



INSTITUTE FOR DEFENSE ANALYSES

**Bayesian Component Reliability Estimation:  
an F-35 Case Study**

Vincent A. Lillard, Project Leader

Rebecca M. Medlin

March 2019

Approved for public release.  
Distribution unlimited.

IDA Document NS D-10561

Log: H 2019-000142

INSTITUTE FOR DEFENSE ANALYSES  
4850 Mark Center Drive  
Alexandria, Virginia 22311-1882



*The Institute for Defense Analyses is a non-profit corporation that operates three federally funded research and development centers to provide objective analyses of national security issues, particularly those requiring scientific and technical expertise, and conduct related research on other national challenges.*

#### About This Publication

This work was conducted by the Institute for Defense Analyses (IDA) under contract HQ0034-14-D-0001, Task 4370, "Data Analysis Support," for the Office of the Director, Operational Test and Evaluation. The views, opinions, and findings should not be construed as representing the official position of either the Department of Defense or the sponsoring organization.

#### Acknowledgments

The IDA Technical Review was conducted by Mr. Robert R. Soule, Director and Dr. Keyla Pagan- Rivera and Dr. Vincent A. Lillard from the Operational Evaluation Division.

#### For more information:

Vincent A. Lillard, Project Leader  
vlillard@IDA.org • (703) 845-2230

Robert R. Soule, Director, Operational Evaluation Division  
rsoule@ida.org • (703) 845-2482

#### Copyright Notice

© 2019 Institute for Defense Analyses  
4850 Mark Center Drive, Alexandria, Virginia 22311-1882 • (703) 845-2000.

This material may be reproduced by or for the U.S. Government pursuant to the copyright license under the clause at DFARS 252.227-7013 (a)(16) [Jun 2013].

INSTITUTE FOR DEFENSE ANALYSES

IDA Document NS D-10561

**Bayesian Component Reliability Estimation:  
an F-35 Case Study**

Vincent A. Lillard, Project Leader

Rebecca M. Medlin



## Executive Summary

---

A challenging aspect of a system reliability assessment is integrating multiple sources of information, such as component, subsystem, and full-system data, along with previous test data or subject matter expert (SME) opinion. A powerful feature of Bayesian analyses is the ability to combine these multiple sources of data and variability in an informed way to perform statistical inference. This feature is particularly valuable in assessing system reliability where testing is limited and only a small number of failures (or none at all) are observed.

The F-35 is DoD's largest program; approximately one-third of the operations and sustainment cost is attributed to the cost of spare parts and the removal, replacement, and repair of components. The failure rate of those components is the driving parameter for a significant portion of the sustainment cost, and yet for many of these components, available estimates of the failure rate are poor. For many programs, the contractor produces estimates of component failure rates based on engineering analysis and legacy systems with similar parts. While these estimates are useful, the actual removal rates

provide a more accurate estimate of the removal and replacement rates the program will experience in future years.

In this document, we show how we applied a Bayesian analysis to combine the engineering reliability estimates with the actual failure data to estimate component reliability. Our analysis technique also allows for us to overcome the problems of cases where few or no failures have been observed. We are able to show that combining the engineering knowledge of reliability with the observed operational reliability results in both a more informed estimate of each individual component's reliability and a more informed estimate of overall F-35 maintenance costs.

The technique presented is broadly applicable to any program where multiple sources of reliability information need to be combined for the best estimation of component failure rates, and ultimately of sustainment costs.



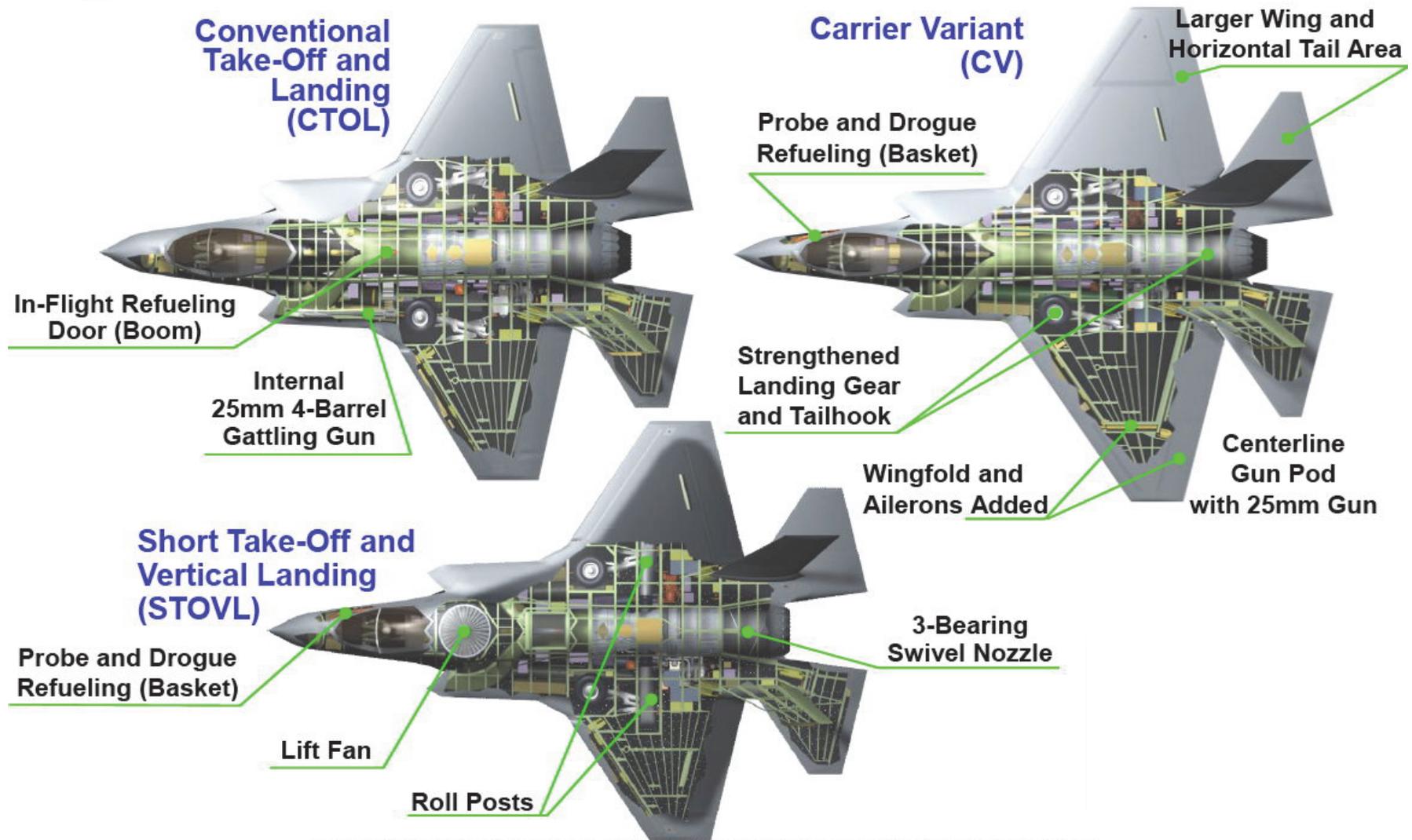
# Bayesian Component Reliability Estimation: an F-35 Case Study

Bram Lillard  
Rebecca Medlin



April 2019

# F-35 is a complex aircraft...



DISTRIBUTION STATEMENT A. Approved for public release; distribution is unlimited.

