NOVEL APPROACH FOR ANALYZING RADAR TRACKING RESIDUALS

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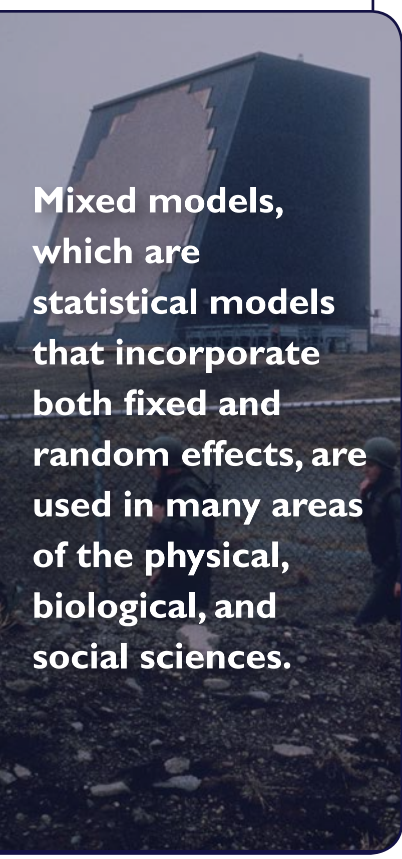
Winner

Mixed models provide a novel approach for analyzing radar tracking residuals by accounting for randomness from different sources. By properly accounting for different sources of randomness, the mixed-models approach can provide greater power to determine the statistical significance of various parameters needed for radar calibration.

The authors, in their article, summarize a mixed-models approach applied to the analysis of radar tracking residuals from calibration satellites observed by the Cobra Dane radar. They found that each of the calibration satellites makes its own idiosyncratic contribution to the observed radar tracking residuals. When properly accounted for, these idiosyncrasies increase the statistical power to determine whether systematic effects exist in the data.



Biases in radar observations can degrade the correlation and fusion of tracks from multiple sensors. They can also degrade the accuracy of project-ahead trajectories based on limited radar observations from a single radar. One way to minimize these problems is to estimate the bias as part of the state vector using augmented state Kalman filters or Schmidt-Kalman filters (Lin, Bar-Shalom, and Kirubarajan 2005; Novoselov et al. 2005). Such approaches can be computationally intensive, and, although the computational difficulties can be reduced by various methods of decoupling the bias estimation from the state estimation, they are not always able to be implemented in real time (Friedland 1969). For real-time applications, the filter can also be subject to being "ill-conditioned" due to limited or redundant tracking observations (Daum and Fitzgerald 1983). These techniques can also be dependent on the nature of the bias.

Mixed models are an alternative for analyzing radar residuals that do not require physics-based modeling of the radar residuals. Mixed models, which are statistical models that incorporate both fixed and random effects, are used in many areas of the physical, biological, and social sciences (Brown and Prescott 2006; Bolker et al. 2009). They are particularly useful when measurements are made repeatedly on the same objects or in cases where the observations can be grouped into clusters. This is exactly the case when radar observations of dedicated calibration satellites are made to detect and remove biases in the observations. A drawback to the mixed models approach is that there is no a priori “right” statistical model. Different mixed models can potentially produce nearly equally good results. They do, however, have the advantage of being well suited to the examination of problems where a large amount of data exists and where there are several factors that might contribute to a data trend or variation in which the specific mechanism that produces these trends is unknown.

COBRA DANE RADAR

Cobra Dane is a single-face, L-band phased array radar located on Shemya Island, Alaska. Preliminary studies of Cobra Dane residual data in late 2004 and 2005 indicated a bias in the azimuth residuals whose magnitude and sign were a function of azimuth angle. This azimuth-dependent bias in the radar residuals manifests itself as an apparent slope across the Cobra Dane radar face. If such a slope or bias is not taken into account and compensated for, the accuracy of Cobra Dane’s radar tracks and, possibly, any missions that these tracks might support could be degraded. Figure 1 shows the bias across the Cobra Dane radar face. This figure uses Cobra Dane radar calibration data from 2005. The x-axis is the relative azimuth position of the calibration satellite relative to the radar boresight. The azimuth boresight is at a relative azimuth of 0 degrees (which is at 319 degrees from true north). The y-axis is the elevation of the calibration satellite on the radar face.

