(\frac{\omega_0}{\omega_1}\sin\omega_1 t - \cos\omega_1 t\right) + 1\right],$$

$$\dot{\xi}_1(t) = \frac{m_1}{m_1 + m_2}\frac{F_0}{k}e^{\omega_0 t}\sin\omega_1 t\left(\omega_0\frac{\omega_0}{\omega_1} + \omega_1\right).$$

Thereafter,

$$\xi_2(t) = e^{\omega_0 t}[A\sin\omega_1 t + B\cos\omega_1 t],$$

$$\dot{\xi}_2(t) = e^{\omega_0 t}[A\omega_0\sin\omega_1 t + B\omega_0\cos\omega_1 t + A\omega_1\cos\omega_1 t - B\omega_1\sin\omega_1 t],$$

where the force is applied for a duration t_1 and

$$A = \frac{m_1}{m_1 + m_2}\frac{F_0}{k}\left[\frac{\omega_0}{\omega_1} + \sin\omega_1 t_1\, e^{-\omega_0 t_1} - \frac{\omega_0}{\omega_1}\cos\omega_1 t_1\, e^{-\omega_0 t_1}\right],$$

$$B = \frac{m_1}{m_1 + m_2} \frac{F_0}{k} \left[-1 + \cos \omega_1 t_1 \, e^{-\omega_0 t_1} + \frac{\omega_0}{\omega_1} \sin \omega_1 t_1 \, e^{-\omega_0 t_1} \right].$$

The maximum spinal compression will occur at t_2 when

$$\tan \omega_1 t_2 = -\frac{\sin \omega_1 t_1 \, e^{-\omega_0 t_1}}{1 - \cos \omega_1 t_1 \, e^{-\omega_0 t_1}}$$

at which time

$$\xi_2(t_2) = e^{\frac{\omega_0}{\omega_1}\left[\tan^{-1}\left(-\frac{\sin \omega_1 t_1 e^{-\omega_0 t_1}}{1 - \cos \omega_1 t_1 e^{-\omega_0 t_1}}\right)\right]} \frac{m_1}{m_1 + m_2} \frac{F_0}{k} \sqrt{1 - 2\cos \omega_1 t_1 \, e^{-\omega_0 t_1} + e^{-2\omega_0 t_1}}\,.$$

For the parameters of interest, DRI would exceed 23 for Δv in excess of approximately 4.6 m/s. Figure 3 shows spinal compression as a function of time if a constant acceleration of a_c is maintained for just over $t_1 = 0.02$ s, which corresponds to $\Delta v = 4.6$ m/s. In this figure, the blue line denotes the regime of force application, and the green line is subsequent passenger motion.

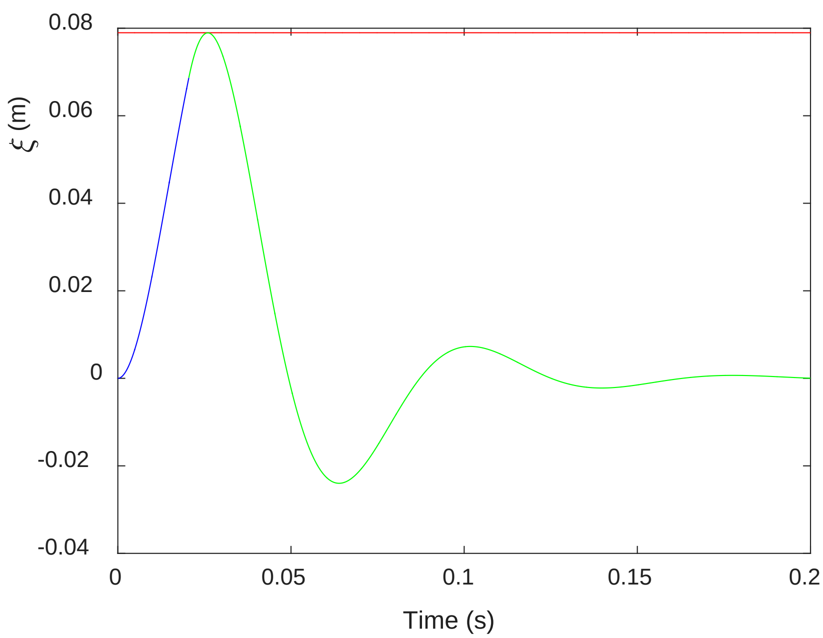

Figure 3. Constant Acceleration Limit DRI < 23

What is the highest Δv survivable under the DRI lethality criterion without seat suspension (i.e., seat damper)? For a fixed lower body velocity, the position of the upper body is governed by

$$m_1 \ddot{x}_1 = -k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2),$$

where the lower body kinematic constraint yields

$$m_1\ddot{x}_1 + b\dot{x}_1 + kx_1 = kt\Delta v + b\Delta v.$$

Applying the initial conditions,

$$\dot{x}_1(0) = x_1(0) = 0,$$

yields

$$x_1 = \Delta v\left(t - \frac{1}{\omega_1}e^{\omega_0 t}\sin\omega_1 t\right),$$

where

$$\omega_0 = -\frac{b}{2m_1},$$

$$\omega_1 = -\omega_0\sqrt{\frac{4km_1}{b^2} - 1}.$$

The resulting spinal stroke is

$$\xi = t\Delta v - x_1 = \frac{\Delta v}{\omega_1}e^{\omega_0 t}\sin\omega_1 t.$$

Maximum compression occurs when

$$\dot{\xi} = 0 = \frac{e^{\omega_0 t}}{\omega_1}(\omega_0\sin\omega_1 t + \omega_1\cos\omega_1 t)$$

at time

$$t_3 = \frac{1}{\omega_1}\tan^{-1}\sqrt{\frac{4km_1}{b^2} - 1}.$$

Setting $\xi(t_3) = 7.9$ cm gives a maximum survivable velocity of 5.7 m/s with no seat suspension (i.e., damper) under the DRI lethality criterion. 5.7 m/s is larger than the 4.6 m/s survivable under a 23 g constant-acceleration damper. In other words, utilization of the DRI criterion leads to a surprising result: a seat without suspension allows for larger applied initial momentum than the one with a 23 g constant-acceleration damper.