# Estimating Component Reliability is essential for Operations and Sustainment



Over 2,000 parts

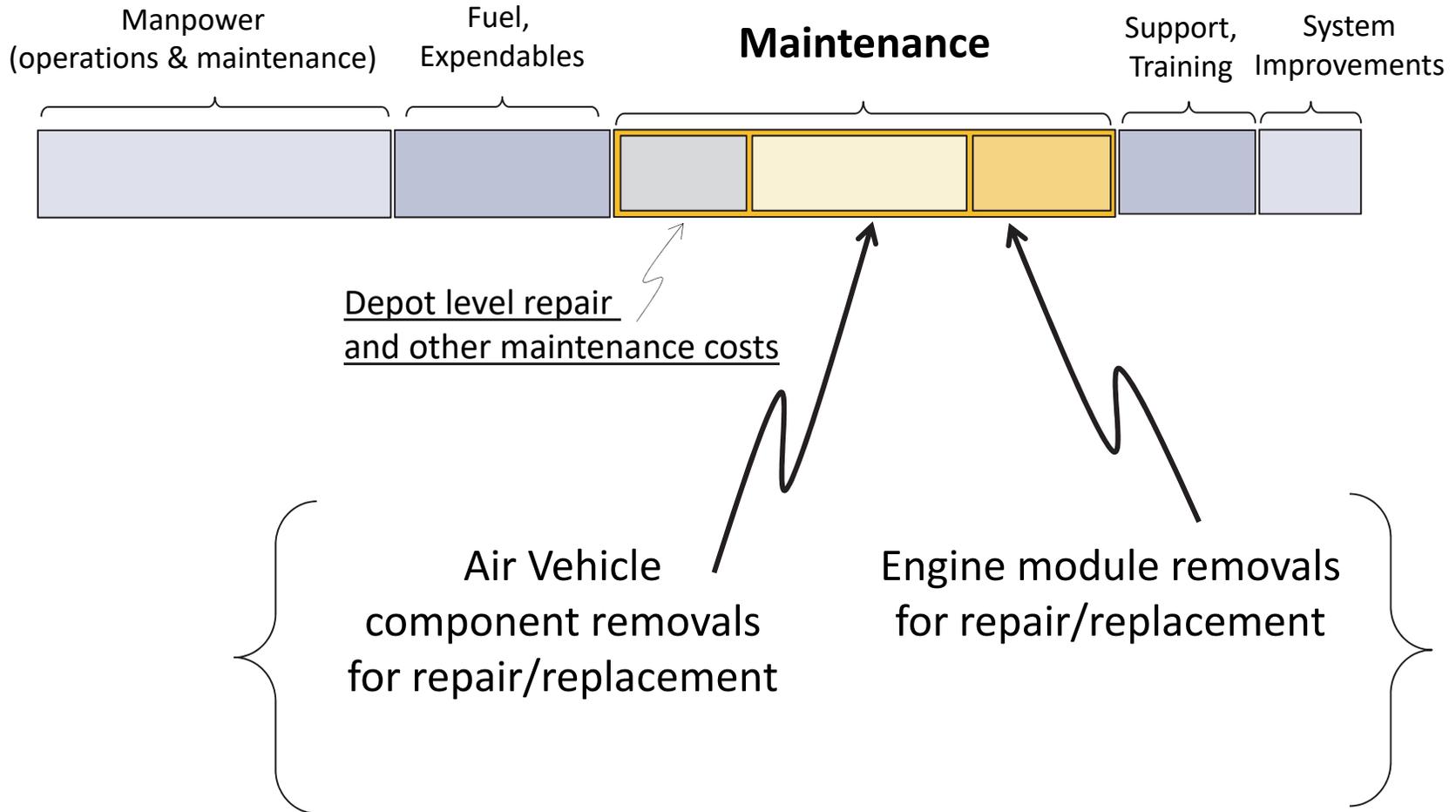


Air Vehicle Systems	# components within category
PWR & THERMAL MGMTSYS (PTMS)	88
270VDC GENERATION AND DIST	28
CONTROL PANELS	26
SENSORS, WPNS BAY, ENG BAY	46
CONTROL SURFACES	49
FUEL SYSTEM	141
ICE DETECTION	5
LANDING GEAR	261
LIGHTING	31
IMU & IEU	16
OXYGEN GEN	7
HELMET AND DATA PROCESSORS	52
PHM AIR VEHICLE	7
VEHICLE SYS PROCESSING (VSP)	16
CNI SYSTEM	70
STANDARD PRACTICES, STRUCTURES	38
DOORS & COVERS	330
FRAME, BULKHEADS	113
STABILIZERS, RUDDER	40
CANOPY	27
STRUCTURE, FARINGS, FLAPS	92
PROPULSION AIRCRAFT INTERFACE	9
THROTTLE	6
DOOR ACTUATORS (STOVL ONLY)	49
RADAR SYSTEM	149
EJECTION SEAT, SYSTEM	34
ELECTRONIC WARFARE	81

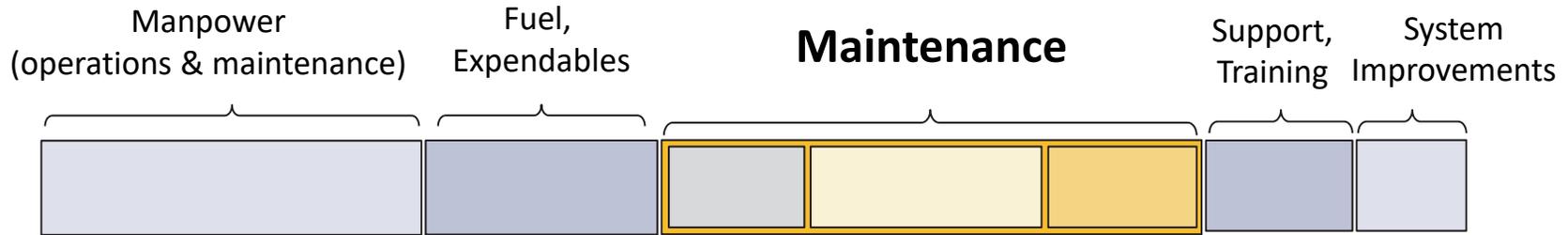
## Reliability estimates drive:

- Spares purchases
- Program budgeting
- Cost estimation
- Readiness

# What comprises F-35 Costs per Flying Hour?



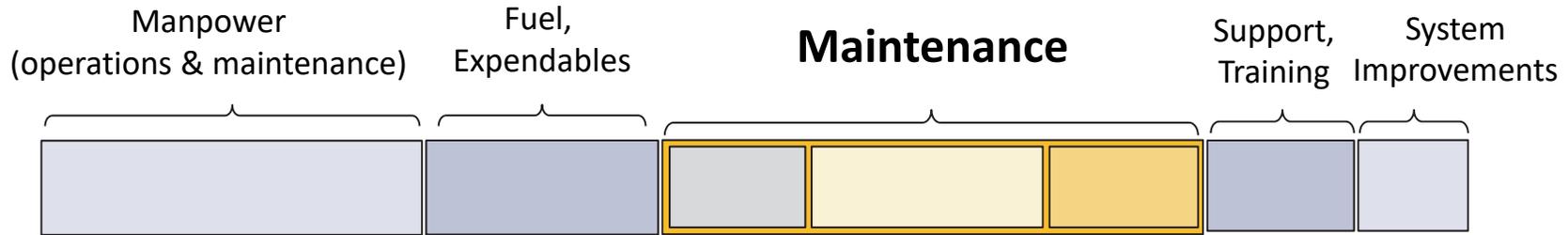
# What comprises F-35 Costs per Flying Hour?



Depot level repair and other maintenance costs

$$CPFH_{Total} = \sum_{i=1}^n \frac{[Replacement\ or\ Repair\ Cost]_i \times (num\ failures)_i}{Total\ Flight\ Hours}$$

# What comprises F-35 Costs per Flying Hour?

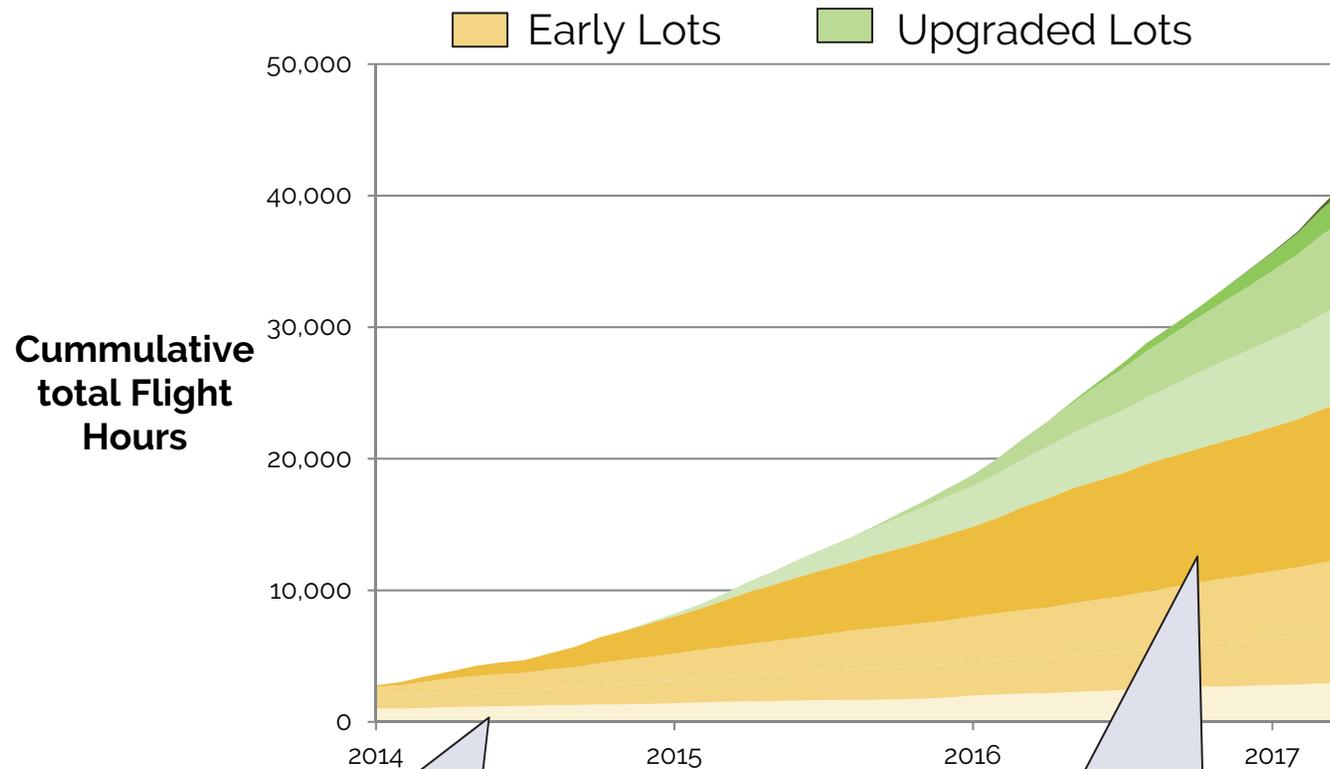


Depot level repair and other maintenance costs

$$CPFH_{Total} = \sum_{i=1}^n \frac{[Replacement\ or\ Repair\ Cost]_i}{MFHBR_i}$$