The radar face is broken up into 5-by-5-degree bins for our analysis. Within each 5-by-5-degree bin in the figure, the average azimuth residual has been computed, and the angle bins have been colored such that black represents either the extreme negative or positive values for the average azimuth residual. The various shades of red represent negative azimuth residuals, and the blues represent positive residuals. This color scheme highlights the transition between negative and positive average azimuth residuals, which is shown by the transition from red to blue. The white angle bins represent locations where no radar calibration satellite was observed. If the radar residual values were random and had zero mean, Figure 1 would have a red/blue speckled appearance. The systematic transition from red to blue (going from left to right) indicates that a slope in the radar residuals exists across the Cobra Dane radar face. The implication is a systematic effect in radar observations across the radar face, which is not sufficient to determine whether this bias is due to the environment or satellite sampling or whether it is inherent to the Cobra Dane radar itself.

RADAR BIAS

Although radar waves propagating through the atmosphere are subject to systematic and random errors in range and elevation angle due to tropospheric turbulence and ionospheric scintillations, it seems unlikely that these effects alone could be the cause of the apparent bias. Such errors are most acute at low elevations (<5 degrees), where

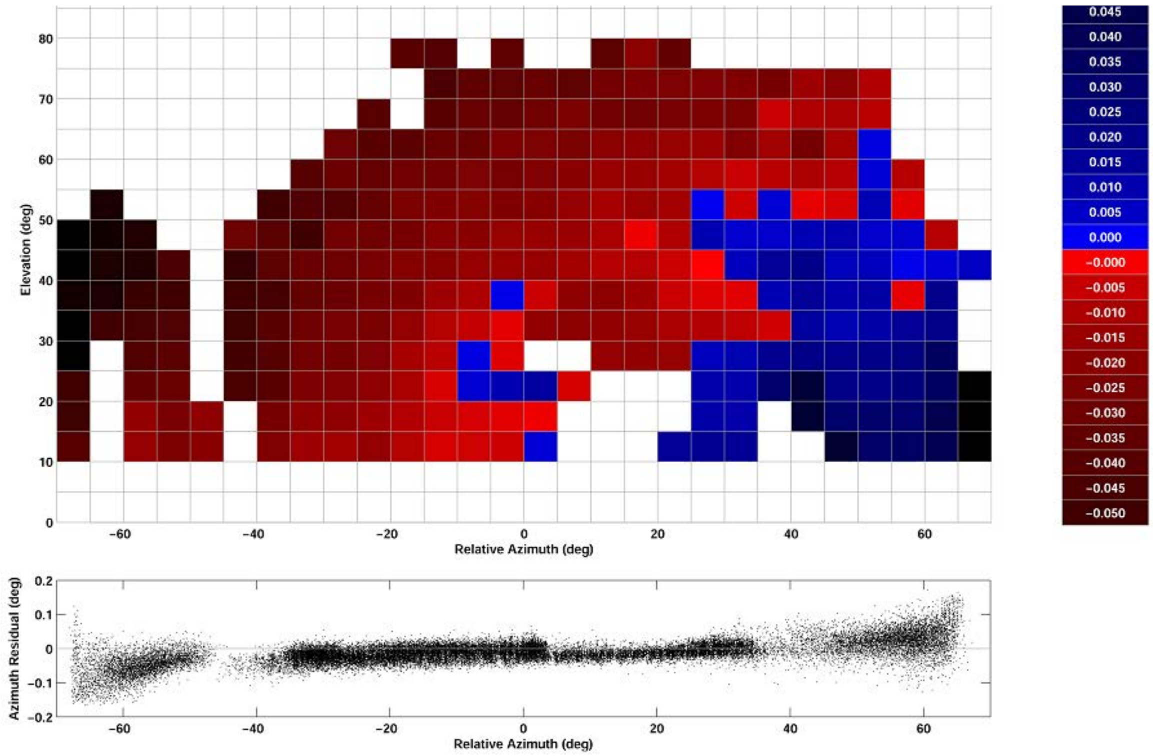


Figure 1. Apparent Bias in Cobra Dane Azimuth Residuals from 2005.

propagation through the lowest layers of the atmosphere is greatest. The apparent azimuthal bias in Figure 1 is, however, observed at elevation angles significantly higher than 5 degrees.