C. DRI-optimized System Evaluated against Acceleration Efficiency

Taking Iluk's DRI-optimal damper and giving a sudden impulse to accelerate the pelvis rapidly toward the shoulders just strongly enough to compress the spine to its maximum safe level followed by a constant acceleration to hold it there, let us compute the acceleration efficiency. From our previous work, the expression for acceleration efficiency can be easily derived as:[8]

$$\eta_a = \frac{\Delta v^2}{2a_c \int_0^{t^*} t a_p(t) dt}.$$

Now we calculate the passenger acceleration as a function of time for the DRI-optimized damper. If the initial impulse gives the lower body an initial velocity of v_0, the body spine stroke dynamics are given by

$$\xi = \frac{v_0}{\omega_1} e^{\omega_0 t} \sin \omega_1 t,$$

where

$$\omega_0 = -\frac{b}{2M}$$

and

$$\omega_1 = -\omega_0 \sqrt{\frac{4kM}{b^2} - 1}.$$

The spine will reach maximum compression when

$$\dot{\xi} = \frac{v_0}{\omega_1} e^{\omega_0 t} (\omega_0 \sin \omega_1 t + \omega_1 \cos \omega_1 t) = 0.$$

This occurs when

$$t_4 = \frac{1}{\omega_1} \tan^{-1} - \frac{\omega_1}{\omega_0}.$$

Then the maximum survivable v_0 is that which produces the maximum allowable ξ at such a time,

$$v_0 = -\omega_0 \sqrt{\frac{4kM}{b^2}} \xi_{\max} e^{-\frac{\omega_0}{\omega_1} \tan^{-1} - \frac{\omega_1}{\omega_0}}.$$

The passenger center-of-mass velocity at the moment of its internal velocity equilibrium $(\dot{\xi} = 0)$, is

$$v_{CM} = \frac{v_0 m_2}{m_1 + m_2}.$$

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8 Teichman and Macheret, "Vehicle Blast Protection Efficiency Analysis and Evaluation." IDA Document D-8170 (Alexandria, VA: Institute for Defense Analyses, forthcoming).

Any remaining acceleration is at $a_c = 23$ g. Thus, the total passenger acceleration profile is given by an impulse at $t = 0$ equivalent to $v_{CM}\delta(t)$ where $\delta(t)$ is the Dirac delta function followed by a constant acceleration of a_c from t_4 to

$$t_5 = \frac{\Delta v - v_{CM}}{a_c} + t_4.$$

If $\Delta v < v_{CM}$, then only the initial impulse is required, and the acceleration efficiency is infinite. Evaluating the acceleration efficiency for higher Δv,

$$\eta_a = \frac{\Delta v^2}{2a_c \int_0^{t^*} t a_p(t) dt} = \left(\frac{\Delta v}{\Delta v - v_{CM}}\right)\left(\frac{\Delta v}{\Delta v - v_{CM} + a_c t_4}\right).$$

The first term must be greater than 1 because $\Delta v > \Delta v - v_{CM}$. The second term will be greater than 1 if $v_{CM} > a_c t_4$, in other words, if the initial impulse confers more momentum than the critical acceleration would confer over the period of spinal stroke closure. Both sides of this inequality can be evaluated. For the given spinal parameters, v_{CM} is just over 4 m/s, and $a_c t_4$ is approximately 3.3 m/s, so $\eta_a > 1$.

Thus, at least in the case of the DRI, a vehicle with passenger protection optimized to accommodate the dynamics of the body will not be penalized with respect to the efficiencies. A vehicle outperforming the limits set by a fixed-acceleration threshold will be properly scored. On the other hand, a vehicle showing efficiency close to 1 may have room for improvement if it is optimized with respect to dynamic lethality criteria.

D. Conclusion

Choice of lethality criterion can affect the level of impulse deemed survivable. Designs at the threshold of lethality with respect to constant-acceleration threshold criterion may be lethal under dynamic lethality criteria like the DRI. Designs optimized for dynamic lethality criteria may demonstrate acceleration efficiencies higher than 1, since the point of comparison is the optimal performance under the constant-acceleration threshold criterion. As impulses get higher and $\Delta v \gg v_{CM}$, however, the maximum acceleration efficiency under the DRI, at least, will approach 1.

Due to the ease of application, it may remain worthwhile to go through initial cycles of design utilizing transparent criteria like the fixed-acceleration threshold, but evaluation against dynamic criteria may be important to ensuring safety.

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14. ABSTRACT

There may be more accurate human lethality criterion than a fixed acceleration threshold (i.e., 23 g). The human body is a dynamic system with rate-dependent material properties and natural frequencies. Since injuries are caused by the internal stresses and strains produced by external forces, a more accurate prediction of injury should account for the body’s dynamic response. This paper examines how the results of the fixed-acceleration threshold analysis relate to more complex criteria. In other words, if a system exhibits high efficiency for the 23 g criterion, how does is perform with respect to the DRI as an exemplar of dynamically sensitive criteria?

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