Accurate component Reliability estimates are essential for cost estimation

# Data is often scarce for reliability estimation



Early in a Program we only have ***Engineering Estimates*** for component reliability (also when a new variant/configuration begins flying)

Later, sufficient failures have occurred, flying hours accumulated, to begin estimating reliability for each component

# Component Reliability Estimates – Many methods

## Three Cases

Lots of failures ( $N > 20$ )



$$MFHBR_i = \frac{\text{Total Flight Hours}}{N_i}$$

(assume failure times follow an Exponential Distribution)

Few failures ( $1 < N < 20$ )



Do we use:

- FH/N (ignore uncertainty)?
- Report a weighted average?
  - E.g.,  $0.3 \cdot (\text{FH}/N) + 0.7 \cdot \text{Eng. Est.}$

No Failures to date ( $N=0$ )



$$MFHBR_i = MFHBR_i^{\text{Engineer Est.}}$$

What if

$\text{FH} \gg MFHBR_i^{\text{Engineer Est.}}$  ?

Do we use the:

- lower CI bound?
- set equal to FH?
- engineering est.?

**Alternatively we can use a Bayesian approach**  
(sliding scale weighted average)

# Bayesian statistics combine “prior” knowledge with observed data to produce an estimate

## Example for Component X:

- Engineering Estimate MFHBR = 990 hours
- Flight Hours flown to date: 40,000 hours
- Observed 2 Failures... traditional methods estimate:
- MFHBR =  $40,000 / 2 = 20,000$  hours

What's the best number to use for MFHBR?

990 or 20,000?

Average the two? (~10,500?)

Weigh one more than the other? Which one?

# One math slide for the presentation...

## Bayesian approach to estimating Relatability

$$\begin{array}{c} \text{Posterior Distribution} \\ \hline \pi(\lambda | \mathbf{x}) \end{array} \propto \begin{array}{c} \text{Likelihood Distribution} \quad \text{Prior Distribution} \\ \hline L(\mathbf{x} | \lambda) \quad \pi(\lambda) \end{array}$$

- **Likelihood Distribution:** Exponential ( $\lambda$ )

- $MFHBR = \frac{1}{\lambda} = \frac{\text{Total Flight Hours}}{n}$

- **Prior Distribution:** Gamma ( $\alpha, \beta$ )

- We can use the engineering estimates to solve for  $\alpha$  and  $\beta$ .

- Inv. Gamma mean =  $MFHBR_{\text{Engineer Est}}$

- Inv. Gamma std. =  $MFHBR_{\text{Engineer Est}} \times p$

- **Posterior Distribution:** Gamma ( $\alpha', \beta'$ )

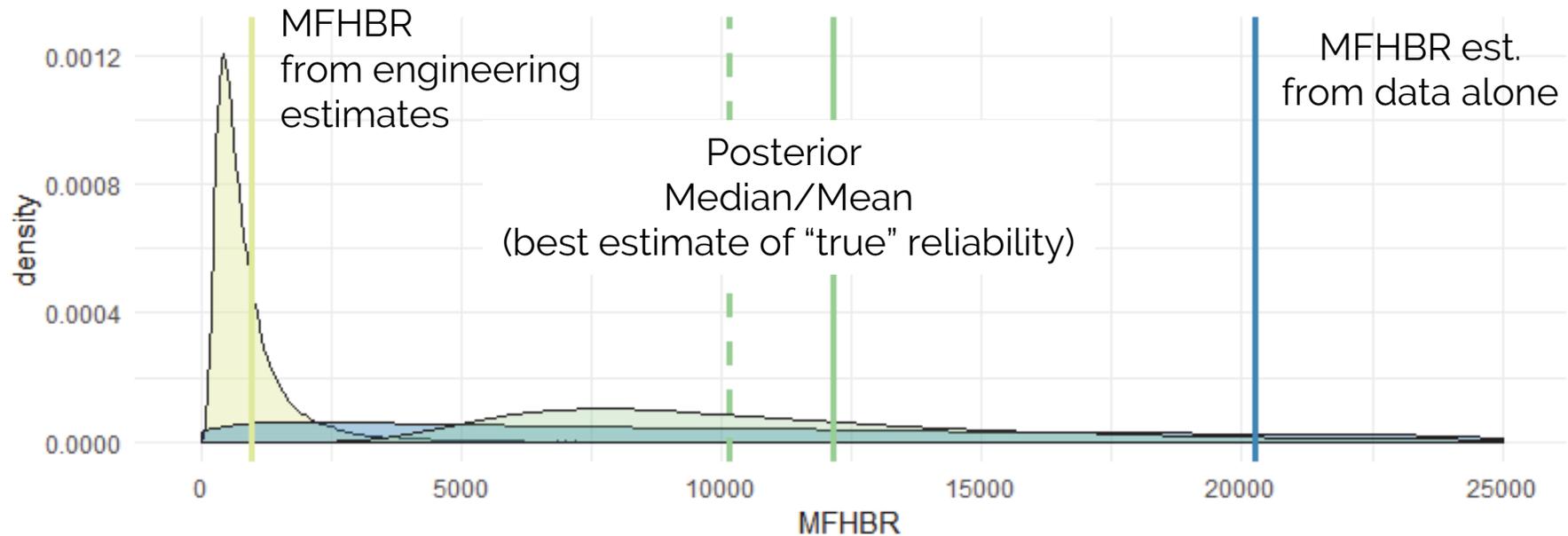
- $\alpha' = N + \alpha$

- $\beta' = \text{Total Flight hours} + \beta$

# Bayesian statistics combine “prior” knowledge with observed data to produce an estimate

Example for Component X:

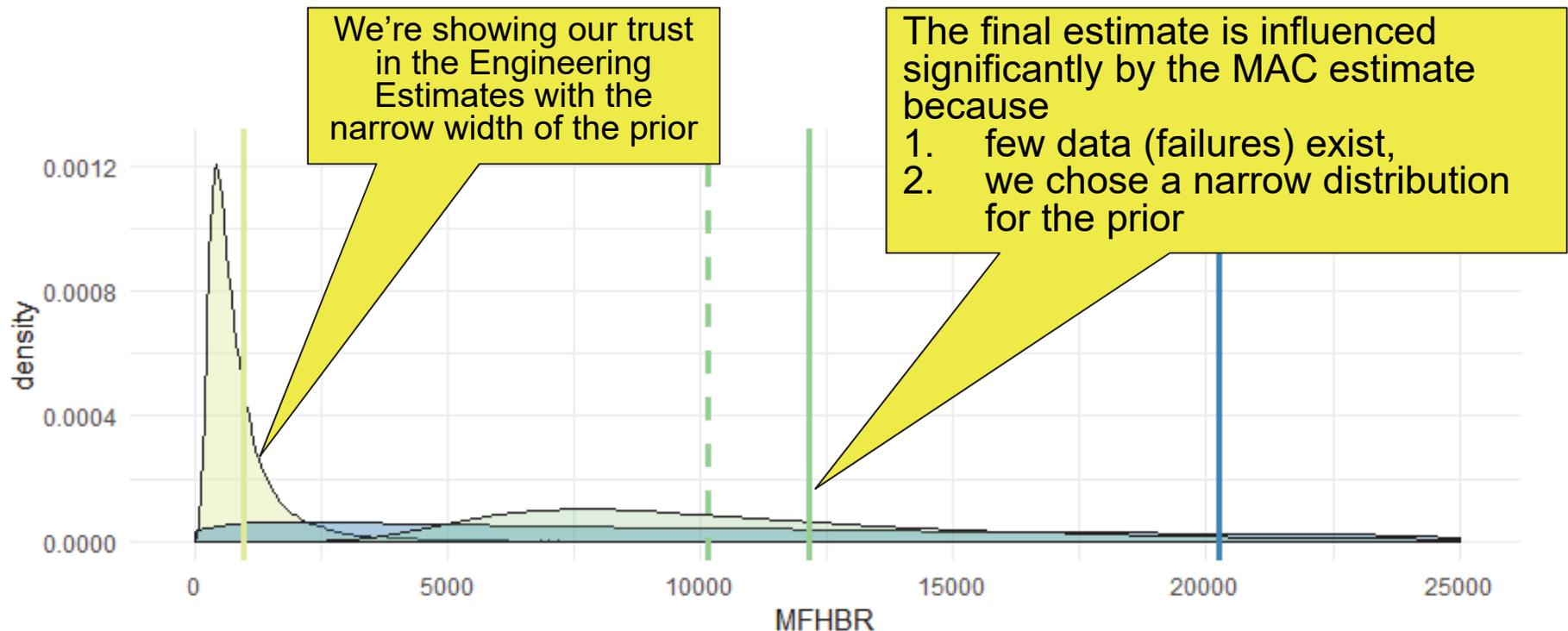
- Engineering Estimate MFHBR = 990 hours (yellow “prior” below)
- Flight Hours flown to date: 40,000 hours
- Observed 2 Failures... traditional methods estimate:  
MFHBR =  $40,000 / 2 = 20,000$  hours (blue “likelihood” below)