If atmospheric and off-boresight effects are unlikely to result in a systematic bias, perhaps the spatial distribution of the ensemble of calibration satellites could be the cause. Since the calibration satellite sample itself is a convenience sample,[1] the data do not uniformly cover the entire Cobra Dane radar face. Figure 1 shows that the sampling of calibration satellites is clearly not uniform across the radar face.

The calibration satellites that are sampled also change over time, thus making it possible to study the apparent bias under different combinations of calibration satellites. It is also possible that a particular combination of satellites, with their varied presentations to the radar, along with different radar cross-sections (both of which can affect the signal-to-noise ratio) coupled with the non-uniform, clustered satellite sampling, could result in the apparent bias seen in Figure 1.

[1] The ensemble is based on satellites for which independent high-precision ephemeris data are available; it is, therefore, a convenience sample. The ensemble of satellites was not selected based on any other criteria, such as spanning the Cobra Dane radar face or having the same radar cross-section (RCS).

COBRA DANE DATA OVERVIEW

Figure 2 provides a graphical look at the entire data set for the 6-year period plotted against all combinations of azimuth residual, elevation residual, relative azimuth, and elevation and grouped by the calibration satellite ID. Histograms of the azimuth residuals, elevation residuals, relative azimuths, and elevations are plotted along the diagonal.

Consider the plots in panels a) and f) of Figure 2. These panels show histograms of the azimuth and elevation residuals with a normal distribution overlay. These panels indicate that the combined residuals (regardless of satellite and time of the observation) in both azimuth and