# Bayesian statistics combine “prior” knowledge with observed data to produce an estimate

Example for Component X:

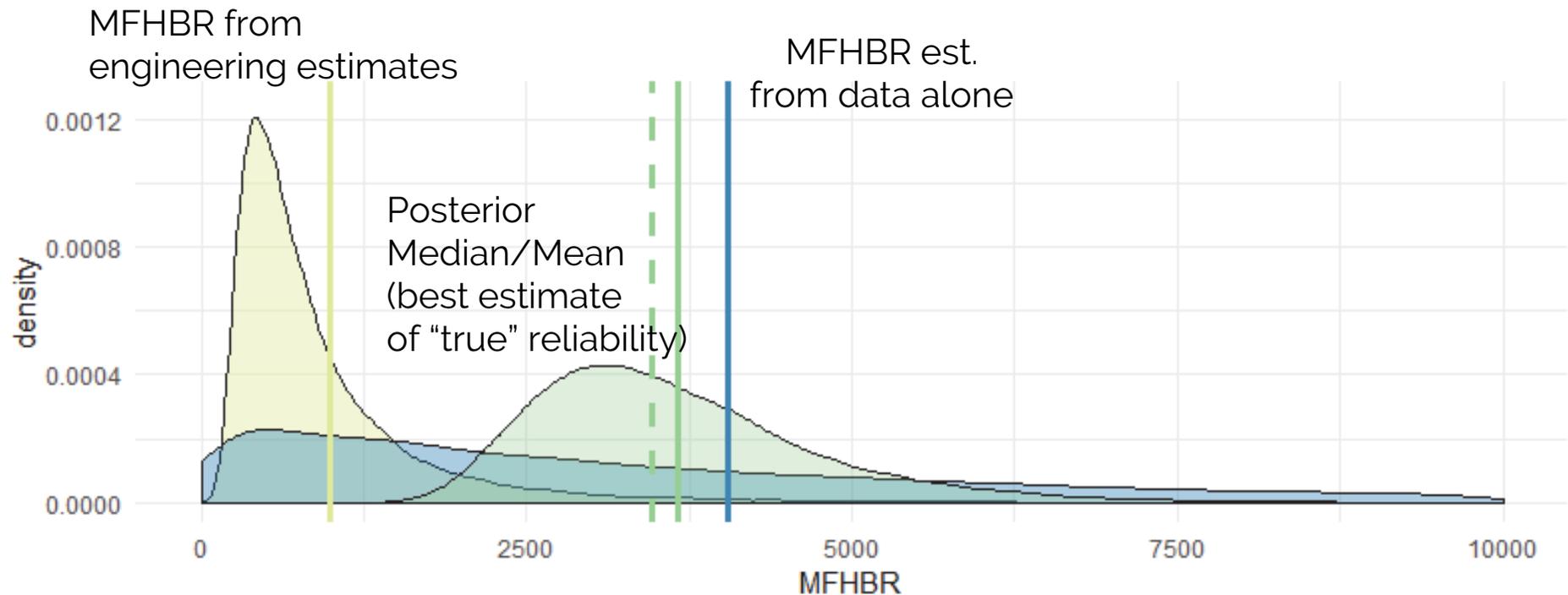
- Engineering Estimate MFHBR = 990 hours (yellow “prior” below)
- Flight Hours flown to date: 40,000 hours
- Observed 2 Failures... traditional methods estimate:
- MFHBR =  $40,000 / 2 = 20,000$  hours (blue “likelihood” below)



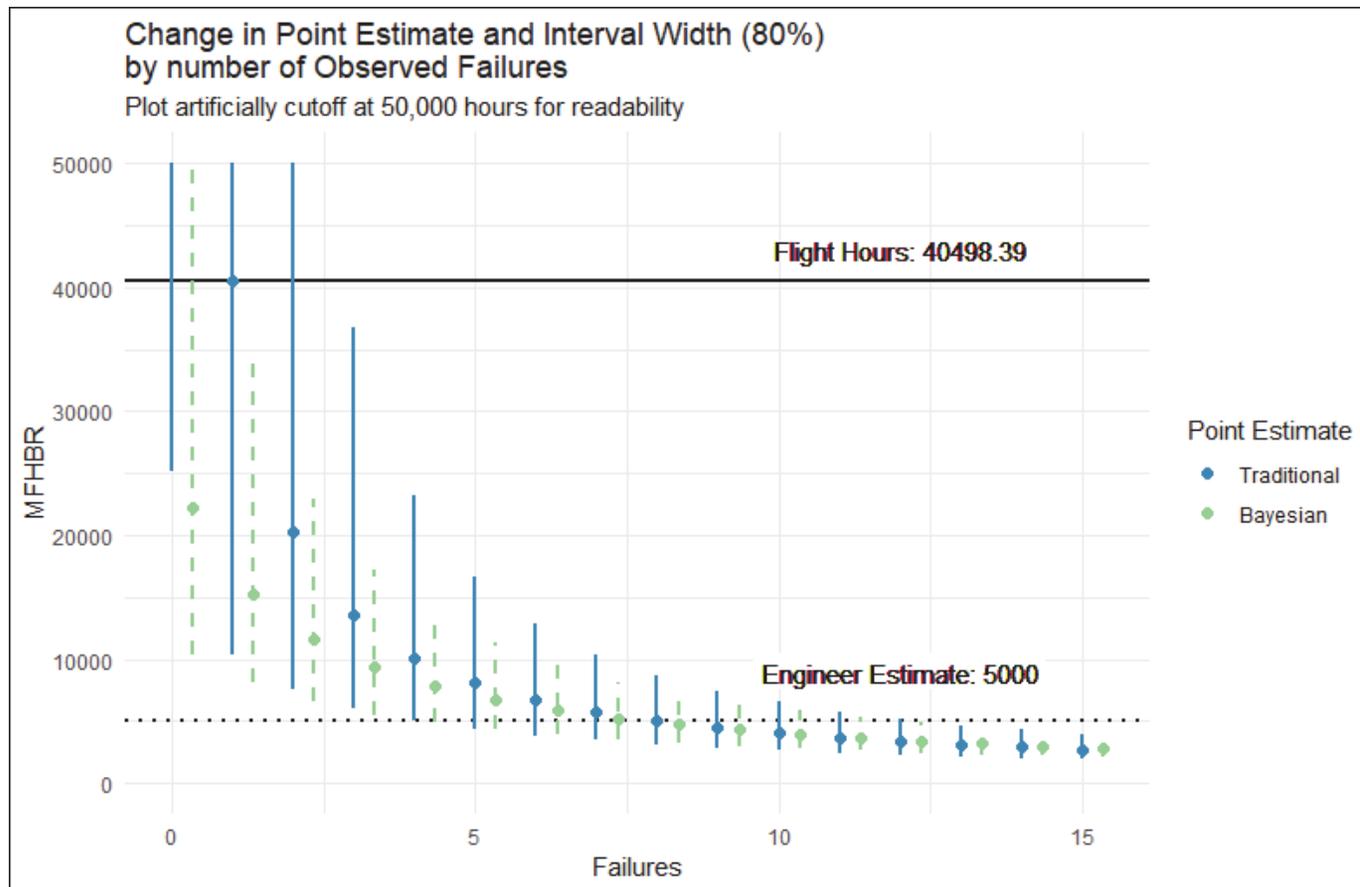
# With more failure data available, the final estimate is less influenced by the MAC value

## Example for Component Y:

- Engineering estimates: MFHBR = 990 hours (yellow prior below)
- Flight Hours flown to date: 40,000
- Observed 10 Failures, so, traditional methods estimate:
- MFHBR =  $40,000 / 10 = 4,000$  hours (blue "likelihood" below)



# A robust methodology for all cases



- Bayesian method appropriately moves MFHBR estimate towards the traditional result as the available data increases
- The approach also automatically handles cases where  $N=0$  (something not satisfactorily handled with traditional approaches)

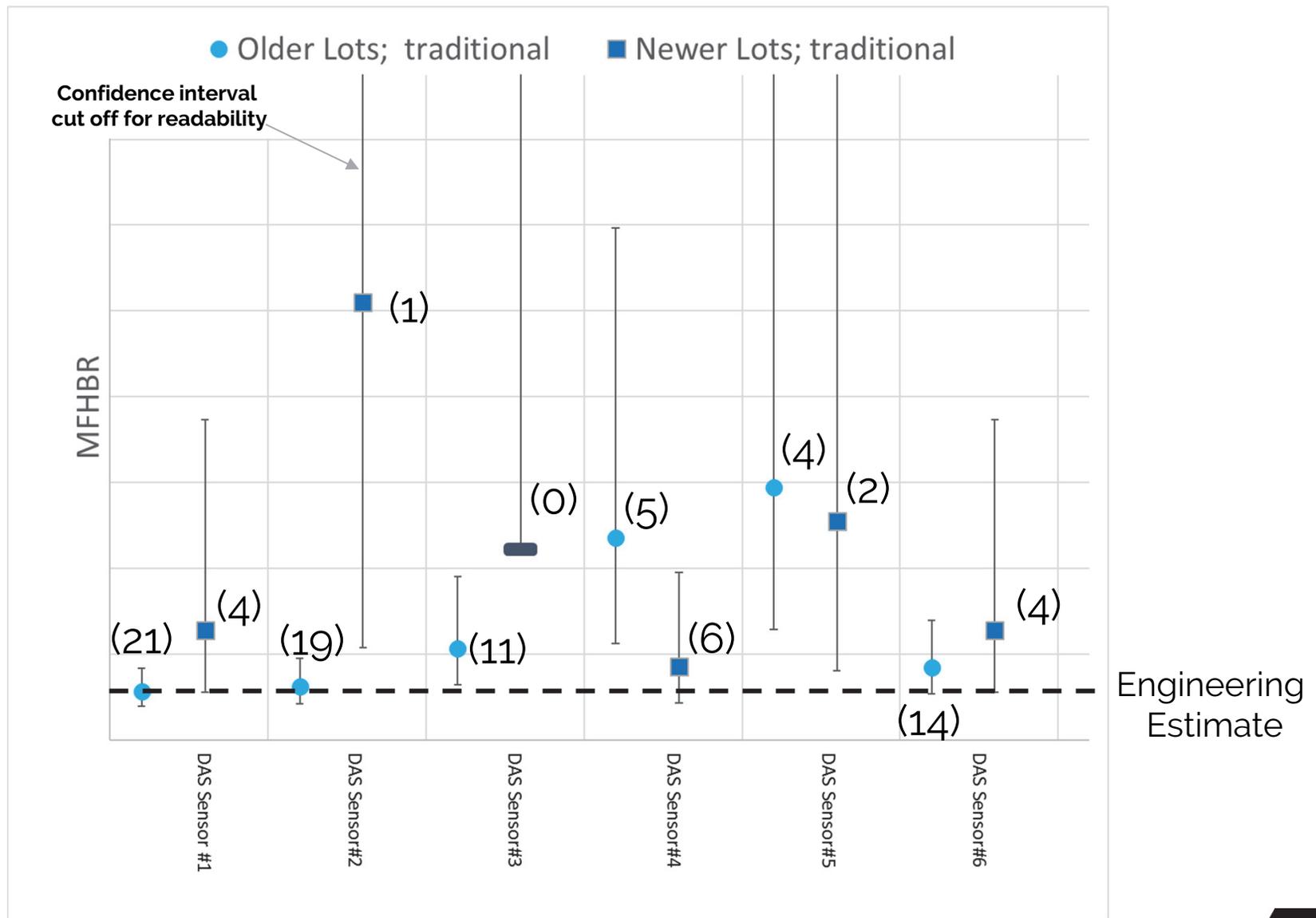
# A frequent debate: How do we estimate MFHBR for a new configuration?

Example for Component Z:

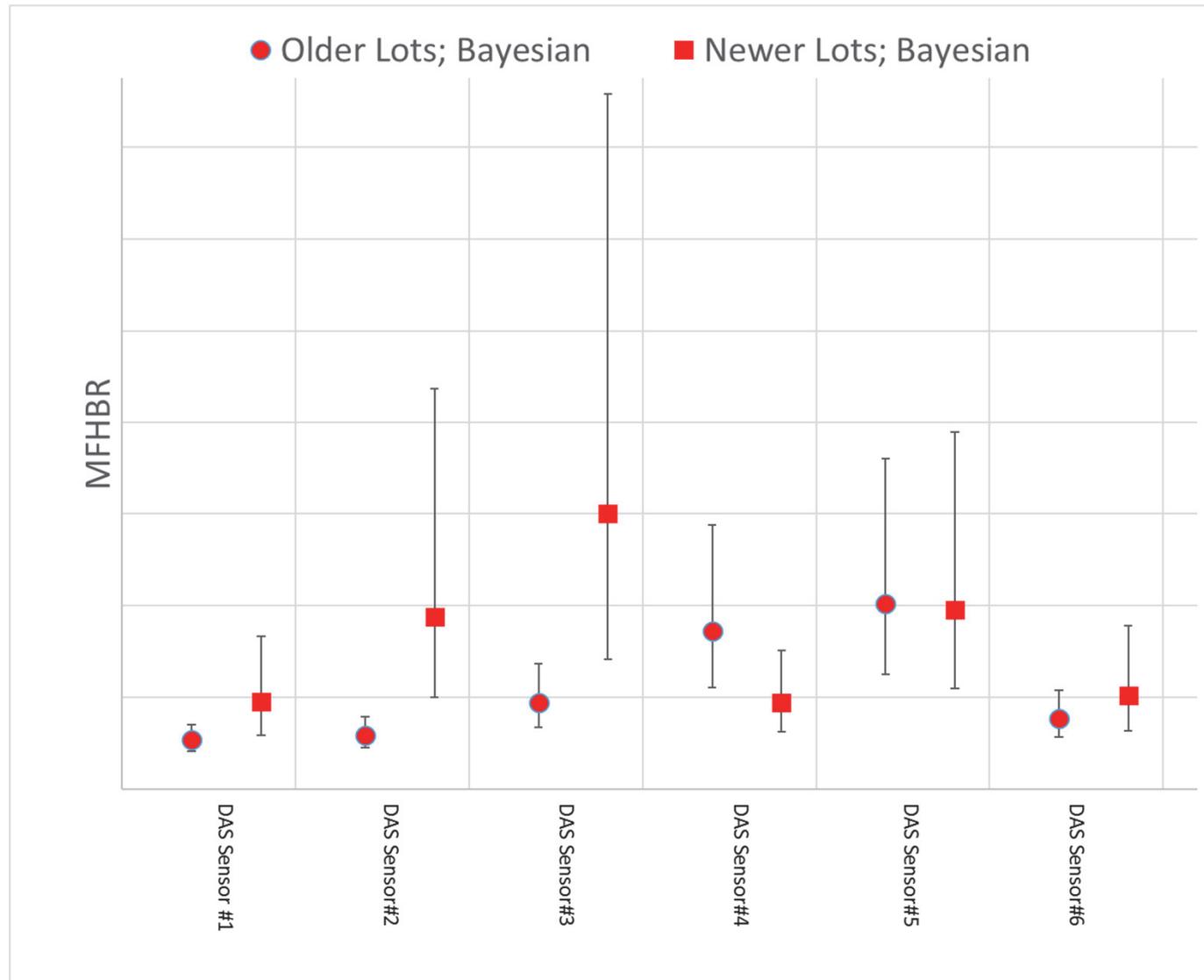
	Older Lots	New Lots (anticipated improvement)
Engineering Estimate	900	900
Flight hours	20,000	10,000
Failures observed	5	0
MFHBR	4,000	3,338? (95% Lower bound) 4,000? (LRIP 2-5 estimate) 900? (Eng. Est.)