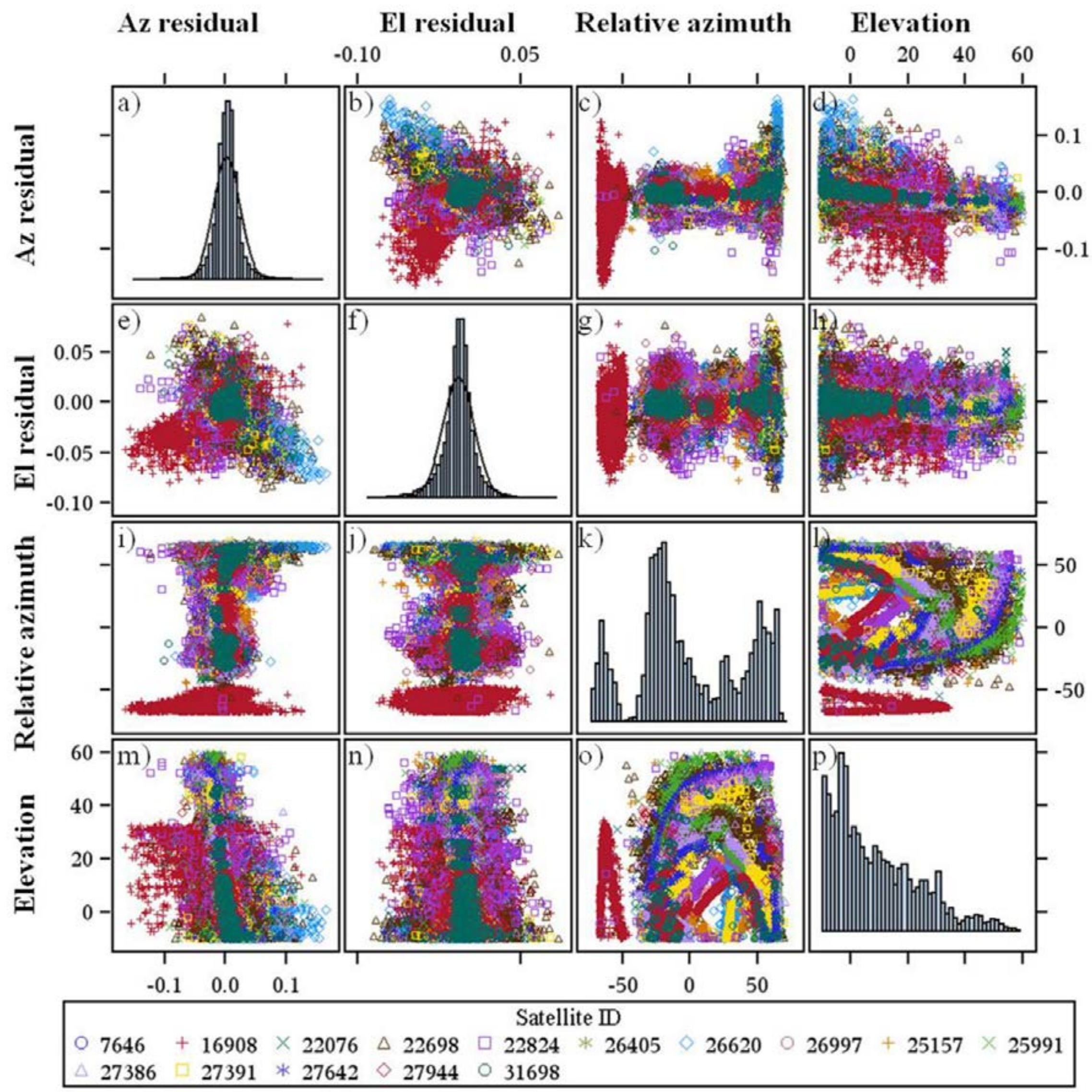


Figure 2. Summary of Cobra Data Calibration Satellite Observations Broken Down by Satellite ID.

elevation appear to be roughly normally distributed, which is beneficial to regression analysis.

Now consider panels k) and p) of Figure 2. These panels show histograms of the relative azimuths and elevations sampled in the dataset. The distributions of calibration satellite observations across the Cobra Dane radar face are clearly not uniform. The azimuth observations are clumped predominantly at the left and right extremes of the field of view (FOV) or are slightly to the left of the azimuth boresight. The elevation observations are mostly at the lower elevations angles and decrease in a nearly linear fashion as elevation angle increases.

Several of the panels (notably panels c), g), i), j), l), and especially o)) also show that the calibration satellite observations are not evenly distributed across the Cobra Dane radar face. Of particular note is satellite 16908 (red plus symbols), which is segregated from the other satellite observations at the extreme left (negative values of relative azimuth) of the azimuth FOV (see panel o)). No other satellites are observed at this extreme azimuth. This satellite also has some of the most negative azimuth residuals observed (see panel c)). Contrast this satellite with satellite 26620 (blue diamonds), which, as panels b), c), d), e), and i) show, has the largest positive values of azimuth residuals observed as well as being observed at the extreme right (positive values of relative azimuth) of the azimuth FOV.

ANALYSIS

Many of the physical parameters of the calibration satellites and the environmental parameters at the time the calibration satellite observations were made, which are needed to develop a physics-based model of the Cobra Dane radar residuals, are not available. For this reason, a statistical model is developed. The model selected to represent these data is a multi-level mixed-effects model, or equivalently a random coefficients model (Schabenberger and Pierce 2002).

The model starts with the following equation

$$azresid_{ij}=b_{0j}+b_{1j}\,realz_{ij}+b_{2j}\,el_{ij}+b_{3j}\,rngrate_{ij}+\epsilon_{ij}, \quad (1)$$

where $azresid_{ij}$ is the i[th] azimuth residual for observations of the j[th] satellite. The first two independent variables, $realz_{ij}$ and el_{ij}, are the relative azimuth and elevation positions where the azimuth residuals were observed (see Figure 1 for an illustration of the realz and el coordinates). Also note that $rngrate_{ij}$ is included in the equation. The $rngrate_{ij}$ variable represents the range rate (in kilometers/second) of the satellite relative to Cobra Dane. The inclusion of $rngrate_{ij}$ produces a slightly better model fit than the same model without $rngrate_{ij}$. The coefficients b_{0j}, b_{1j}, and b_{2j} are random variables, and b_{1j} and b_{2j} also depend on the calendar quarter in which the calibration satellite is observed. The b_{0j}, b_{1j}, b_{2j}, and b_{3j} are given by