- Bayesian method provides an ideal (and defensible) calculation method for this case
- MFHBR for Older Lots serves as the *new prior estimate* for the New Lots calculation
  - Appropriately using the available data as a starting point, but allowing the available New data to dictate how much the final estimate is moved
- Bayesian results for New Lots : MFHBR = 7,576

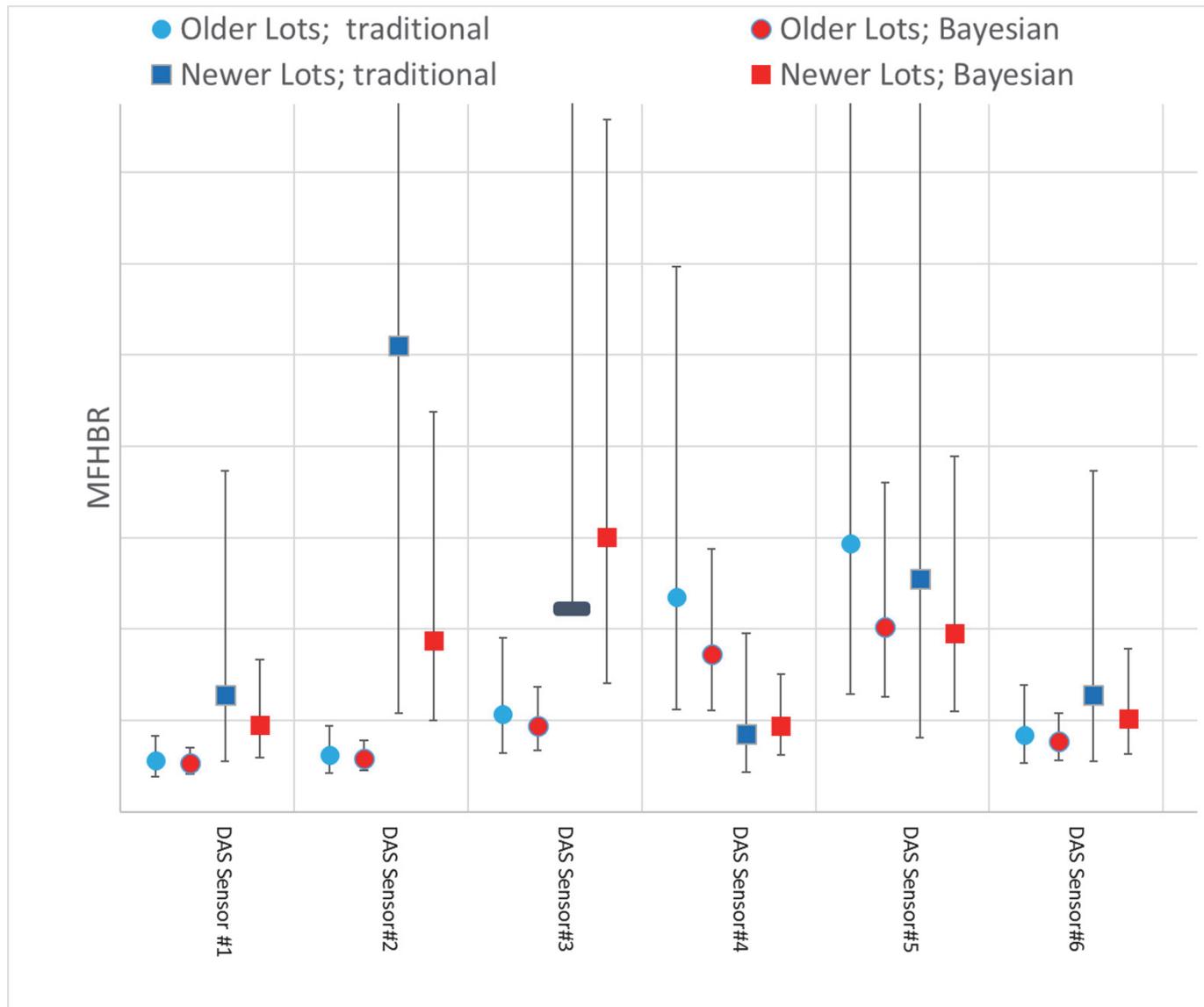
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



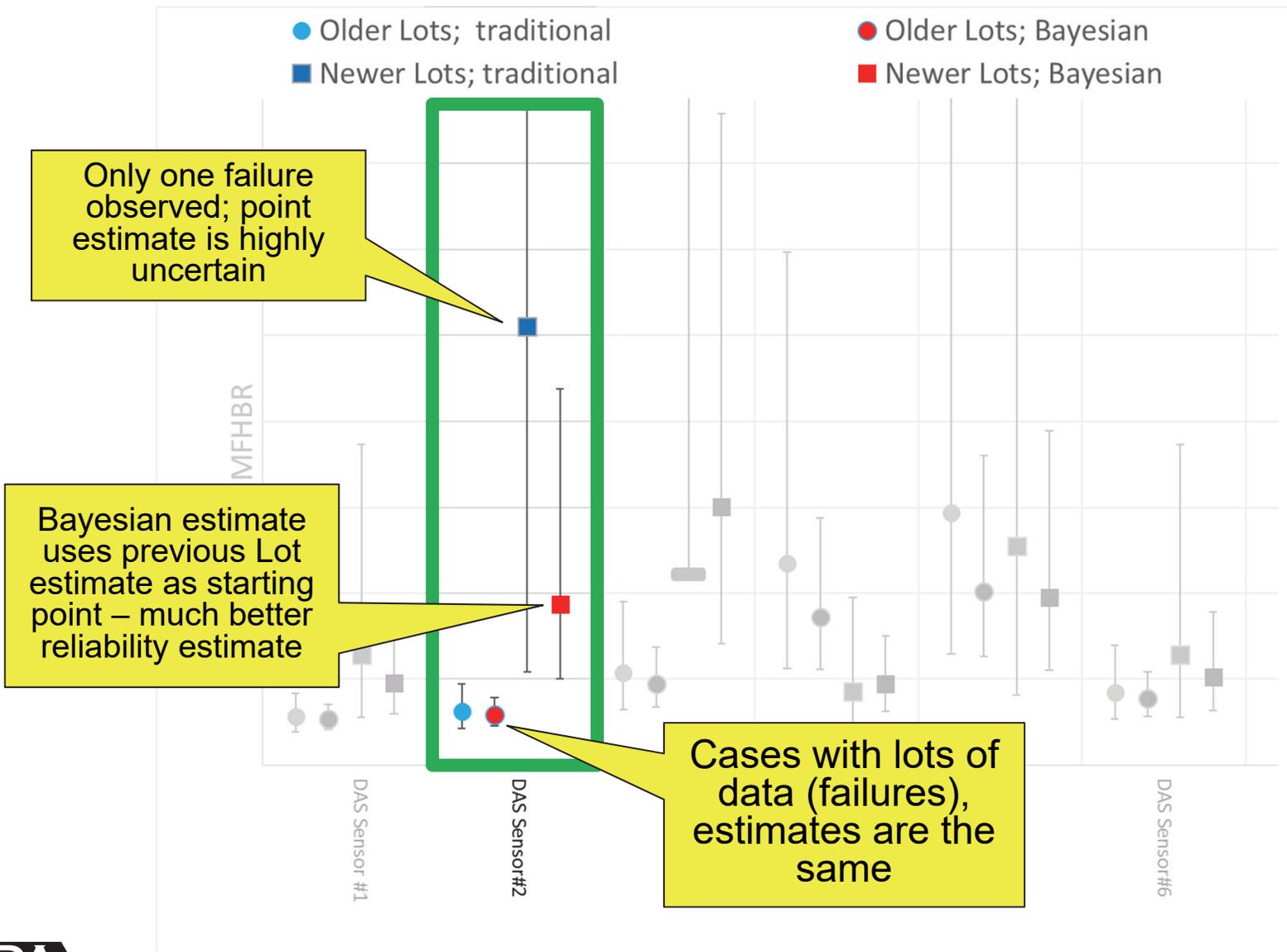
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



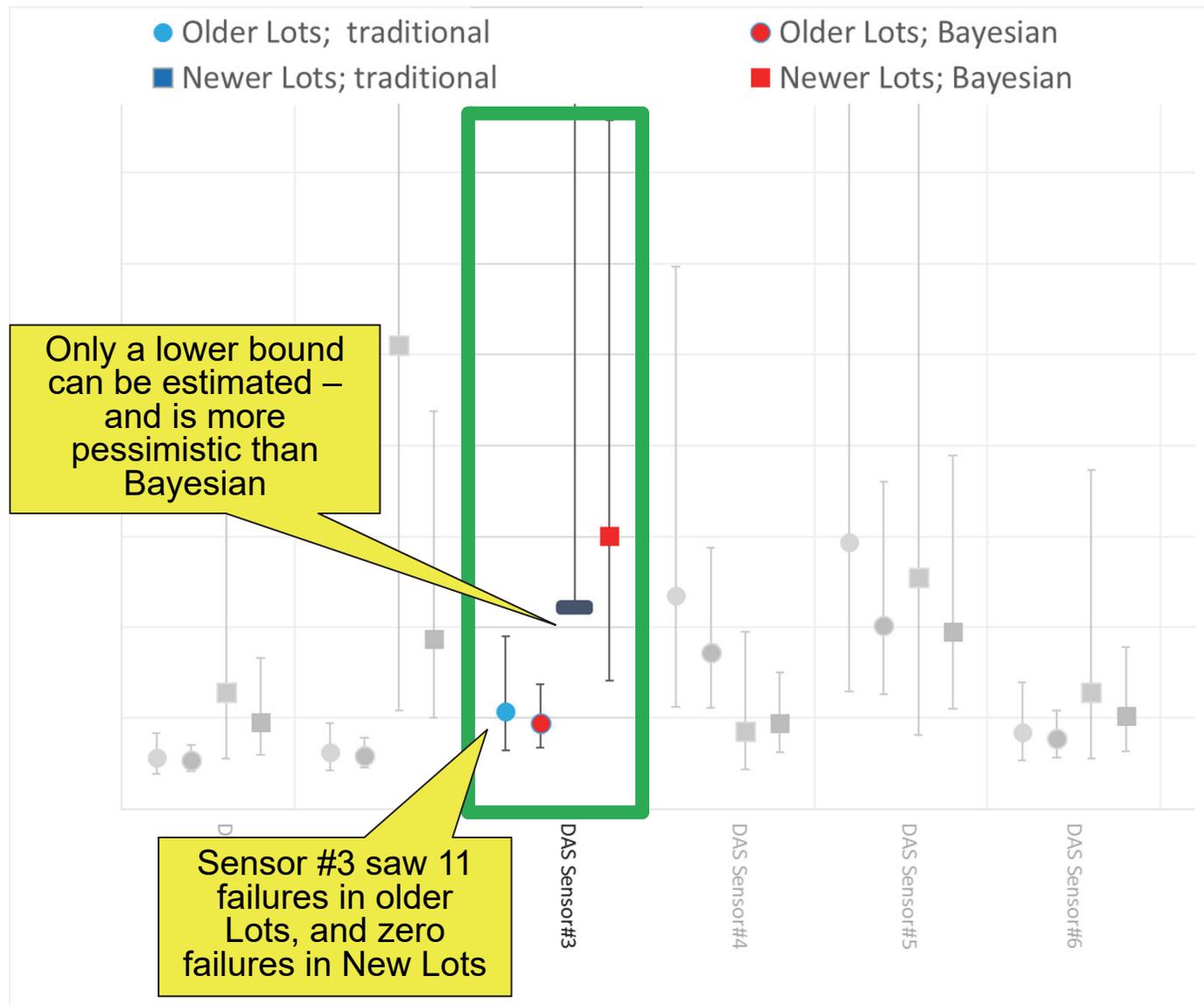
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



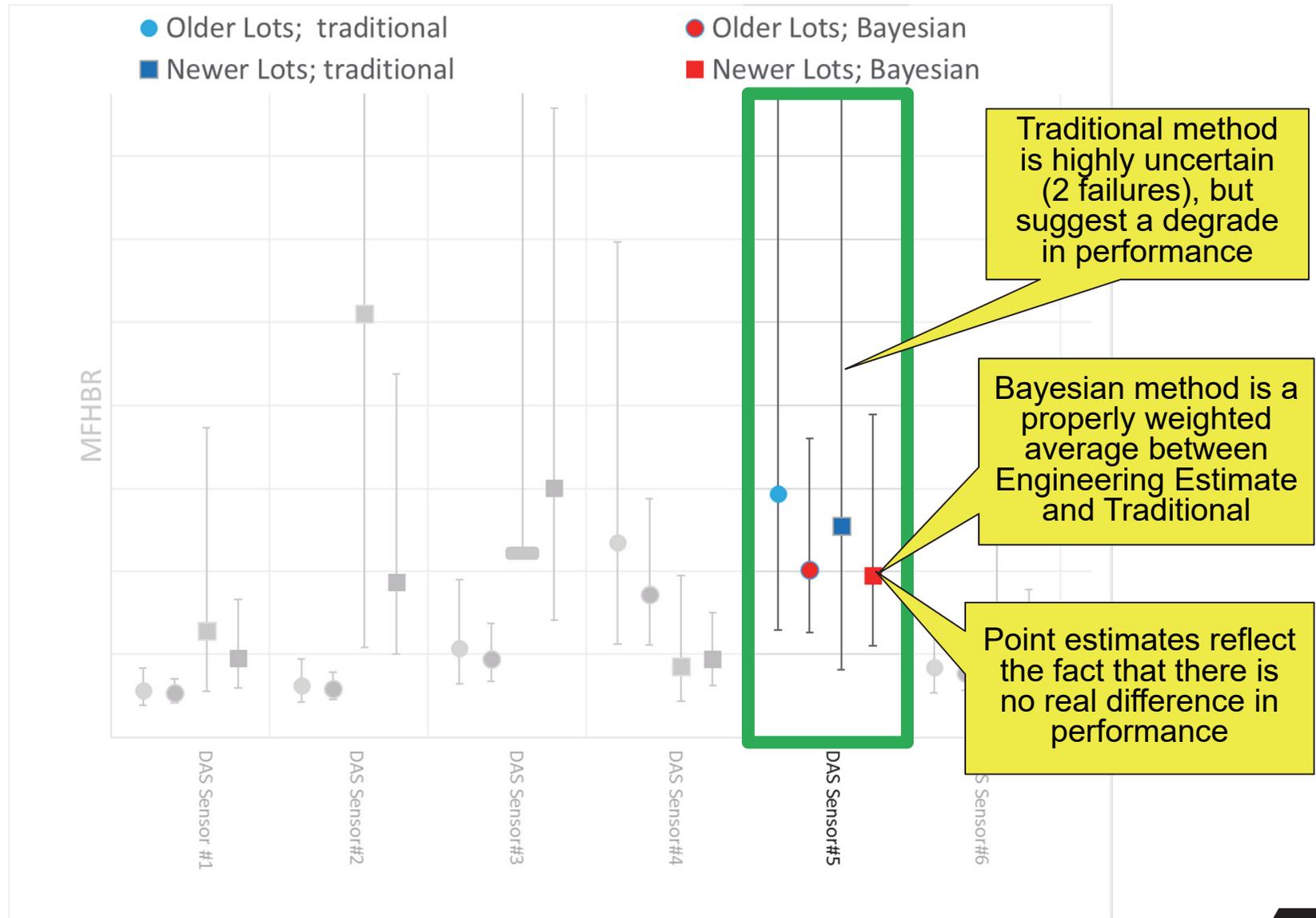
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



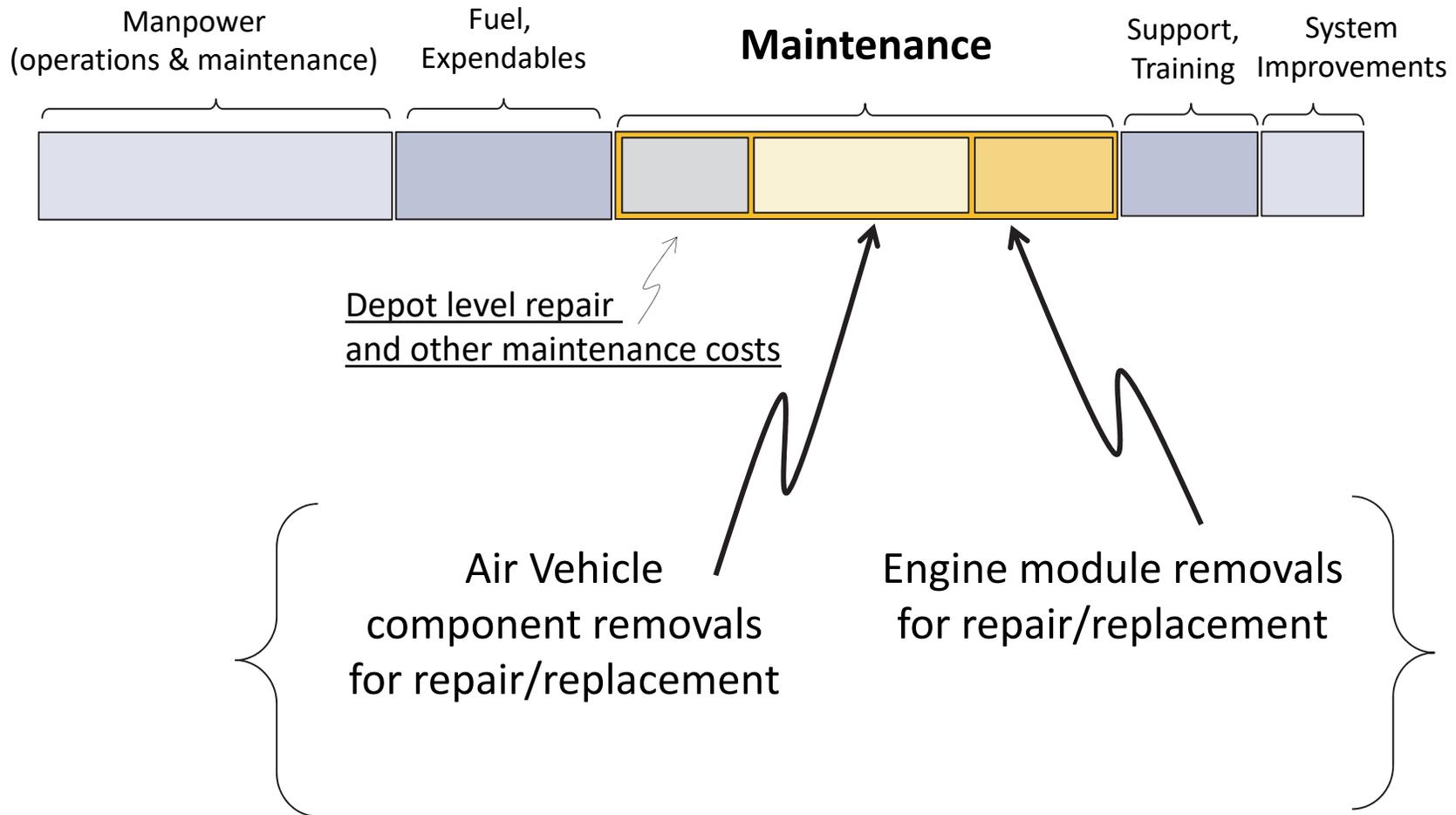
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