$$b_{0j}=\beta_{00}+b_{0j}^*,$$

$$b_{1j}=\beta_{10}+\sum_{(n=1)}^{23}\beta_{1n}\;qtr_{nij}+b_{1j}^*,$$

$$b_{2j}=\beta_{20}+\sum_{(n=1)}^{23}\beta_{2n}\;qtr_{nij}+b_{2j}^*,\text{and}$$

$$b_{3j}=\beta_{30}, \quad (2)$$

where β_{00}, β_{10}, β_{20}, β_{30}, β_{1n} and β_{2n} (where n goes from 1 to 23 and represents the quarter in which the observation was made) are estimated fit parameters, and b_{0j}^*, b_{1j}^*, and b_{2j}^* are random variables that take on different values for each satellite. In this manner, the random effects due to each satellite are incorporated into the model.

The b_{0j} term can be interpreted as the random intercept for $azresid_{ij}$. It is random because of the b_{0j}^* term in Eq. (2). Thus, each satellite has its own idiosyncratic intercept value. The b_{1j} term is the slope for the relative azimuth term, and it, too, is a random variable because of the b_{1j}^* term. In addition, b_{1j} also varies for each quarter because of the $\beta_{1n}\, qtr_{nij}$ term. The way time is modeled here allows each quarter to have its own independent effect on the azimuth residuals. No autocorrelated effects are explicitly accounted for in this model. A similar interpretation applies to b_{2j}. Note that since b_{3j} does not have a random component, it is therefore not a random variable.

The SAS® statistics package was used to compute the parameters in the mixed model.

RESULTS

Solving for the parameters in the mixed model presented in Eqs. (1) and (2) showed that each calibration satellite has its own idiosyncratic slope and intercept, some of which are dramatically different from the others. By properly accounting for these random effects, a larger portion of the variance in the azimuth residual data is explained, thus increasing the statistical power to determine whether a time-varying slope, or bias, exists. After taking into account these random satellite effects, the slope of the azimuth residuals as a function of azimuth position is found to vary depending on time. This time-varying azimuth slope decreases over time. In 2010, the magnitude of the slope was an order of magnitude smaller than it was in 2005. The cause of this decrease has not been determined but could be due to effects either external to the radar (environmental effects) or internal to the radar (such as system upgrades or software updates). If the cause is external to the radar, the model might benefit by including environmental parameters as either fixed or random effects.

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REFERENCES

Bolker, Benjamin M., Mollie E. Brooks, Connie J. Clark, Shane W. Geange, John R. Poulsen, M. Henry H. Stevens, and Jada-Simone S. White. 2009. "Generalized Linear Mixed Models: A Practical Guide for Ecology and Evolution." *Trends in Ecology & Evolution* 24 (3) (March): 127–135.

Brown, Helen, and Robin Prescott. 2006. *Applied Mixed Models in Medicine.* West Sussex, England: John Wiley & Sons.

Daum, Frederick E., and Robert J. Fitzgerald. 1983. "Decoupled Kalman Filters for Phased Array Radar Tracking." *IEEE Transactions on Automatic Control* 28 (3) (March): 269–283.

Friedland, Bernard. 1969. "Treatment of Bias in Recursive Filtering." *IEEE Transactions on Automatic Control* 14 (4) (August): 359–367.

Lin, Xiangdong, Y. Bar-Shalom, and T. Kirubarajan. 2005. "Multisensor-Multitarget Bias Estimation for General Asynchronous Sensors." *IEEE Transactions on Aerospace and Electronic Systems* 41 (3) (July): 899–921.

Novoselov, Roman Y., Shawn M. Herman, Sabino M. Gadaleta, and Aubrey B. Poore. 2005. "Mitigating the Effects of Residual Biases with Schmidt-Kalman Filtering." In *Proceedings of the 2005 8th International Conference on Information Fusion, Vol. 1*, 358–365, IEEE.

Schabenberger, Oliver, and Francis J. Pierce. 2002. *Contemporary Statistical Models for the Plant and Soil Sciences.* Boca Raton, FL: CRC Press LLC.