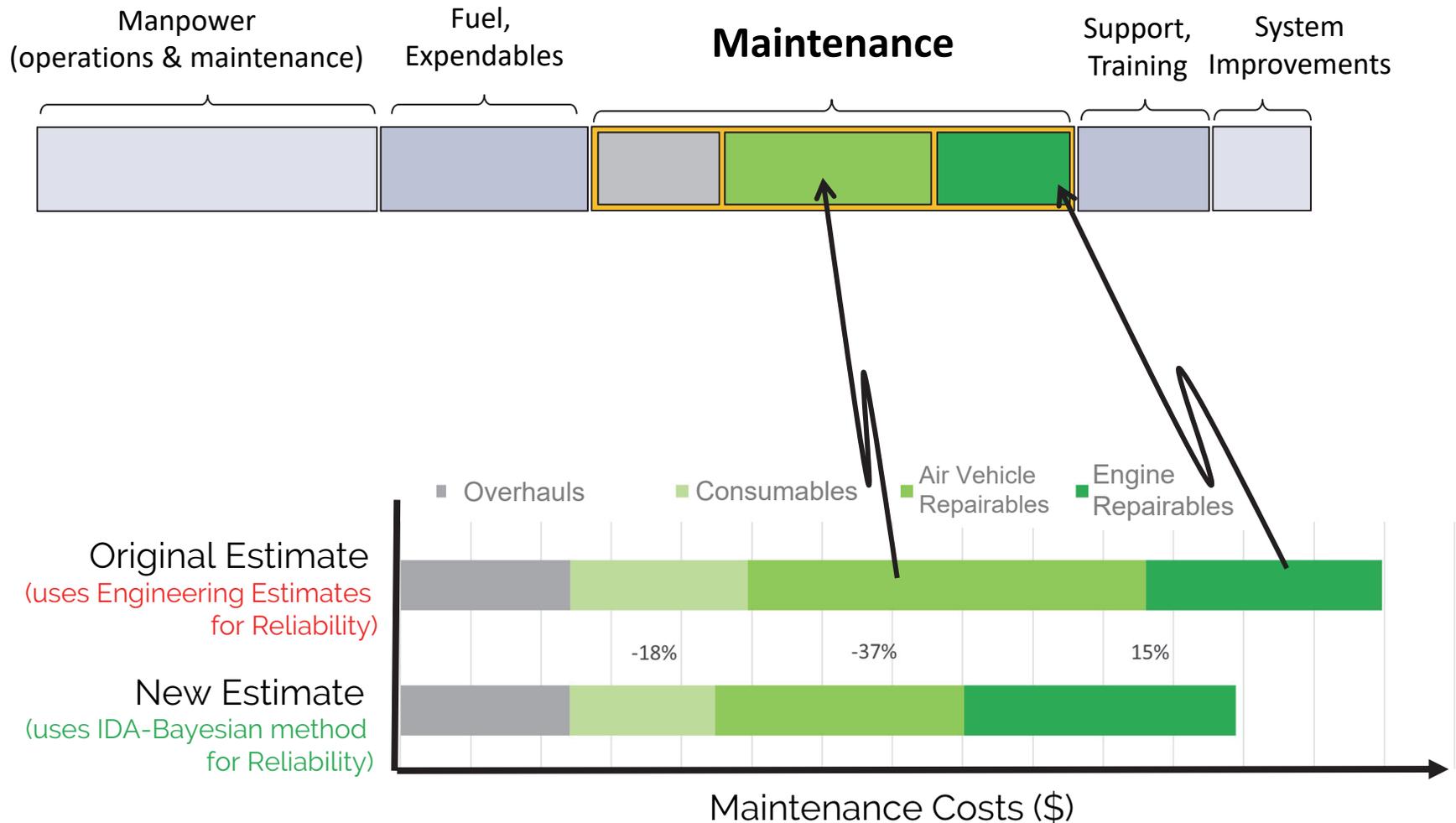
# Distributed Aperture System Sensors' reliability show the benefit of the Bayesian approach



# What comprises F-35 Costs per Flying Hour?



# Bayesian Reliability results in a more informed estimate of maintenance costs



# Conclusion

Bayesian methods provide a means to combine available knowledge of reliability with operational data to estimate component reliability, resulting in a more informed estimate of F-35 maintenance costs.

- Updated from early engineering estimates
- Updated from previous system/variant data
- Handles cases with few data (even no failures!)

**REPORT DOCUMENTATION PAGE**

*Form Approved*  
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.  
**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

<b>1. REPORT DATE (DD-MM-YYYY)</b> 03-2019		<b>2. REPORT TYPE</b> OED Draft		<b>3. DATES COVERED (From - To)</b>	
<b>4. TITLE AND SUBTITLE</b>  Bayesian Component Reliability Estimation: an F-35 Case Study				<b>5a. CONTRACT NUMBER</b> HQ0034-14-D-0001	
				<b>5b. GRANT NUMBER</b>	
				<b>5c. PROGRAM ELEMENT NUMBER</b>	
<b>6. AUTHOR(S)</b>  Institute for Defense Analyses 4850 Mark Center Drive Alexandria, Virginia 22311-1882				<b>5d. PROJECT NUMBER</b> BA-9-4370	
				<b>5e. TASK NUMBER</b> 4370	
				<b>5f. WORK UNIT NUMBER</b>	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  Institute for Defense Analyses 4850 Mark Center Drive Alexandria, Virginia 22311-1882				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  D-10561-NS  H 2019-000142	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  Office of Secretary of Defense–Office of Cost Assessment Program Evaluation (OSD/CAPE) 1800 Defense Pentagon Washington, DC 20301				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> DOT&E	
				<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b> OSD/CAPE	
<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b>  Approved for public release. Distribution is unlimited.					
<b>13. SUPPLEMENTARY NOTES</b>  Project Leader: Vincent Lillard (OED)					
<b>14. ABSTRACT</b> A challenging aspect of a system reliability assessment is integrating multiple sources of information, such as component, subsystem, and full-system data, along with previous test data or subject matter expert (SME) opinion. A powerful feature of Bayesian analyses is the ability to combine these multiple sources of data and variability in an informed way to perform statistical inference. This feature is particularly valuable in assessing system reliability where testing is limited and only a small number of failures (or none at all) are observed. The F-35 is DoD's largest program; approximately one-third of the operations and sustainment cost is attributed to the cost of spare parts and the removal, replacement, and repair of components. The failure rate of those components is the driving parameter for a significant portion of the sustainment cost, and yet for many of these components, available estimates of the failure rate are poor. For many programs, the contractor produces estimates of component failure rates based on engineering analysis and legacy systems with similar parts. While these estimates are useful, the actual removal rates provide a more accurate estimate of the removal and replacement rates the program will experience in future years. In this document, we show how we applied a Bayesian analysis to combine the engineering reliability estimates with the actual failure data to estimate component reliability. Our analysis technique also allows for us to overcome the problems of cases where few or no failures have been observed. We are able to show that combining the engineering knowledge of reliability with the observed operational reliability results in both a more informed estimate of each individual component's reliability and a more informed estimate of overall F-35 maintenance costs. The technique presented is broadly applicable to any program where multiple sources of reliability information need be combined for the best estimation of component failure rates, and ultimately of sustainment costs.					
<b>15. SUBJECT TERMS</b>  Bayesian; F-35; Joint Strike Fighter; Aircraft Reliability; Depot Level Repairables					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b>
<b>a. REPORT</b>	<b>b. ABSTRACT</b>	<b>c. THIS PAGE</b>			Vincent Lillard (OED)
Unclassified	Unclassified	Unclassified	Unlimited	30	<b>19b. TELEPHONE NUMBER (Include area code)</b> (703) 845-2230

